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Published in:
The IET Irish Signals and Systems Conference 2007

Publication date:
2007

Document Version
Publisher's PDF, also known as Version of record

On the Generation of a Robust Residual for Closed-loop Control systems that Exhibit Sensor Faults

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Abstract — **This paper presents a novel design methodology, based on shaping the system frequency response, for the generation of an appropriate residual signal that is sensitive to sensor faults in the presence of model uncertainty and exogenous unknown (unmeasured) disturbances. An integrated feedback controller design and robust frequency-based fault detection approach is proposed for Single-Input/Single-Output systems. The efficiency of the proposed method is demonstrated on a Single Machine Infinite Bus (SMIB) power system that achieves a coordinate power system stabilizer with satisfactory sensor fault detection capabilities.**

Keywords — **Model-based fault detection, Robust residual signal, Power system stabilizer (PSS).**

I Introduction

The goal of reliability and fault tolerance in control system design requires that fault detection modules perform well under a variety of internal/external conditions. Model-based Fault Detection and Isolation (FDI) has been the subject of significant attention in recent years [4, 6] and references therein. The main objective of a model-based FDI paradigm is to generate a so-called *residual signal* that is sensitive to an exogenous fault vector. In this context, the question of joint disturbance decoupling and robustness of the attendant residual signal in the presence of significant plant uncertainty is the specific question that is considered in this paper. A great deal of the published research on this issue concentrates on observer-based and parameter estimation methods [4, 6, 9, 12]. However, the authors feel that such methods provide solutions that do not yield an easy interpretation that can manage the trade-off between disturbance decoupling and fault detection in a closed-loop configuration. Moreover, the necessary on-line algorithms that are required for parameter estimation are time-consuming, and can lead to a significant increase in the complexity of the design.

The focus of this work therefore, is a "frequency domain" approach wherein the system frequency response is used to provide an insight into the necessary design trade off between disturbance decoupling and fault detection. To address this issue, several robust FDI techniques, motivated by the H_∞/H_2, H_∞/μ and H_∞/LMI paradigms, have been presented in [6, 10, 13, 16, 17]. However, the inherent conservatism within such a frequency domain H_∞-based approach can lead to high order designs, quite possibly without any guarantees about *a priori* levels of robust performance.

Having determined an appropriate residual signal, the next step in an FDI technique is the so-called *residual evaluation* in order to make detection and isolation decisions, [4, 6]. Due to the inevitable existence of noise and model errors, these residuals are never zero, even if there is no fault and the disturbance is decoupled perfectly. Therefore a detection decision requires that residuals be compared with a so-called *threshold* signal, obtained empirically (generally) or theoretically. Again a significant literature exists relating to the determination of such an appropriate threshold [4, 6, 8, 17]. Most of these aforementioned references are presented for open-loop systems however, and as industrial systems, usually of necessity, work under feedback control, the FDI algorithm should offer the capacity of being applied in such a scenario. In [13, 16], a H_∞-based methodology for such an integrated closed-loop FDI system has been presented.

In this paper a novel two-degree-of-freedom Sensor Fault Detection and Isolation (SFDI) technique is presented for Single-Input/Single-Output (SISO) closed-loop systems. The disturbance decoupling and the robust residual generation are sequentially addressed via the following two stage procedure: In step (1) the effects of exogenous disturbances and model uncertainty are minimized by feedback controller $G(s)$. In step (2) a SFDI filter that tracks the pre-specified residual reference model is synthesised. A well-known residual evaluation function is then utilized to determine the occurrence of false alarms [8, 17].

This paper is organized as follows. In Section II the specific fault detection scheme to be considered and the objectives of the paper are outlined. The design of the feedback compensator is discussed in section III. The Detection filter design stage is presented in section IV. Section V considers the numerical evaluation of the residual and finally the efficiency of the proposed method is demonstrated on a Single Machine Infinite Bus (SMIB) power system in section VI.

II System description

Fig. 1 illustrates a block diagram of the design methodology considered in this paper. $P(s)$ represents a SISO linear(ised) plant transfer function within the uncertainty region $\{\mathcal{P}\}$. $d_1(t)$ and $d_2(t)$ denote unknown exogenous input and output disturbances, and $f(t)$ represents the sensor fault affecting the system. $y_m(t)$ represents the sensor output to be compared with reference signal, $c(t)$.

The objective is to design an appropriate residual signal, $r(t)$, which is sensitive to $f(t)$ and is robust against the aforementioned uncertain factors, [10, 17].

Throughout the paper it is assumed that:

Assumptions:

1. $P(s)$ belongs to RH_∞, real rational functions with $||P||_\infty = sup_\omega(P(j\omega)) < \infty$.
2. The exogenous disturbances and model uncertainty are bounded.
3. The reference input is known completely and the class of failures is given.
4. The reference signal excites the system at the start of the detection window and that a failure occurs within the detection window.
5. The fault is detectable.

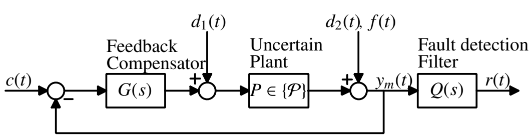

Fig. 1: Two-degree-of-freedom SFD structure.

III Design of feedback compensator $G(s)$

Initially, $G(s)$ is primarily designed to achieve a satisfactory level of robust stability *and* robust performance in the presence of model uncertainty and disturbance when the system is fault free. A variety of different robust control methods, for instance H_∞ [7], quantitative feedback theory (QFT) [11] can be used in this stage. It must be noted that the design of $G(s)$ in Fig. 1 is not the focus of this paper. The interested reader might consider [1] where an alternative method of feedback compensator design is introduced based on a QFT loop-shaping technique that neatly complements the approach presented here.

IV Design of filter $Q(s)$

Having designed an appropriate $G(s)$, step 2 of the procedure is the synthesis of a SFDI filter $Q(s)$ to generate a robust residual signal $r(t)$. The basis for the work relies on the assumption that it is feasible to construct a reference (i.e. desired) model for the residual in the presence of sensor fault as proposed in [17]. Such residual reference models are denoted here by $G^d_{rf}(s)$. The *objective* is then to design $Q(s)$ such that the transfer function from $f(t)$ to the actual residual, $r(t)$, becomes matched to the predefined residual reference model $G^d_{rf}(s)$. Formally, the design objective can therefore be stated as follows:

Design $Q(s)$ such that the following inequality holds:

$$\left|G^d_{rf}(j\omega) - \frac{Q(j\omega)}{1 + P(j\omega)G(j\omega)}\right| \leq |E_d(j\omega)|, \tag{1}$$

$$for\ P \in \{\mathcal{P}\}\ and\ \omega \in \Omega,$$

$E_d(s)$ describes the desired dynamic behaviour of the error between the residual reference model and corresponding actual models denoted by $G_{rf}(s)$. Ω represents the frequency region where the energy of the fault is likely to be concentrated.

a) *Residual reference model*

The method proposed in [17] is adopted to obtain the residual reference models $G^d_{rf}(s)$.

Theorem 1: Consider a nominal system given by (2):

$$\begin{aligned} \dot{x} &= Ax + Bu + B_d d_1, \\ y &= Cx + Du + D_f f + D_d d_2. \end{aligned} \tag{2}$$

where x is the state vector, u is the control signal, f is the sensor fault signal and d_1 and d_2 are the unknown (input/output) disturbances. A, B, C, D are the system matrices for the open loop system and the remaining matrices (B_d, D_f, D_d) are fault/disturbance distribution matrices. Suppose Assumptions 1, 3 and 5 hold true. Then the corresponding reference residual models can be obtained by using the following state-space model:

$$\begin{aligned} \dot{x}_f &= (A - H^*C)x_f - H^*D_f f + B_d d_1 - H^* D_d d_2, \\ r_f &= V^*Cx_f + V^*D_f f + V^*D_d d_2, \end{aligned} \tag{3}$$

where

$$H^* = (B_d D_d^T + YC^T)X^{-1}, \tag{4}$$

$$V^* = X^{-1/2}, \tag{5}$$

and $X = D_d D_d^T$ and $Y \geq 0$ is a solution of the algebraic Riccati equation:

$$\bar{A}^T Y + Y\bar{A} - Y\bar{B}X^{-1}\bar{B}^T Y + \bar{Q} = 0, \tag{6}$$

where,

$$\begin{aligned} \bar{A} &= (A - B_d D_d^T X^{-1} C)^T, \\ \bar{B} &= C^T, \\ \bar{Q} &= B_d(I - D_d^T X^{-1} D_d)^2 B_d^T. \end{aligned}$$

Proof: See [17]. ■

b) design of the filter $Q(s)$ to detect the sensor fault

To address this issue, a log-polar coordinate system is used to transform (1) into a set of quadratic inequalities with known coefficients over the uncertainty region, [2, 5].

Theorem 2: Consider the closed-loop system as shown in Fig. 1. Assume that $G(s)$ has been designed *a priori* to reduce the effects of disturbance and plant uncertainty. Moreover, the sensor residual reference model $G^d_{rf}(s)$ is obtained via Theorem 1. Then, in order to achieve a predefined level of SFDI given by (1) over the frequency range of Ω, it is sufficient to find a $Q(s)$ which satisfies the following quadratic inequality for a finite set of frequencies over Ω:

$$Aq^2 + Bq + C \geq 0 \tag{7}$$

where

$$A = -x^2,$$
$$B = 2xm\cos(\phi_x + \phi_q - \phi_m),$$
$$C = -m^2 + e_d^2.$$

m, x, e_d, q, ϕ_m, ϕ_x, ϕ_{e_d} and ϕ_q are defined as follows:

$$G^d_{rf}(j\omega_i) = m\, e^{j\phi_m},$$
$$1/(1 + P(j\omega_i)G(j\omega_i)) = x\, e^{j\phi_x},$$
$$E_d(j\omega_i) = e_d\, e^{j\phi_{e_d}},$$
$$Q(\omega_i) = q\, e^{j\phi_q} \tag{8}$$
$$\omega_i \in \Omega. \tag{9}$$

Proof: Assume that a finite set of frequencies $\widetilde{\omega} = \{\omega_1, \omega_2, \cdots, \omega_n\}$ is selected over the frequency range Ω. By substituting (8) into (1), it can be shown that (1) is transformed into the following inequality at each design frequency $\omega_i \in \widetilde{\omega}$:

$$\Big(m\cos(\phi_m) - xq\cos(\phi_x + \phi_q)\Big)^2 +$$
$$\Big(m\sin(\phi_m) - xq\sin(\phi_x + \phi_q)\Big)^2 \leq e_d^2, \tag{10}$$

where $G^d_{rf}(s)$, $1/(1 + P(s)G(s))$ and $E_d(s)$ are known and $Q(s)$ is the unknown parameter to be tuned. By the extension, (10) can then be rewritten as (7). ■

Equation (7) should be computed and solved for a family of selected plants over the uncertainty region and for all $\omega_i \in \widetilde{\omega}$. The solution of (7) for q for a given plant case and design frequency, and over $\phi_q \in [-360, 0]$ will divide the complex plane of Q into acceptable and unacceptable regions that greatly reduces the computational burden of filter design and conservatism, [2, 5]. The intersection of these regions provides an exact bound for the design of filter. $Q(s)$ should then be designed to lie within these bounds at each frequency ω_i.

Remark 1:

a) Such a solution implicitly captures phase information and can hence be applied to both minimum and non-minimum phase plants as well as time delay systems, [2, 5].

b) By defining $S(s) = 1/(1 + P(s)G(s))$ as a sensitivity function of the closed-loop system, it is clear that: "the smaller the sensitivity transfer function, the better the robustness that is provided against exogenous disturbances." However, (1) indicates that an extreme reduction of the sensitivity function results in an extra cost on $Q(s)$ when trying to achieve the desired error $E_d(s)$. Therefore, there is always a trade off between the disturbance decoupling and SFDI.

c) One candidate $Q(s)$ for the SFDI can be selected (trivially) as:

$$Q(s) = G^d_{rf}(s)(1 + P_0(s)G(s)) \tag{11}$$

for a sensor fault. However, this $Q(s)$ will generally fail to provide an acceptable performance over the desired uncertainty region.

V Residual evaluation

Suppose that $G(s)$ and $Q(s)$ have been designed to meet or exceed a set of design constraints. To generate an appropriate fault alarm, the following evaluation function is subsequently introduced on the residual signal:

$$\|r\|_2 = \left[\int_{t_1}^{t_2} r^2(t)dt\right]^{1/2} \tag{12}$$

where

$$r(t) = r_c(t) + r_{d_1}(t) + r_{d_2}(t) + r_f(t), \tag{13}$$

and $r_c(t)$, $r_{d_i}(t)$ for $i = 1, 2$, and $r_f(t)$ are respectively defined as follows:

$$r_c(t) = r(t)|_{d_i=0,\ f=0}$$
$$r_{d_i}(t) = r(t)|_{c=0,\ f=0}$$
$$r_f(t) = r(t)|_{c=0,\ d_i=0}.$$

Note that a satisfactory level of tracking performance has been achieved *a priori* by the design of $G(s)$. By assuming that $r_c(t) - c(t) \approx 0$, the bias of the reference input, i.e., $r_c(t)$ can be ignored by using a feed forward of $c(t)$ on the residual signal $r(t)$ as shown in Fig. 2. $\tilde{r}(t)$ is then used for the residual evaluation according to Fig. 2.

$$\tilde{r}(t) = r_{d_1}(t) + r_{d_2}(t) + r_f(t). \tag{14}$$

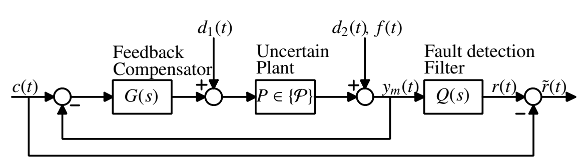

Fig. 2: Improved SFDI structure to eliminate the bias of reference signal $c(t)$ in the residual evaluation function.

To determine an appropriate fault alarm, a threshold, J_{th}, is selected. $\|\tilde{r}\|_2$ should be less than J_{th} in the absence of a fault. Moreover a failure should be declared if $\|\tilde{r}\|_2$ exceeds J_{th}. The common standard practice is to select J_{th} as the upper bound on the residual signal in the absence of any fault signal. Based on the above statements the minimum required J_{th} is subsequently chosen as follows:

$$J_{th} = \sup_{P \in \{\mathcal{P}\}} \|r_{d_1}(t)\|_2 + \|r_{d_2}(t)\|_2 \tag{15}$$

Remark2:

Since an evaluation of a residual signal over the whole time range is impractical [8], it is desired that the faults be detected as early as possible within the detection window $\tau = t_2 - t_1$, see Assumption 5 . Note that in the presence of noise, τ must be large enough to separate that noise signal from a genuine sensor failure signal. The selection of such a detection window has been discussed in [8].

VI Illustrative example

A Single Machine Infinite Bus (SMIB) power system is now considered to illustrate the methodology. Fig. 3 shows the functional diagram for the SMIB equipped with a conventional excitation control system. The excitation voltage, E_{fd}, is supplied by the exciter and is controlled by an Automatic Voltage Regulator (AVR) to keep the terminal voltage equal to reference voltage. Although the AVR is very effective during steady state operation, it can have a negative influence on the damping of the low frequency electromechanical oscillations that naturally occur in such a plant. For this reason a supplementary control loop, known as the Power System Stabilizer (PSS), is often added as shown in Fig. 3, in order to achieve an overall improvement in damping of these electromechanical modes, [3].

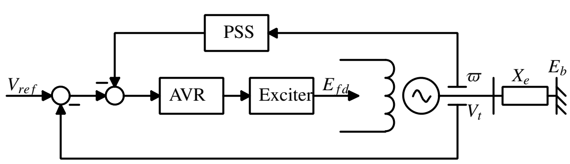

Fig. 3: Conventional excitation control system.

By linearizing the system around any given steady-state operating condition, the generator and excitation control system can be modeled as a fourth-order system in state space form (2) where,

$$x = \begin{pmatrix} \Delta\delta \\ \Delta\varpi \\ \Delta e'_q \\ \Delta E_{fd} \end{pmatrix}; u = \Delta V_{ref}; y = \Delta\varpi$$

$$A = \begin{pmatrix} 0 & \omega_B & 0 & 0 \\ -\frac{K_1}{2H} & 0 & -\frac{K_2}{2H} & 0 \\ -\frac{K_4}{T'_{do}} & 0 & -\frac{1}{T'_{do}K_3} & \frac{1}{T'_{do}} \\ -\frac{K_A K_5}{T_A} & 0 & -\frac{K_6 K_A}{T_A} & -\frac{1}{T_A} \end{pmatrix}; B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}; \ D = 0$$

(16)

The system matrix $\mathbf{A}$ contains uncertain variables K_i, for $i \in \{1, \cdots, 6\}$ whose values are determined by the particular operating conditions at hand. The related equations to compute K_i, for $i \in \{1, \cdots, 6\}$ are provided in Appendix A. An operating condition is determined from the value of active power, P_m, reactive power, Q_m, and the impedance of the transmission line, X_e. In order to represent model uncertainty, it is assumed that these parameters vary independently over the range P_m : 0.4 *to* 1.0, Q_m : -0.2 *to* 0.5, and X_e : 0.2 *to* 0.7. A random model in the specified range is arbitrarily selected as the nominal plant. The system data used for this example is given in Appendix B.

The necessary SFDI structure, incorporating sensor fault and fault detection filter is represented by the unity feedback system as shown in Fig. 4, in which the effect of changes in the terminal voltage is treated as an input disturbance to the system.

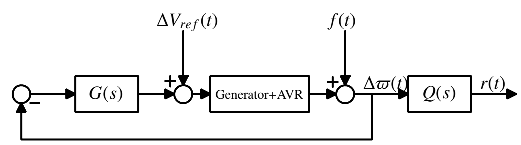

Fig. 4: Block diagram of the coordinate PSS and SFD structures for the SMIB plant.

a) Feedback controller design

The use of a feedback compensator (17) achieves the following performance characteristics :

- the effects of the changes in the terminal voltage is reduced to less than $-10dB$.
- a lower gain margin of $K_M = 1.833 = 5.26(dB)$ and a phase margin angle of $\phi_M = 49.25°$ are achieved thereby guaranteeing a satisfactory level of robust stability.

$$G(s) = -18\frac{10s}{1+10s}\frac{(1+8.4s)^2}{(1+33s)^2} \tag{17}$$

b) Design of $Q(s)$ to detect sensor faults

The design procedure is now repeated using the procedure outlined by Theorem 2. The residual reference model for detection of sensor faults is then computed using the following matrices as the nominal plant:

$$B_d = B, \ D_f = 1, \ D_d = 1. \tag{18}$$

To investigate the effect of a sensor fault, D_f and D_d have been set to unity so as to only quantify the actual effects of fault and noise on the output measurement. Equation (19) gives the transfer function of the obtained residual reference signal.

$$G_{rf}^{d} = \frac{(s^2 + 20.41s + 123.7)(s^2 + 0.4183s + 52.35)}{(s^2 + 20.55s + 128.4)(s^2 + 0.2802s + 50.43)} \quad (19)$$

An appropriate engineering interpretation for the resulting G_{rf}^{d} is that, the magnitude of residual signal, should track (in DC gain terms) the actual value of a particular fault.

The desired fault detection error $E_d(s)$ is selected so as to guarantee a zero steady state error between a residual and an actual fault as follows:

$$E_d(s) = \frac{0.25s}{(s + 0.5)(s + 5)} \quad (20)$$

A finite set of $\tilde{\omega} = \{0.2, 0.5, 1, 5\}(rad/s)$, appropriate for the problem at hand, is selected to generate the filter design bounds. Fig. 5 illustrates the resulting constraints. A suitable low order, low bandwidth filter that satisfies these bounds is found to be (21):

$$Q(s) = \frac{10}{(s + 10)}. \quad (21)$$

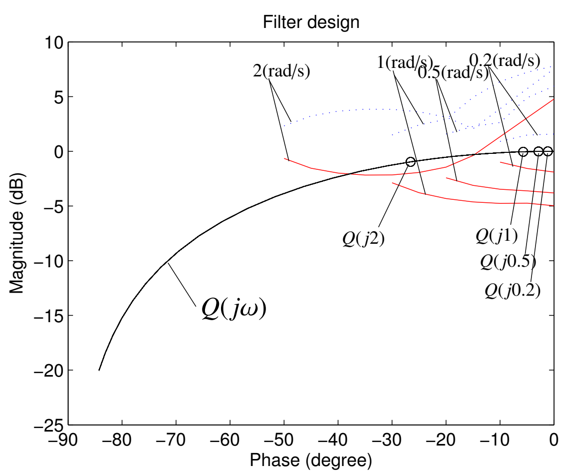

Fig. 5: Design of $Q(s)$ so that SFDI filter lies inside permitted bounds. Upper bounds 'dot', and Lower bounds 'solid'.

c) Performance analysis

To evaluate system performance, a 5% step disturbance at the voltage reference input of the AVR is now considered, i.e, $\Delta V_{ref}(t) = 0.05(pu)$. $f(t)$ is simulated as a pulse of unit amplitude that occurs in the time window $10(s)$ to $20(s)$ (and is zero otherwise). In the simulations, the detection window has been selected as $\tau = 20(s)$. It is assumed that a band-limited white noise with power of 10^{-3} (zero-order hold with sampling time $0.1(s)$) affects the measured signal $y_m(t)$. The simulation is repeated for a selection of plants described in Table 1.

Fig. 6 illustrates $r(t)$ for this family of plants. It confirms (1) has been satisfied by the design specifications. In addition, the effect of disturbance signals have been fully decoupled *and* damped. To select an appropriate threshold J_{th}, the residual evaluation function (15) is computed where a sensor fault is absent. Fig. 7 shows the residual evaluation signal for the family of plants with and without $f(t)$. The figure illustrates that with a selection of $J_{th} = 0.1$, the fault will be detected as soon as it occurs.

Table 1: Three example plants covering the uncertainty region, [15].

	P_m (pu)	Q_m (pu)	X (pu)
case1	0.8	0.4	0.2
case2	0.8	0.0	0.6
case3	1.0	0.5	0.7

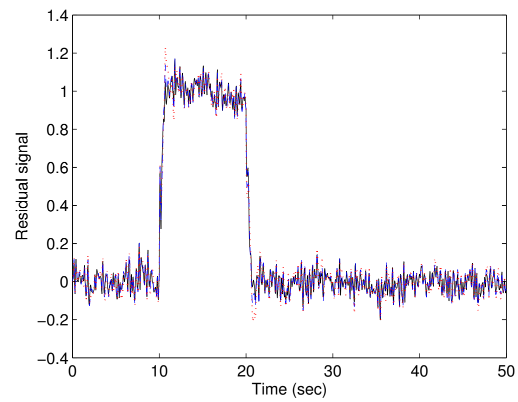

Fig. 6: Residual signal $r(t)$ in the presence of sensor fault and sensor noise, for the family of plants in Table 1.

d) Comparison with open-loop SFDI technique

A comparison of the proposed methodology with an open-loop SFDI technique which is not robust, is presented. It is shown that although the open-loop SFDI approach can result in an appropriate residual reference model, the resulting residual reference model can not solely be used as the fault detection filter in this case. A widely used open-loop fault detection filter with respect to the system given by (2), is proposed by the following equation, [17]:

$$\begin{aligned} \dot{\hat{x}} &= (A - HC)\hat{x} + (B - HD)u + Hy, \quad (22) \\ \hat{y} &= C\hat{x} + Du \\ r &= V(y - \hat{y}). \end{aligned}$$

where A, B, C, D are the system matrices for the open loop system. $\hat{x}$ and $\hat{y}$ represent the state and output estimated vectors, respectively. u denotes the control signal which is considered as a known input signal in open-loop SFDI. The observer gain matrix H and residual weighting matrix V should be designed such that the proposed fault detection objectives are achieved. It should be noted that Theorem 1 also provides a mechanism whereby open-loop fault

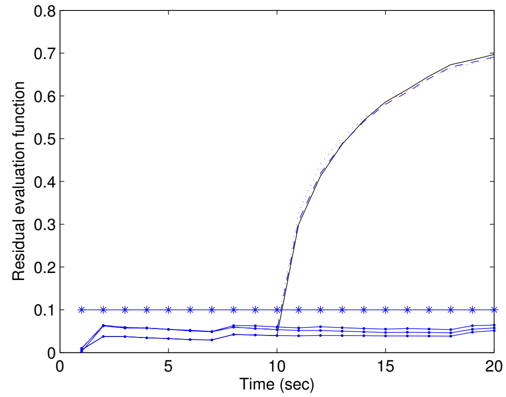

Fig. 7: Residual evaluation signal $\|\tilde{r}(t)\|_2$ with and without sensor fault $f(t)$, for the family of plants in Table 1, in the presence of sensor noise. '-.-' represents the residual evaluation signal without $f(t)$. '-*-' represents the threshold J_{th}.

detection can be achieved. Fig. 8 illustrates a comparison of this approach with the proposed closed-loop SFDI technique. In the figure both residual signals that are generated by Case 3 of Table 1, under open loop *and* closed-loop SFDI are presented. The figure demonstrates that the residual reference generated by (19) cannot reliably distinguish between a fault and, say, disturbance and/or model uncertainty. It is therefore necessary to simultaneously consider some measure of disturbance decoupling within the design procedure to ensure the fidelity of the algorithm in the presence of the model uncertainty.

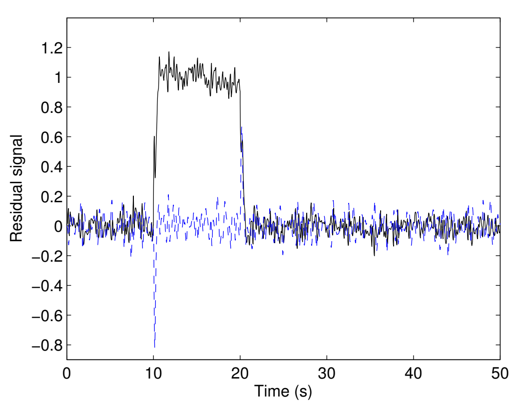

Fig. 8: Comparison of residual signals in the presence of sensor fault using closed-loop FDI 'solid' and open-loop FDI 'dashed'.

VII Conclusion

A novel design methodology has been presented that generates robust residual signals in the presence of sensor faults has been presented for the SISO systems. A two-degree-of-freedom design methodology based on a shaping of frequency response has been introduced that provides an integrated control and detection filter that is robust toward uncertainties as well as disturbances. As the proposed technique captures the exact phase information, it is an effective design tool for both minimum and non-minimum phase plants. A Single Machine Infinite Bus (SMIB) power system highlights the effectiveness of the proposed approach.

VIII Appendix

A System dynamic equations

Suppose the active power, P_m, reactive power, Q_m, and terminal voltage, V_t are given. Then K_1 to K_6 are computed by following equations:

$$
\begin{aligned}
V_d &= P_m V_{t_0} / \sqrt{P_m^2 + (Q_m + V_{t_0}/X_q)^2} \\
V_q &= \sqrt{V_{t_0}^2 - V_d^2} \\
V_t &= \sqrt{V_d^2 + V_q^2} \\
I_d &= (P_m - I_q V_q)/V_d \\
I_q &= V_d / X_q \\
e' &= V_q + X'_d I_d \\
V_{od} &= V_d + X_e I_q \\
V_{oq} &= V_q - X_e I_d \\
E_b &= \sqrt{V_{od}^2 + V_{oq}^2} \\
\delta_0 &= \tan^{-1}(V_{od}/V_{oq})
\end{aligned}
$$

$$
\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_q \end{bmatrix} + \begin{bmatrix} \frac{E_b \sin\delta_0}{X_e + X'_d} & \frac{E_b \cos\delta_0}{X_e + X_q} \\ \frac{1}{X_e + X'_d} & 0 \end{bmatrix} \begin{bmatrix} (X_q - X'_d) I_q \\ E'_q + (X_q - X'_d) I_d \end{bmatrix}
$$

$$
\begin{bmatrix} K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} \frac{X_e + X'_d}{X_e + X_d} \\ \frac{X_e - X'_d}{X_e + X'_d} E_b \sin\delta_0 \end{bmatrix}
$$

$$
\begin{bmatrix} K_5 \\ K_6 \end{bmatrix} = \begin{bmatrix} 0 \\ V_q / V_t \end{bmatrix} + \begin{bmatrix} \frac{E_b \sin\delta_0}{X_e + X'_d} & \frac{E_b \cos\delta_0}{X_e + X_q} \\ \frac{1}{X_e + X'_d} & 0 \end{bmatrix} \begin{bmatrix} -X'_d V_q / V_t \\ X_q V_d / V_t \end{bmatrix}
$$

where

Subscript 0	Steady state value
Δ	Small deviation
δ	Rotor angle
ω	Rotor angular speed
e'_q	Voltage proportional to field flux linkage
E_{fd}	Field voltage
ω_B	Base speed
V_{ref}	AVR reference input
K_A	AVR gain
T_A	AVR time constant
H	Rotor inertia constant
V_t	Generator terminal voltage
T'_{do}	d-axis transient open circuit time
X'_d	d-axis transient reactance
X_d, X_q	d and q axes synchronous reactances
I_d, I_q	d and q axes generator currents
V_d, V_q	d and q axes generator voltages
E_b	Infinite bus voltage
T_m	Mechanical torque

B System data

The system data are given by Table 2.

Table 2: Physical parameters used for the example.

Parameters	**Value**
X_d	2.0 (pu)
X'_d	0.244 (pu)
X_q	1.91 (pu)
T'_{do}	4.18 (sec)
E_b	1.0 (pu)
H	3.25 (sec)
ω_B	314.15 (rad/sec)
K_A	50.0
T_A	0.05 (sec)

Acknowledgment

This work was supported by Science Foundation Ireland under grant No. 05RFPCMS0048. Special thanks are extended to Prof. Jakob Stoustrup, *Department of Electronic Systems, Aalborg University* for kindly giving his time to support the development of this research.

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