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Blind Quality Estimation for Corrupted Source Signals Based on A-Posteriori Probabilities

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Abstract — A novel approach is presented for assessing the quality of transmission systems, comprising quantized source signals and APP source decoders, via Monte-Carlo simulation. A-posteriori probabilities are exploited in order to obtain an unbiased estimate of both the symbol error probability and the expected distortion for the transmission system; knowledge of the transmitted source signal is not necessary. Compared to the conventional method this blind quality estimation has a smaller estimation variance.

Summary

The bit error rate estimation based on a-posteriori probabilities (APPs) was shown to be superior to the conventional one based on hard decisions [1]. In this paper this method is extended to symbol error rate (SER) and distortion estimation. Let us assume a simplified transmission model, where a real-valued source signal \( U \) is quantized to quantization indices \( I, i \in \mathbb{I} \), which are transmitted over a communication channel. Based on the channel observations \( Y \) the receiver generates APPs \( \Pr(I = i|y) \) [2], which are exploited to obtain estimates \( \hat{U} \) and \( \hat{I} \) of \( U \) and \( I \).

Typically quality evaluation via Monte-Carlo simulation is based on a comparison of the transmitted source data \((u, i)\) to their reconstructed versions \((\hat{u}, \hat{i})\) with respect to appropriate quality measures, such as the symbol error rate \( P_e \) or distortion \( D \). For a transmission of \( K \) source symbols, the corresponding quality samples can be used to compute the hard SER estimate \( \hat{z}^{(K)}_H \) and hard distortion estimate \( \hat{d}^{(K)}_H \) as

\[
\hat{z}^{(K)}_H := \frac{1}{K} \sum_{k=1}^{K} \hat{z}_{H,k} \quad \text{and} \quad \hat{d}^{(K)}_H := \frac{1}{K} \sum_{k=1}^{K} \hat{d}_{H,k}.
\]

Obviously, \( z^{(K)}_H \) and \( d^{(K)}_H \) rely on the knowledge of \( u \) and \( i \), from which it follows that the conventional Method H is not suitable for application in practical transmission systems. Thus, we consider now the case, where knowledge of \( u \) and \( i \) is not available. I.e., only the source statistics, the estimates \( \hat{u} \) and \( \hat{i} \), and the set of APPs \( p_{HA} = \{\Pr(I = i|y) | i \in \mathbb{I}\} \) may be used for quality estimation. These restrictions lead to a novel approach for the evaluation of \( P_e \) and \( D \), referred to as Method S in the following:

Method S: We define the soft SER sample as \( \hat{z}_S := \Pr(I \neq I | \hat{I} = i, P_A = p_A) \), which can be computed as \( \hat{z}_S = 1 - \Pr(I = i|y) \), and we define the soft distortion sample \( \hat{d}_S := \mathbb{E}[(U - \hat{U})^2|P_A = p_A] \), which is given by the a-posteriori expectation of the mean-squared error according to \( \hat{d}_S = \sum_{u \in \mathbb{U}} \mathbb{E}[(u - \hat{u})^2|I = i] \cdot \Pr(I = i|y) \) for a given \( \hat{u} \). Considering again the transmission of \( K \) source symbols, the soft SER estimate \( \hat{z}^{(K)}_S \) and the soft distortion estimate \( \hat{d}^{(K)}_S \) for Method S are given by

\[
\hat{z}^{(K)}_S := \frac{1}{K} \sum_{k=1}^{K} z_{S,k} \quad \text{and} \quad \hat{d}^{(K)}_S := \frac{1}{K} \sum_{k=1}^{K} d_{S,k}.
\]

For comparison of both methods we regard the hard and the soft SER and distortion samples as random variables \( Z_H, Z_S \) and \( D_H, D_S \). From their definitions and since the estimates are sample means, it follows that \( \mu_H = \mathbb{E}[Z_H] = \mathbb{E}[Z_S] = P_t \) and \( \mu_D = \mathbb{E}[D_H] = \mathbb{E}[D_S] = D \). Thus, both estimates are unbiased for both the SER and the distortion estimation.

An appropriate figure-6merit is the estimation variance. The variance of the hard SER sample \( Z_H \) can be written as

\[
\sigma^2_{Z_H} = E[Z_H^2] - \mu^2_h = E[Z_H] - \mu_H^2 = \mu(1 - \mu), \quad \mu = \mathbb{E}[Z_H] = \mathbb{E}[Z_S] = P_t
\]

and is upper bounded by \( \mu(1 - \mu) \), thus \( Z_S \leq Z_H \) and \( \mathbb{E}[Z_S^2] \leq \mathbb{E}[Z_H^2] \). Equality holds for the uninteresting cases \( Z_S = 0 \) and \( Z_S = 1 (\sigma^2_{Z_S} = 0) \). For all other cases we have a lower bound on the ratio of variances \( \sigma^2_{Z_H} / \sigma^2_{Z_S} \) and \( \sigma^2_{D_H} / \sigma^2_{D_S} \) of the SER samples:

\[
\frac{\sigma^2_{Z_H}}{\sigma^2_{Z_S}} > 1, \quad \frac{\sigma^2_{D_H}}{\sigma^2_{D_S}} > 1, \quad \text{(3)}
\]

resulting directly from (1) and (2).

A similar bound on the ratio of variances \( \sigma^2_{Z_H} \) and \( \sigma^2_{Z_S} \) of the distortion samples can be derived by applying Jensen’s inequality to the a-posteriori expectation of \( D_H \):

\[
E[D_H^2|P_A = p_A] \geq E[D_H|P_A = p_A]^2 = D_H^2, \quad \text{(4)}
\]

where the identity \( D_H = (u - \hat{u})^2 \) and the definition of the soft distortion sample \( D_S \) is exploited. It follows from (4) that \( E[D_H^2] \geq E[D_S^2] \), where again equality holds for \( \sigma^2_{D_S} = 0 \) (see above), and otherwise

\[
\frac{\sigma^2_{D_H}}{\sigma^2_{D_S}} > 1, \quad \text{(5)}
\]

which represents a lower bound on the ratio of variances \( \sigma^2_{D_H} \) and \( \sigma^2_{D_S} \) of the distortion samples.

The bounds in (3) and (5) prove that the hard SER sample as well as the hard distortion sample have always (except for \( P_t = 0 \)) a larger variance than the soft SER sample and the soft distortion sample, respectively. This reveals the superiority of the proposed Method S to the conventional Method H. In numerical results for Gauss-Markov sources the gain with respect to the estimation variance turned out to be even larger than predicted.

References
