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Published in:
SDBE 2007

Publication date:
2007

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):

Heiselberg, P., Brohus, H., Hesselholt, A. T., Rasmussen, H. E. S., Seinre, E., & Thomas, S. (2007). Application of Sensitivity Analysis in Design of Sustainable Buildings. In *SDBE 2007: Sustainable Development of Building and Environment* Chongqing University.

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Application of Sensitivity Analysis in Design of Sustainable Buildings

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ABSTRACT: Building performance can be expressed by different indicators as primary energy use, environmental load and/or the indoor environmental quality and a building performance simulation can provide the decision maker with a quantitative measure of the extent to which an integrated design solution satisfies the design requirements and objectives.

In the design of sustainable Buildings it is beneficial to identify the most important design parameters in order to develop more efficiently alternative design solutions or reach optimized design solutions.

A sensitivity analysis makes it possible to identify the most important parameters in relation to building performance and to focus design and optimization of sustainable buildings on these fewer, but most important parameters. The sensitivity analyses will typically be performed at a reasonably early stage of the building design process, where it is still possible to influence the important parameters. The methodology is presented and an application example is given for design of an office building in Denmark.

Keywords: Sustainable buildings, energy performance, sensitivity analyses, integrated design

1. INTRODUCTION

The European Union has taken a strong leadership role in promoting energy efficiency in buildings. This is among other things highlighted by the Directive on the Energy Performance of Buildings [1], which is designed to promote the improvement of energy performance of buildings in member states.

One of the benefits of this directive is that it provides an integrated approach to different aspects of buildings energy use, which until now only a few member states were doing, and that all aspects are expressed in simple energy performance indicators. The integrated approach allows flexibility regarding details, giving designers greater choice in meeting minimum standards.

In order to achieve a certain degree of harmonisation of assessment of buildings for designers and users throughout the EU, a common methodology based on an integrated approach is established and includes the following aspects:

- a. thermal characteristics of the building;
- b. heating installation and hot water supply
- c. ventilation and air-conditioning installation;
- d. built-in lighting installation;
- e. position and orientation of buildings, including outdoor climate;

- f. passive solar systems, solar protection, natural ventilation and natural lighting;
- g. indoor climatic conditions, including the designed indoor climate.
- h. active solar systems and other heating and electricity systems based on renewable energy sources; district or block heating and cooling systems.

Building performance are expressed by different indicators as primary energy use, environmental load and/or the indoor environmental quality and a building performance simulation provide the decision maker with a quantitative measure of the extent to which an integrated design solution satisfies the design requirements and objectives.

In order to fulfil the Directive new requirements for primary energy consumption in new buildings including heating, cooling, domestic hot water, ventilation and lighting (not included for residential buildings) entered into force in Denmark in April 2006. The total primary energy use has to be calculated by a new developed software programme BE06, which applies a simplified method for calculation of energy use based on mean monthly average values for climate data, heat loads and occupation schedules.

The primary energy consumption for all buildings (except residences) must not exceed:

$$(95 + \frac{2200}{A}) kWh/m^2 year$$

where A is the heated floor area of the building.

In the calculation of the primary energy use heating energy is multiplied by a factor of 1,0 while electricity use is multiplied by a factor of 2,5.

Besides requirements on the energy consumption the building regulations also puts requirements on airtightness (1,5 l/s m² floor area at a pressure difference of 50Pa) and heat loss (6W/m² envelope, except windows and doors, at a temperature difference of 32 K). The latter means that the average U-value for the building envelope must not exceed 0,19 W/m²K.

The new requirements implied a reduction of 25-30% from the previous requirements and plans for the future include similar reductions in 2010 and 2015.

In order to achieve such reductions of the energy use in new buildings it will require development of new construction solutions, new types of building envelopes, and development of new building materials. It will also require the development of more holistic building concepts, sustainable buildings where an integrated design approach is needed to ensure a system optimization and to enable the designer(s) to control the many design parameters that must be considered and integrated.

Therefore, in the design of sustainable buildings it will be very beneficial to be able identify the most important design parameters in order to develop more efficiently alternative design proposals and/or reach optimized design solutions. This can be achieved by applying sensitivity analysis early in the design process.

A sensitivity analysis makes it possible to identify the most important design parameters in relation to building performance and to focus design and optimization of sustainable buildings on these fewer, but most important parameters. A sensitivity analysis will typically be performed at a reasonably early stage of the building design process, where it is still possible to influence the selection of important parameters.

The objective of the present paper is to present the methodology of sensitivity analysis and by an application example of the design of an office building in Denmark to demonstrate the benefits achieved in a design process and an example of what design parameters contribute significantly to sustainable building energy performance in Denmark.

2. SENSITIVITY ANALYSIS

A sensitivity analysis determines the contribution of the individual design variable to the total performance of the design solution. Sensitivity analysis can be grouped into three classes: screening methods, local sensitivity methods and global sensitivity methods.

Screening methods are used for complex situation which are computational expensive to evaluate and/or have a large number of design parameters as in sustainable building design. It is an economical method that can identify and rank qualitatively the design parameters that control most of the output variability, i.e. energy performance. The methods are often so-called OAT-methods (One-parameter-At-a-Time) in which the impact of changing the values of each design parameter is evaluated in turn (partial analysis). A performance estimation using "standard values" is used as control. For each design parameter, usually two extreme values are selected on both sides of the standard value. The differences between the result obtained by using the standard value and using the extreme values are compared to evaluate, which design parameters the building energy performance is significantly sensitive to.

Local sensitivity methods are an OAT approach, where evaluation of output variability is based on the variation of one design parameter, while all other design parameters are held constant. This method is useful for comparison of the relative importance of various design parameters. The input-output relationship is assumed to be linear and the correlation between design parameters is not taken into account.

Global sensitivity methods are an approach, where output variability due to one design parameter is evaluated by varying all other design parameters as well, and where the effect of range and shape of their probability density function is incorporated.

The basic six steps in a sensitivity analysis, see Figure 1, include:

1. Identification of questions to be answered by the analysis, define output variable(s)
2. Determine input parameters to be included by an initial screening analysis
3. Assign probability density functions to each parameter
4. Generate an input vector/matrix (maybe considering correlation)
5. Create an output distribution
6. Assess the influence of each input parameter on the output variable(s)

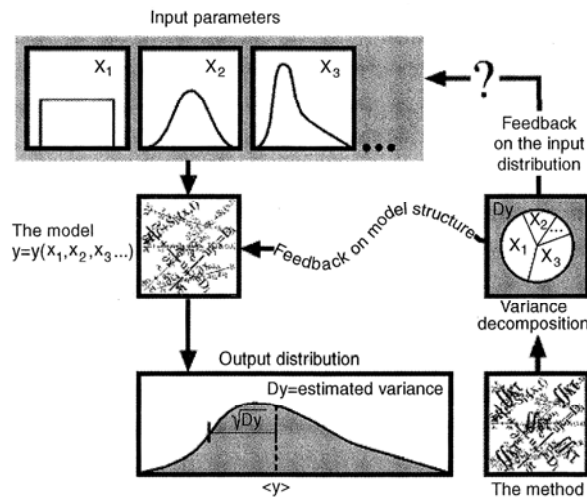


Figure 1. Basic steps in sensitivity analyses.
From Saltelli et al., 2000, p. 7.

A number of different mathematical methods for sensitivity analysis can be found in the literature (Saltelli et al. 2000a,b; Hamby, 1994; Lam and Hui, 1996; Lomas and Eppel, 1992; Morris, 1991). Based on the available information the Morris method (Morris, 1991) is evaluated as the most interesting for sensitivity analysis in sustainable building design as:

- The method is able to handle a large number of parameters
- It is economical – the number of simulation is few compared to the number of parameters
- It is not dependent on assumptions regarding linearity and/or correlations between parameter and model output
- Parameters are varied globally within the limits
- Results are easily interpreted and visualised graphically.
- Indicates if parameter variation is non-linear or mutually correlated.

Sensitivity analyses can in principle be used for all kinds of projects, however, the more spread found in the various design parameters and the higher the sensitivity to those parameters, the more benefit will be gained from the analyses. The sensitivity analyses will typically be performed by consulting engineers preferably at a reasonably early stage of the building design process, where it is still possible to influence the important parameters.

The sensitivity analysis makes it possible to identify the most important design parameters for building performance and to focus the

building design and optimization on these fewer parameters.

The main barrier for application of sensitivity analysis in building performance assessment is the increase in calculation time and complexity. Even if the Morris method is relative effective about 500 calculations of output variables are needed for an investigation of 50 variable design parameters.

3. DESCRIPTION OF METHOD

The first step in a sensitivity analysis is to identify the question(s) to be answered by the analysis, i.e. define the output variable. Often the analyses will focus on the building energy performance (e.g. kWh/(m² year)) and/or the indoor environmental quality (e.g. average/cumulated PPD, number of hours exceeding a certain predefined temperature etc.). The building costs may be linked to the sensitivity analyses and form an integrated part of the entire decision process.

The second step is by a screening method to determine which design parameters should be included in the analysis. A design parameter can be considered to be sensitive, if its value can vary considerably. These parameters are the ones selected for the initial screening. If variation of the parameter results in considerable variation in the building energy performance, the parameter is considered to be important and it is included in the further analysis.

The third step is to assign a probability density functions to each parameter, which are used in the investigation and quantification of their importance for energy performance. In most cases it is possible to estimate the limits for the variation of a design parameter, to estimate the most probable value of the parameter within the limits and to choose the most appropriate probability density function. Sensitivity analysis results generally depend more on the selected ranges than on the assigned distributions. Typically three different probability density functions are used, see Figure 2:

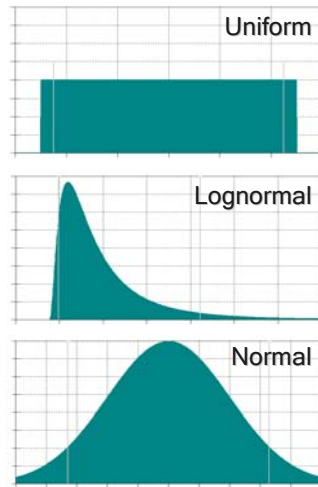


Figure 2. Probability density distributions usually applied in sensitivity analysis.

The fourth step is to generate an input vector/matrix. Based on the probability density functions of each parameter random samples of design parameters are generated. Various sampling procedures exist among which are: random sampling, Latin hypercube sampling and quasi-random sampling. Control of correlation between variables within a sample is extremely important and difficult, because the imposed correlations have to consistent with the proposed variable distribution. A method proposed by Morris (1991) and Saltelli (2000) is applied in this work. The method comprises a number of individually randomised one-factor-at-a-time samples of design parameters where all parameters are varied within their variable space in a way that spans the entire space to form an approximate global sensitivity analysis (Morris 1991, Saltelli 2000).

The fifth step is to create an output variable for each sample of design parameters. This can be achieved by a simulation model.

The last step is the sensitivity analysis, where the influence of each design parameter on the expected value and the variance of the output parameter(s) is estimated. A number of different techniques can be used, like rank transformation, regression analysis and scatter plots, yielding different measures of sensitivity, Saltelli et al. (2000a).

The method of Elementary Effects (Morris 1991, Saltelli 2000) is applied in this work. The method, which can be seen as an extension of a derivative-based screening method, can be characterised as a method with global characteristics. The method has been applied in several areas of building sciences e.g. natural night ventilation (Breesch 2004) and thermal

building simulation (de Wit 1997). The method determines the so-called elementary effect EE of a model $y = y(x_1, \dots, x_k)$ with input factors x_i . The Elementary Effect for the i^{th} input factor in a point x is:

$$EE(x_1, \dots, x_k) = \frac{y(x_1, x_2, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(x_1, \dots, x_k)}{\Delta}$$

A number of elementary effects EE_i of each factor are calculated based on the generated samples of design parameters. The model sensitivity to each factor is evaluated by the mean value and the standard deviation of the elementary effects:

$$\mu = \sum_{i=1}^r |EE_i| / r$$

$$\sigma = \sqrt{\sum_{i=1}^r |EE_i - \mu|^2 / r}$$

where μ is the mean value of the absolute values of the elementary effects determining if the factor is important, and σ is the standard deviation of the elementary effects which is a measure of the sum of all interactions of x_i with other factors and of all its nonlinear effects. r is the number of elementary effects investigated for each factor.

The result of the sensitivity analysis is a list of important parameters and a ranking of the parameters by the strength of their impact on the output.

4. DESIGN EXAMPLE

In order to illustrate the application of the method described, it is demonstrated on a 7 storey office building. The building consists of a ground floor, which is larger than the six upper floors. An atrium is placed in the southern façade from ground floor to the roof, see figure 3. The ground floor measures 24,0 m \times 32,4 m and is mainly used for an entrance hall, restaurant, office, conference room, cafe and technical appliances. Stairways, toilets and elevators are placed in the centre of the building. The upper levels, measuring 24,0 m \times 24,0 m, are conceptually made in the same way although there are slight differences. The 2nd, 4th and 5th floor are designed with the same layout, see Figure 3.

The height of each story is 3,5 m, which gives a building height of 24,5 m. Brutto floor area is $A_{\text{brutto}} = 4233,6 \text{ m}^2$. The heated floor

area is $A = 3910,8 \text{ m}^2$. The areas of the zones in the building on each floor are listed in Table 1.

The ground floor includes a restaurant with a large glazed area. On the south facade of the building the atrium facade has glazing running continuously from the ground level to the 6th floor. In Table 2 the total window area of each floor, including the atrium, is listed as well as the orientation of the windows. The window area compared to the heated floor area 17 %.

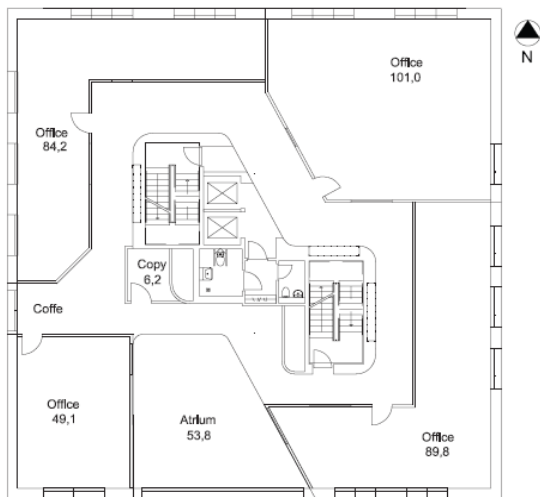
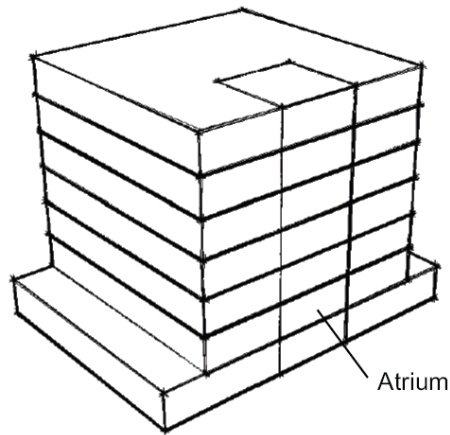


Figure 3. Sketch and Plan drawing of building.

Table 1. Floor area of building zones.

	Gr.	1 st	2 nd	3 rd	4 th	5 th	6 th
Conference room	100,1	0	0	0	0	0	0
Offices	112,1	237,6	324,1	315,5	324,1	324,1	182,8
Copy rooms	6,5	12,8	6,2	12,8	6,2	6,2	12,8
Café	52,3	0	0	0	0	0	128,4
Restaurant	126,0	38,6	0	0	0	0	0
Kitchen	40,8	37,0	0	0	0	0	0
Technical rooms	7,0	0	0	6,2	0	0	6,2
Administration	22,1	0	0	0	0	0	0
Corridors	235,7	121,2	116,9	112,7	116,9	116,9	117,0
Toilets	7,4	7,4	7,4	7,4	7,4	7,4	7,4
Building core	67,6	67,6	67,6	67,6	67,6	67,6	67,6
Σ	777,6	522,2	522,2	522,2	522,2	522,2	522,2

Table 2. Window area and direction

Floor	North [m ²]	East [m ²]	South [m ²]	West [m ²]
Ground floor	29,7	12,8	47,3	20,8
1. floor	20,2	12,8	46,1	14,2
2. floor	19,2	12,8	46,1	12,8
3. floor	20,9	12,8	46,1	12,8
4. floor	19,2	12,8	46,1	12,8
5. floor	19,2	12,8	46,1	12,8
6. floor	20,2	12,8	46,1	12,8
Σ	148,6	89,6	323,9	99,0

The total energy use for heating, ventilation, cooling and lighting in the reference building is calculated by the software programme BE06 to be $E = 107.4 \text{ kWh/m}^2 \text{ year}$ (heating $45,9 \text{ kWh/m}^2 \text{ year}$, ventilation $33,5 \text{ kWh/m}^2 \text{ year}$, cooling $0 \text{ kWh/m}^2 \text{ year}$ and lighting $28,0 \text{ kWh/m}^2 \text{ year}$). This is above the present requirements ($95 \text{ kWh/m}^2 \text{ year}$) and some changes in the actual design is necessary in order to reduce the energy use.

5. RESULTS OF SENSITIVITY ANALYSIS

The sensitivity analysis was performed to identify the important design parameters to change in order to reduce the energy use in the reference building. In the analysis a series of parameters were changed and the effect of the changes on the demand for heating, cooling and total energy were evaluated by the software package BE06.

Table 3 shows the design parameters included in the analysis and for each parameter the defined range and distribution. For some design parameters the probability density function is given as a normal distribution defined by its mean value and the standard deviation. For other design parameters a uniform distribution is defined by four discrete values. For each design parameter 4 different elementary effects are used, i.e. that for each parameter 4 values of the output variable (energy use) are obtained. With 21 parameters, the minimum number of simulations is (using *Morris' Randomized OAT Design as a Factor Screening Method for Developing Simulation Metamodels*, 2004):

$$N = r \cdot (k + 1) = 4 \cdot (21 + 1) = 88$$

where

N is the number of simulations

r is the number of elementary effects per factor

k is the number of design parameters

Instead of using the minimum number of simulations 10 paths through the design parameter space are explored to give more accurate results. With this number of elementary effects for each parameter, the number of simulations becomes 220.

The design parameters given in the unit percent in Table 3 the discrete values are the percentage of the values used in the reference building. The usage factor and the installed power concern the lighting zones in the building. The numbers 1 to 4 for the lighting control system refer to None control, Manual control, Automatic control and Continuous control in BE06.

Table 3. Design parameters for sensitivity analysis, their range and distribution.

Parameter	Unit	Discrete values				μ	σ
1 heat capacity	$\left[\frac{Wh}{K \cdot m^2}\right]$					120	10
2 U (climate shield)	[%]	100	83,3	66,7	50		
3 Line loss	[%]	100	66,7	33,3	0		
4 U (windows)	$\left[\frac{W}{m^2 \cdot K}\right]$	1,5	1,3	1,1	0,9		
5 g-value	[n.d.]	0,7	0,6	0,5	0,4		
6 Shading	[-]	0,8	0,6	0,4	0,2		
7 Overhang	[°]	45	30	15	0		
8 q_m	$\left[\frac{l}{s \cdot m^2}\right]$	0	1	2	3		
9 η_{VGV}	[-]	0,7	0,75	0,8	0,85		
10 q_n	$\left[\frac{l}{s \cdot m^2}\right]$	0	0,15	0,3	0,45		
11 $q_{i,n}$	$\left[\frac{l}{s \cdot m^2}\right]$	0	0,1	0,2	0,3		
12 SFP	$\left[\frac{W}{m^2}\right]$	2,1	1,7	1,3	0,9		
13 $q_{m,s}$	$\left[\frac{l}{s \cdot m^2}\right]$	0	1	2	3		
14 $q_{n,s}$	$\left[\frac{l}{s \cdot m^2}\right]$	0	1	2	3		
15 $q_{m,n}$	$\left[\frac{l}{s \cdot m^2}\right]$	0	1	2	3		
16 $q_{n,n}$	$\left[\frac{l}{s \cdot m^2}\right]$	0	1	2	3		
17 Heat loads	$\left[\frac{W}{m^2}\right]$					14	2
18 Lighting power	$\left[\frac{W}{m^2}\right]$	7	8	10	11		
19 Daylight factor	[-]					2	0,5
20 Light control	[-]	1(N)	2(M)	3(A)	4(C)		
21 Usage factor	[-]	1,0	0,9	0,8	0,7		

The ventilation parameters in Table 3 are:

- q_m is the mechanical rate during daytime in winter
- η_{VGV} is the efficiency of the heat recovery
- q_n is the natural ventilation rate during daytime in winter
- $q_{i,n}$ is the infiltration rate during nighttime in winter
- SFP is the specific fan power
- $q_{m,s}$ is the mechanical rate during daytime in summer
- $q_{n,s}$ is the natural ventilation rate during daytime in summer
- $q_{m,n}$ is the mechanical rate during nighttime in summer
- $q_{n,n}$ is the natural ventilation rate during nighttime in summer

The results of the sensitivity analysis are shown in Figure 4. The figure shows the mean value, μ , of the absolute values of the elementary effects determining if the factor is important, and the standard deviation, σ , of the elementary effects which is a measure of the sum of all interactions of x_i with other factors and of all its nonlinear effects.

The dotted wedge in the figure shows the following relation between the mean value and the standard deviation:

$$\sigma = \frac{\mu \sqrt{r}}{2}$$

where

σ is the mean value of the elementary effect ($kWh/m^2 \cdot year$)

r is the number of elementary effects per design parameter

μ is the standard deviation of the elementary effect ($kWh/m^2 \cdot year$)

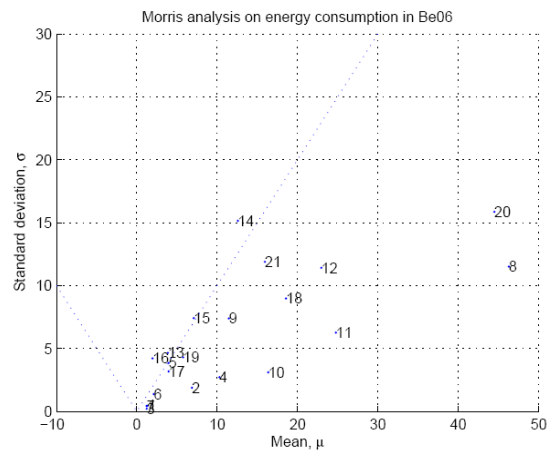


Figure 4. Influence of design parameters on the total energy use.

The location of a point compared to the wedge given by the above equation provides information about the characteristics of that design parameter. If the point is placed inside the wedge the design parameter has mainly a correlated or/and a non-linear impact on the output (energy use). If the point for a design parameter is placed outside the wedge the impact can be considered as linear and a change in the design parameter would give a proportional change of the output (energy use). If the point is located close to the lines of the wedge it is combination of the two cases.

From the results shown in Figure 4 it is possible to estimate the influence of each of the design parameters and a ranking of the design parameters influence on the sensitivity of the energy use is listed in Table 4. From the ranking

it can be concluded, that especially design parameters related to artificial lighting and ventilation of the building in the winter (heating) season have a significant mean value. Also the U-values have a notable influence.

Table 4. Ranking of design parameters according to their impact on primary energy use.

Rank	Parameter	μ
1	8 q_m	46,30
2	20 Lighting control	44,84
3	11 $q_{i,n}$	24,78
4	12 SFP	22,96
5	18 Lighting power	18,56
6	10 q_n	16,36
7	21 Usage factor	15,94
8	14 $q_{n,s}$	12,58
9	9 η_{VGV}	11,46
10	4 U (windows)	10,28
11	15 $q_{m,n}$	7,08
12	2 U (climate shield)	6,86
13	19 Daylight factor	5,76
14	5 g-value	3,96
14	17 Heat loads	3,96
16	13 $q_{m,s}$	3,86
17	6 Shading	2,12
18	16 $q_{n,n}$	1,96
19	1 Heat capacity	1,36
20	3 Line loss	1,24
21	7 Overhang	1,22

The sensitivity analysis was also performed with the heating demand as the output parameter. The results of this are shown in Figure 5.

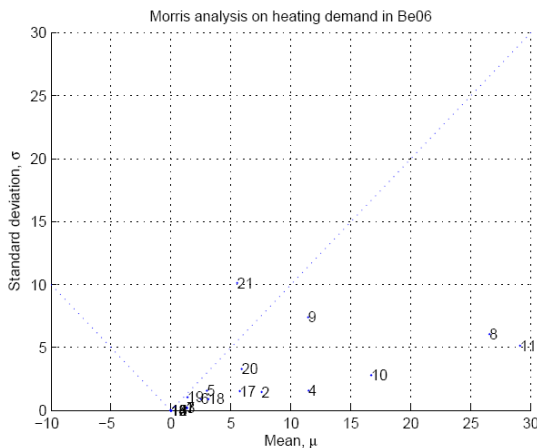


Figure 5. Influence of design parameters on the heating demand.

DISCUSSION

The calculation of the primary energy use of the reference office building it was shown that the heating demand (45,9 kWh/m² year) was dominating while the ventilation (33,5 kWh/m² year) and lighting (28,0 kWh/m² year) demand was slightly lower and no demand for cooling existed.

The sensitivity analysis shows which design parameters are the most important ones to change in order to reduce the energy consumption. The results show that lighting control and the amount of ventilation during winter are the two most important parameters that will have the largest effect on the energy use. This means that introduction of lighting control according to daylight levels and demand controlled ventilation in the heating season are two technologies that should be considered in the next design step.

It can also be seen that even if the heating demand is dominating the ranking of design parameters reducing the heat loss from the building are quite low. If an analysis is done for the heating demand alone in stead of for the total energy use it can be seen from figure 5 that the ranking of parameters change a lot. Now design parameters related to ventilation in the heating season is the most important ones, while improvement of insulation levels and cold bridges still only will have a minor influence.

As the cooling demand does not exist it can also be seen that design parameters influencing the heat load of the building naturally have the lowest ranking.

It can be concluded that a sensitivity analysis in the early stages of the design process can give important information about which design parameters to focus on in the next phases of the design as well as information about the unimportant design parameters that only will have a minor impact on building performance.

The sensitivity analysis will improve the efficiency of the design process and be very useful in an optimization of building performance.

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