Bounds on information combining for the accumulator of repeat-accumulate codes without Gaussian assumption

Land, Ingmar; Sayir, Jossy; Peter A., Hoeher

Published in:
Proceedings on the international symposium of information theory, 2004

DOI (link to publication from Publisher):
10.1109/ISIT.2004.1365480

Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Bounds on Information Combining for the Accumulator of Repeat-Accumulate Codes without Gaussian Assumption

Ingmar Land, Peter A. Hoeher
Information and Coding Theory Lab
University of Kiel, Germany
{il,ph}@tf.uni-kiel.de
www.tf.uni-kiel.de/ict

Jossy Sayir
Telecom. Research Centre (ftw.)
Vienna, Austria
jossy@ftw.at
www.ftw.at

Abstract — **Information combining is applied to an accumulator to obtain bounds on the extrinsic information transfer (EXIT) functions without relying on Gaussian or other approximations for the distribution of the a-priori messages.**

I. Introduction

In [1], information combining was introduced to provide bounds on the mutual information of coupled channels expressed solely as a function of the mutual informations of the single channels. This method was extended in [2, 3] to bound the extrinsic information of repetition codes and single parity-check codes. Using this approach, bounds on extrinsic information transfer (EXIT) functions of low-density parity-check (LDPC) codes were computed without assuming a specific a-priori channel, i.e., without Gaussian or equivalent approximations.

In this paper, information combining is applied to an accumulator, i.e., a rate-1 recursive convolutional encoder defined by the generator function $g(D) = 1/(1+D)$. Bounding the EXIT function for an accumulator is interesting in itself because the accumulator is a component of repeat-accumulate (RA) codes. Furthermore, our analysis provides an indication of how information combining may be applied to general convolutional codes, opening the way to bound the EXIT functions of turbo codes.

II. Information Combining

Figure 1 shows the factor graph of an accumulator whose output $X_1, X_2, \ldots$ is transmitted over a noisy communication channel to yield observations $Y_1, Y_2, \ldots$. The observations of the encoded digits $U_1, U_2, \ldots$ obtained from previous iterations are modeled as outputs of $V_1, V_2, \ldots$ of an "a-priori" channel. No specific assumption on the a-priori channel is made.

We define $f_2(x_1, x_2) := 1 - h\left((1-\epsilon_1)\epsilon_2 + \epsilon_1(1-\epsilon_2)\right)$ with $\epsilon_1 = h^{-1}(1-x_1)$, $\epsilon_2 = h^{-1}(1-x_2)$, $h(.)$ is the binary entropy function and $h^{-1}(.)$ is its inverse over $[0, 1/2]$. The following theorem bounds the extrinsic information for the accumulator using information combining:

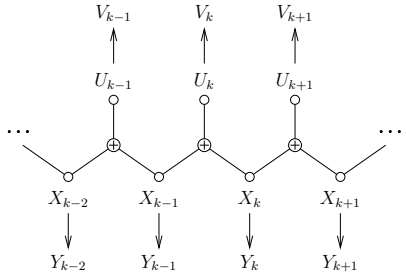

Figure 1: Part of the factor graph of the accumulator.

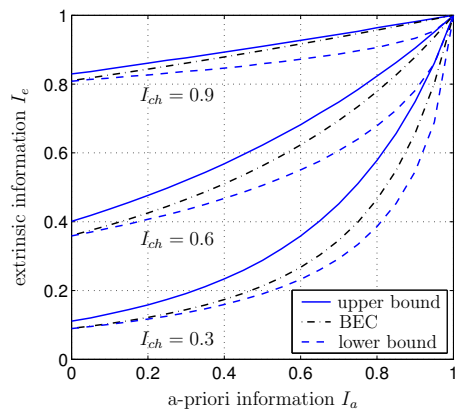

Figure 2: Bounds on the EXIT functions of the accumulator and exact EXIT functions for the BEC-case.

Theorem 1 *The extrinsic information* $I_{e,k} = I(U_k; \mathbf{Y}\mathbf{V}_{\setminus k})$ *is bounded as*

$$I_{\min} \cdot I_{\min} \;\leq\; I_e \;\leq\; f_2(I_{\max}, I_{\max}),$$

where $I_{\min}$ *is the minimum value and* $I_{\max}$ *is the maximum value* $I \in [0,1]$ *fulfilling simultaneously*

$$I \;\geq\; I_{ch} + I_a \cdot I - f_2(I_{ch}, I_a \cdot I) \text{ and}$$
$$I \;\leq\; I_{ch} + f_2(I_a, I) - I_{ch} \cdot f_2(I_a, I),$$

with $I_{ch} = I(X_k; Y_k)$ *and* $I_a = I(U_k; V_k)$.

The theorem is proved by applying Theorem 1 from [1] and Theorem 1 from [3] recursively and assuming that stationarity conditions on mutual informations hold for sufficiently long encoded sequences.

The resulting bounds on the EXIT functions are depicted in Figure 2, along with the exact EXIT curves for binary erasure channels (BECs). The bounds are close to each other only for small and for large values of the a-priori information. This is due to the fact that the upper and lower bounds in Theorem 1 from [1] and Theorem 1 from [3] are tight for BECs and for binary symmetric channels (BSCs) respectively, but the lower and the upper bounds are applied recursively in the proof of Theorem 1 in the present paper.

References

[1] I. Land, S. Huettinger, P. A. Hoeher, and J. Huber, "Bounds on Information Combining," in *Proc. Int. Symp. on Turbo Codes and Rel. Topics,* Brest, France, Sept. 2003, pp. 39-42.

[2] I. Sutskover, S. Shamai (Shitz), and J. Ziv, "Extremes of information combining," in *Proc. Allerton Conf. on Communications, Control and Computing,* Monticello, Illinois, USA, Oct. 2003.

[3] I. Land, P. A. Hoeher, and J. Huber, "Bounds on information combining for parity-check equations," in *Proc. Int. Zurich Seminar on Communications (IZS),* Zurich, Switzerland, Feb. 2004.