

# Numerical Calculation of Interface Bond Fracture

Henrik Myhre Jensen

Department of Civil Engineering  
Aalborg University, Aalborg, Denmark  
e-mail: hmj@civil.aau.dk

**Summary** A description of two methods for numerical prediction of crack propagation through an adhesive layer is presented. The first method is based on a fracture mechanical approach where the edge of the bond region is treated as an interface crack front. Along the front, the energy release rate and the mode I, II and III stress intensity factors are calculated. A method for predicting quasi-static crack growth is presented by introducing a crack growth criterion. The shape of the crack front and the critical applied load to propagate the crack is obtained. The second method is based on a cohesive zone description of the adhesive layer. Comparisons of results based on the two approaches are shown.

## Introduction

Examples of plate or shell structures, which are adhesively bonded, include composite structures applied in the aeroplane, automotive and the wind turbine blade industry. Traditional methods for calculating the failure strength of adhesive bonds include the model of Volkersen [1] and the model of Goland and Reissner [2]. Both models are stress based and they are used as simple design tools for dimensioning single lap joints. The theory has later been expanded to other geometries as described by Adams [3].

Fracture mechanical models for predicting bond failure have been developed more recently. Fracture mechanical solutions for initiation of failure in spot welds have been formulated in Radaj [4] and Zhang [5] and in Jensen [6], [7] for initiation and propagation of fracture in adhesive joints. A thin adhesive layer can be analyzed as an external interface crack front. Assuming linear elastic fracture mechanics, the energy release rate  $G$  at the crack front is given by the effective crack tip loads. The fracture mechanical model uses a mixed mode interface fracture criterion coupled with a propagation formulation, embedded in an outer finite element model.

In the cohesive zone model, the adhesive bond region is represented by non-linear springs used to model the fracture process. The cohesive zone is embedded in a finite element model of the adherends. Cohesive zone models have been applied to model fracture in elastic-plastic solids in *e.g.* Tvergaard and Hutchinson [8]. Plastically deforming adhesive joints in Modes 1 and 2 loading conditions have been modeled using a cohesive zone representation of the bond region in Wei and Hutchinson [9] and Yang and Thouless [10]. In Feraren and Jensen [11] the cohesive zone model predictions were compared to fracture mechanical predictions of the crack front shape during the process of interface bond failure.

The basic joint geometry considered consists of two partly overlapping shells, bonded along a thin adhesive layer. In this case the thickness of the adherends are required to be significantly higher than that of the adhesive layer, and it is assumed that the fracture process is limited to the bond region. The significance of fracture process zone parameters is investigated.

## Fracture mechanics

The edge of the bond zone is regarded as an interface crack front, which is subject to combined mode I, II and III loading. The energy release rate,  $G$ , and the mode I, II and III contributions to  $G$

can be calculated by the coupling of an inner, fracture mechanics based solution close to the crack tip with an outer solution for the stress state in the adherends.

The relation between the energy release rate and the stress intensity factors  $K_I$ ,  $K_{II}$  and  $K_{III}$  is given by

$$G = \frac{1}{\cosh^2(\pi\varepsilon)} \frac{1}{2} \left( \frac{1}{\bar{E}} + \frac{1}{\bar{E}_s} \right) (K_I^2 + K_{II}^2) + \frac{1}{2} \left( \frac{1+\nu}{E} + \frac{1+\nu_s}{E_s} \right) K_{III}^2 \quad (1)$$

The subscript ( )<sub>s</sub> refers to the lower plate, which may have elastic properties  $E_s$  and  $\nu_s$  different from those of the top plate. In (1)  $\varepsilon$  denotes the bimaterial index.

A family of interface fracture criteria formulated in Jensen *et al.* [12] is applied in the form

$$G_I + \lambda_2 G_{II} + \lambda_3 G_{III} = G_{Ic} \quad (2)$$

where  $\lambda_2$  and  $\lambda_3$  denote parameters between 0 and 1 adjusting the relative contributions of mode II and III to the fracture criterion, and  $G_{Ic}$  is the mode I fracture toughness of the bond.

The criterion (2) has been applied to thin film debonding problems in *e.g.* Jensen *et al.* [12] and Jensen and Thouless [13]. The fracture criterion captures the mixed mode dependence of interface fracture toughness due to plastic deformation at the crack tip [8] or rough crack faces contacting under mode II and III dominant loading conditions (Evans and Hutchinson [14] and Jensen [15]).

The application of (2) requires a separation of the energy release rate into mode I, II and III components, This follows from the definitions of the phase angles of loading  $\psi$  and  $\phi$  introduced in Jensen *et al.* [12]

$$\tan \psi = \frac{\text{Im}((K_I + iK_{II})h^{i\varepsilon})}{\text{Re}((K_I + iK_{II})h^{i\varepsilon})}, \quad \cos \phi = \sqrt{\frac{G_{III}}{G}} \quad (3)$$

where  $i$  is the imaginary unit ( $i = \sqrt{-1}$ ),  $\varepsilon$  is the bimaterial constant and  $h$  is the thickness of the top plate. The results below are presented for the case of a large difference in bottom and top plate thickness but this is not a restriction on the method.

The fracture mechanical approach to interface crack propagation works by increasing the load incrementally until (2) is exceeded. A crack growth criterion point wise along the crack front during further incremental loading is assumed of the type

$$C_{i+1} = C_i + \tilde{\varepsilon} \mathbf{n} \left( G \left( 1 + (1 - \lambda_2) \sin^2 \psi \sin^2 \phi + (1 - \lambda_3) \cos^2 \phi \right) - G_{Ic} \right)^p \quad (4)$$

Here,  $C_i$  denotes the crack front curve at increment number  $i$ , which has the unit normal vector  $\mathbf{n}$  and  $\tilde{\varepsilon}$  and  $p$  are parameters chosen so that during incremental loading the fracture criterion is satisfied along the propagating part of the crack front, and the fracture criterion is not exceeded elsewhere.

## Results and discussion

Results are presented below for the case of planar plates subject to in-plane loads. The following parameters are introduced

$$k = \frac{2\lambda_3}{(1-\nu)(1+(\lambda_2-1)\sin^2 \omega)}$$

$$\sigma_c = \left( \frac{2EG_{1c}}{(1-\nu^2)h(1+(\lambda_2-1)\sin^2 \omega)} \right)^{1/2} \quad (5)$$

The angle  $\omega$  is a weak function of the elastic mismatch in the system. It has been tabulated in Suo and Hutchinson [16] and for most systems  $40^\circ < \omega < 60^\circ$ .

The stress,  $\sigma_c$ , has the interpretation as being the critical stress required to propagate a plane strain edge crack under steady-state conditions.

In Fig. 1 shapes of initially circular bond regions during fracture are shown. Results are presented for three values of parameters in the fracture criterion (2) characterised by  $k$  in (5).

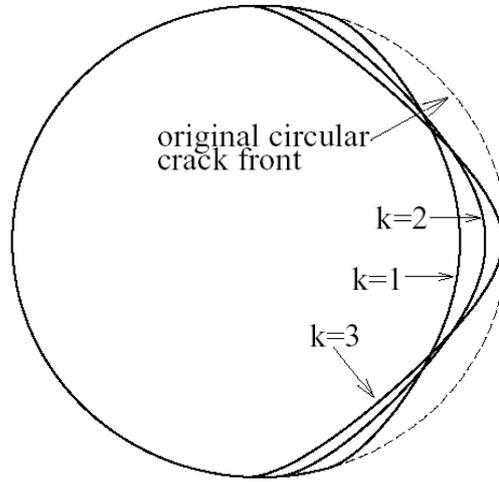


Fig. 1. Shapes of circular bond during failure for three different fracture criteria.

Initial stable crack propagation in the bond region is predicted indicating a significant residual strength of the bond after initial failure. The failure strength of the bond is denoted by  $\sigma_0$  and is obtained as part of the numerical predictions. The initial stable crack propagation following initiation is illustrated in Fig. 2 where the stress required to propagate the crack is shown as a function of the relative area change of the bond region, which is introduced as a measure of the amount of crack growth.

The bond strength,  $\sigma_0$ , is written as

$$\sigma_0 = \frac{1}{\sqrt{hF_p}} \sqrt{\frac{2EG_{1c}}{(1-\nu^2)(1+(\lambda_2-1)\sin^2 \omega)}} \quad (6)$$

where  $F_p$  denotes the peak value of the left hand side of (2) along the crack front for a given applied external load.

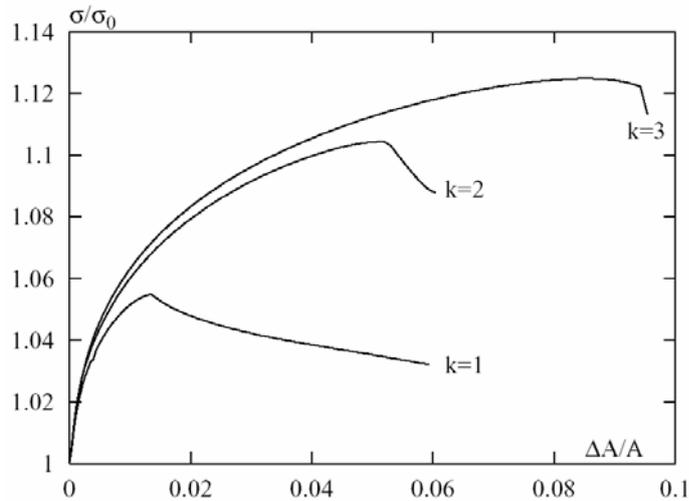


Fig. 2. Stress for crack propagation as a function of relative area change of bond region.

As seen in Fig. 2 the residual strength of the bond is sensitive to the interface fracture criterion. The classical mode independent Griffith fracture criterion corresponds to  $k = 3$ .

As described, the cohesive zone model assumes the bond region to be described by non-linear springs. A tri-linear relationship between crack surface tractions and crack opening displacements is assumed (Feraren and Jensen [11]). A measured traction separation law is shown in Fig. 3 for a glass fibre epoxy beam.

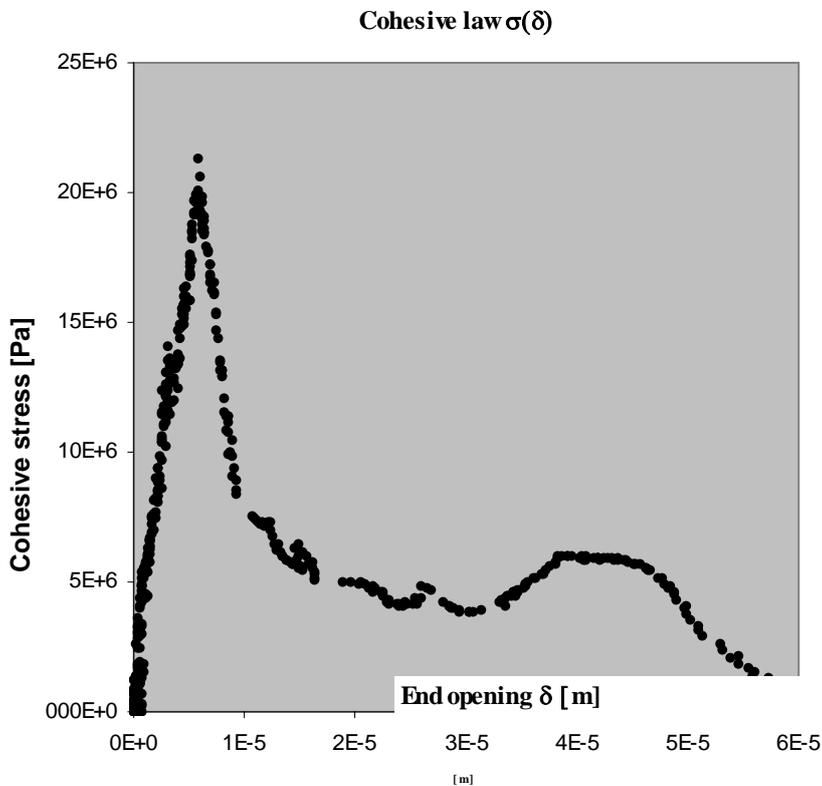


Fig. 3. Measured traction vs. separation law.

A comparison between calculated shapes of the crack front based of the fracture mechanics approach and the cohesive zone model is shown in Fig. 4. A good agreement is observed.

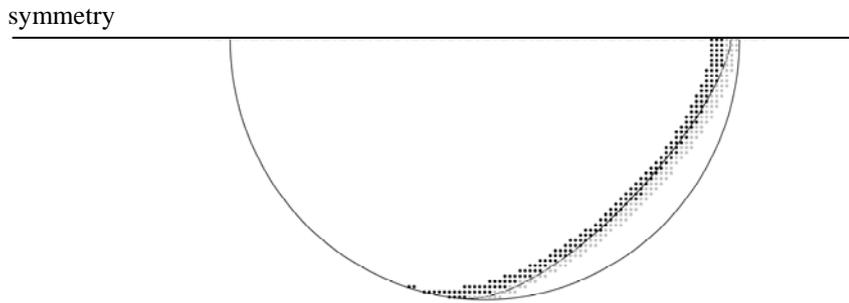


Fig. 4. Crack front predicted by fracture mechanics and cohesive zone model.

The advantage of the cohesive zone model over the fracture mechanical model is that large curvature of the crack front is allowed for. Also plastic deformation in large scale in the adherends can be taken into account. A realistic situation in adhesive bond problems is the occurrence of trapped air-bubbles or flaws, which reduces the strength of the bond. In Fig. 5 the shape of a crack propagating at an interface and interacting with a circular flaw is shown.

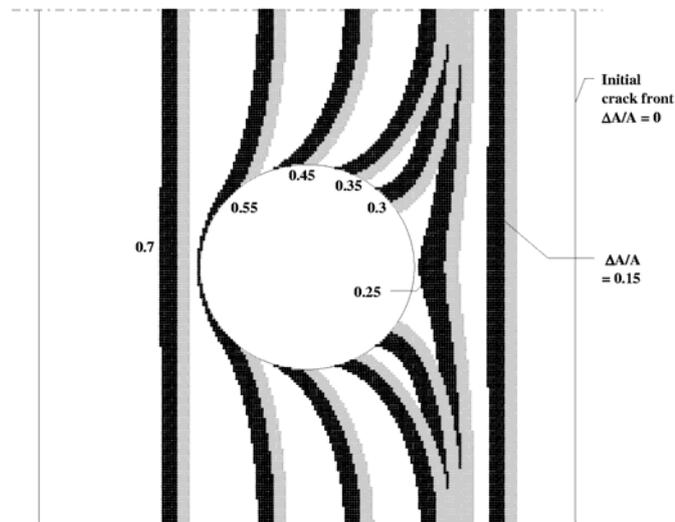


Fig. 5. Predicted crack front shape around a circular interface flaw.

The stress vs. relative area change prediction corresponding to Fig. 5 are shown in Fig. 6 by which the reduction in bond strength can be predicted. The three curves in Fig. 6 denoted A, B and C correspond to three cohesive laws with the same toughness but different strength.

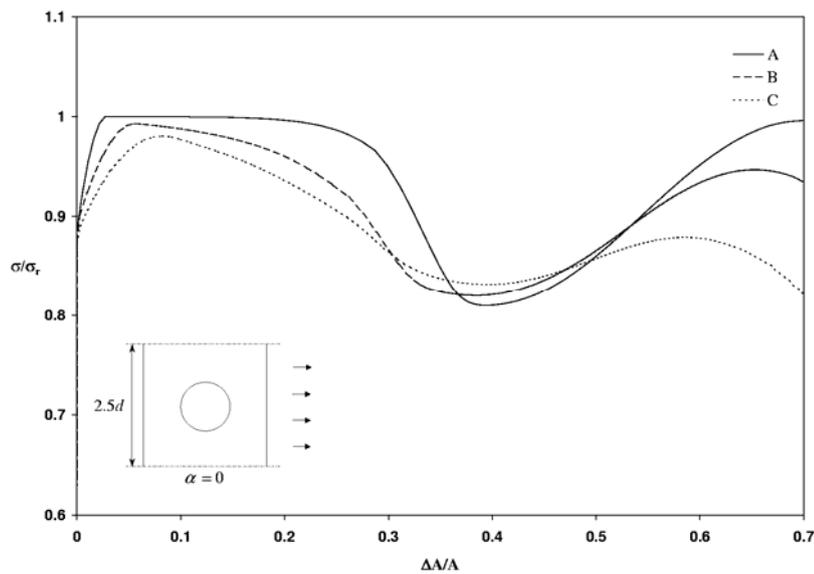


Fig. 6. Stress vs. relative area change for three cohesive laws.

**Acknowledgements** Support from the Danish Technical Research Council through the Framework Programme "Interface Design of Composite Materials" (STVF fund no. 26-03-0160) is gratefully acknowledged.

## References

- [1] O. Volkersen. Die nietkraftverteilung in zugbeanspruchten nietverbindungen mit konstanten laschenquerschnitten. *Luftfahrtforschung* **15**, 41–47 (1938).
- [2] M. Goland and E. Reissner. The stresses in cemented joints. *Journal of Applied Mechanics* **66**, A17–27 (1944).
- [3] R.D. Adams. Theoretical stress analysis of adhesively bonded joints. *Joining Fibre Reinforced Plastics*, 185–226 (1987).
- [4] D. Radaj. Local fatigue strength characteristic values for spot welded joints. *Engineering Fracture Mechanics* **37**, 245–250 (1990).
- [5] S. Zang. Fracture mechanics solutions to spot welds. *International Journal of Fracture* **112**, 247–274 (2001).
- [6] H.M. Jensen. Three-dimensional numerical investigation of brittle bond fracture. *International Journal of Fracture* **114**, 153–165 (2001).
- [7] H.M. Jensen. Crack initiation and growth in brittle bonds. *Engineering Fracture Mechanics* **70**, 1611–1621 (2003).

- [8] V. Tvergaard and J.W. Hutchinson. The relation between crack growth resistance and fracture process parameters in elastic–plastic solids. *Journal of the Mechanics and Physics of Solids* **40**, 1377–1397 (1992).
- [9] Y. Wei and J.W. Hutchinson. Interface strength, work of adhesion and plasticity in the peel test. *International Journal of Fracture* **93**, 315–333 (1989).
- [10] Q.D. Yang and M.D. Thouless. Mixed-mode fracture analyses of plastically-deforming adhesive joints. *International Journal of Fracture* **110**, 175–187 (2001).
- [11] P. Feraren and H.M. Jensen, Cohesive Zone Modelling of Interface Fracture near Flaws in Adhesive Bonds. *Engineering Fracture Mechanics* **71**, 2125-2142, (2004).
- [12] H.M. Jensen, J.W. Hutchinson and K.-S. Kim. Decohesion of a cut prestressed film on a substrate. *International Journal of Solids and Structures* **26**, 1099–1114 (1990).
- [13] H.M. Jensen and M.D. Thouless. Effects of residual stresses in the blister test. *International Journal of Solids and Structures* **30**, 779-795 (1993).
- [14] A.G. Evans and J.W. Hutchinson. Effects of non-planarity on the mixed mode fracture resistance of bimaterial interfaces. *Acta Materialia* **37**, 909-916 (1989).
- [15] H.M. Jensen. Mixed mode interface fracture criteria. *Acta Materialia* **38**, 2637-2644 (1990).
- [16] Z. Suo and J.W. Hutchinson. Interface crack between two elastic layers. *International Journal of Fracture* **43**, 1-18 (1990).