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Yield-Line Theory and Material Properties of Laterally Loaded Masonry Walls

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The behaviour of masonry walls subjected to lateral loads has been studied by means of a large number of special tests. The fracture process has been studied in order to find the answer to an important question: does masonry show any ductile properties that may justify the application of yield-line theory as a design method for laterally loaded masonry walls? From the results of the tests it is concluded that the answer must be in the affirmative. The masonry material shows distinctly ductile properties with respect to forces imposed by lateral loads, and stress-strain relationships are well described by an elastoplastic model. The fracture criteria, describing which combinations of moments and vertical in-plane forces give rise to failure, appears to be approximately fracture criteria of the Coulomb type. The test series and results reported in this paper are described in detail in Brincker [12].

INTRODUCTION

The tests described in the following derive from the problem of designing laterally loaded masonry walls and, especially, the discussion regarding the application of yield-line theory to such walls. As our starting point let us consider a masonry wall as shown in Figure 1. The wall is primarily subjected to lateral forces acting horizontally, but there may also be some vertical forces acting in plane (dead load). The problems involved in designing such masonry walls have not yet been satisfactorily solved.

If the members adjacent to the wall under consideration are sufficiently stiff or if in-plane extensions are in some way prevented, we can assume arch action, which provides great resistance to lateral loads. It is usually easy to prove the necessary strength against lateral loads in such cases. However, in the many cases in which arch action cannot be assumed, serious design problems arise, and it is these we shall be dealing with in the following.

Design methods based on elastic solutions have been proposed, see Falconer [1], Bradshaw and Entwisle [2], Francis [3], and Hallquist [5], but are generally not easy to handle, and the load determined on the basis of the theory of elasticity often represents a gross underestimation of the strength. Modified methods such as the strip method have been proposed, see Baker [6] and Hendry [8]. Here, too, the results are often too conservative, and it is generally difficult to take account of different support conditions and special configurations such as holes.

The yield-line theory has therefore been proposed by some authors, see Losberg and Johansson [4], Satti [7], Haseltine [9], Hendry and Kheir [10], Haseltine, West and Tutt [11], and Cajdert [13]. The yield-line theory is a flexible design method, especially when orthotropic and inhomogeneous properties, holes and special support conditions, have to be taken into account. According to the above references, the yield-line theory shows good agreement with experimental results, but one problem is that there is at present no rational justification for its use.

The main aim of the tests performed has been to focus on the physical properties of the masonry material against such sectional forces occurring in laterally loaded masonry walls. The crack patterns of such walls at failure develop very much like the yield-line patterns of reinforced concrete slabs. If the wall under consideration is simply supported along its four edges, like that shown in Figure 1, both horizontal and oblique yield lines will develop.

![Lateral load.](image)

The material's resistance to the actual sectional forces has been studied in a horizontal and an oblique yield line. In the case of the horizontal yield line, the criteria describing which combinations of vertical in-plane forces and bending moments give rise to failure have been determined by experiments with eccentrically loaded brick piers in which the tensile strength of the masonry was assumed to be zero (cracked section assumed).

In the oblique yield line, the conditions are much more interesting and have therefore been studied in far greater detail. The fracture properties of the material in an oblique yield have been studied by means of an advanced test in which a specially designed test specimen is subjected to a combination of bending, torsion and vertically acting in-plane forces. This test is described in detail later.
In the programme tests were performed on four combinations of materials, two qualities of mortar and two types of brick. The bricks were Danish normal size 55x108x228 mm; one of them was solid, while the other had 55 holes distributed in five rows parallel with the longest edge of the brick. The solid brick had a compression strength of 48 MPa, and the brick with holes a compression strength of 28 MPa. The two mortars were both relatively weak. One was a pure lime mortar with a lime/sand ratio of 100/1200 (by weight), and the other was a lime-cement mortar, with a lime/cement/sand ratio of 50/50/750. The compression strength of the mortars, determined by means of 40x80 cylindrical mortar specimens, was found to be 1.0-1.7 MPa for the lime mortar and 3.0-5.2 MPa for the lime-cement mortar. The four combinations of materials have each been given a code as shown in Table 1.

Typical strains of the line of action of \( K \) is given by

\[ \varepsilon = K / b \]

and the bending moment is given by

\[ M = K e \]

The strain \( \varepsilon_K \) corresponding to the force \( K \) (the strain of the line of action of \( K \)) is given by

\[ \varepsilon_K = \varepsilon_1 - \frac{e_2 - e_1}{2} \frac{h}{b} \]  

Typical \( \sigma, \varepsilon_K \) stress-strain relationships for the LC-S combinations of materials (see Table 1) are given in Figures 3 to 7.

The last points on the curves, which show strongly stochastic behaviour, should not be paid too much attention, since the determination of the strain \( \varepsilon_K \) given by equation (3) is based on the assumption that plane sections remain plane during deformation. This is not true when the bricks fail.

On the basis of the tests performed it can be concluded that the stress-strain relations - and this applies to all four combinations of materials - can be described by dividing the relationship into three phases: first a linear elastic phase, then a plastic phase with strain-hardening, and last, a fracture phase, with values equal to or less than zero for the slope \( d\sigma / d\varepsilon_K \).

According to the stress-strain relationships the point of failure is defined as the first point between a phase 2 and a phase 3, see figure 6. The modulus of rupture \( \sigma_c = K_c / b \) found in this way may therefore sometimes be less than the ultimate breaking stress \( \sigma_{um} \). Table 2 shows mean values and standard deviations of the failure load \( K_c \) for the different cases.

The eccentric compression tests to investigate the strength of the material in respect of sectional forces acting in a yield line were performed as shown in Figure 2, using the following four load distributions:

- type 1: \( h = t \Rightarrow e = 0 \)
- type 2: \( b = 1/2 \Rightarrow e = 1/4 t \)
- type 3: \( b = 1/4 \Rightarrow e = 3/8 t \)
- type 4: \( b = 1/8 \Rightarrow e = 7/16 t \)

As a main rule each test was repeated five times.

During each test corresponding values of the axial load \( K \) and the strains \( \varepsilon_1 \) and \( \varepsilon_2 \), defined in Figure 2 were measured. The uniform stress \( \sigma \) is given by:

\[ \sigma = \frac{K}{l_0} \]  

(1)

TABLE 1 - CODE NUMBERS AND COMBINATIONS OF MATERIALS USED IN THE TEST PROGRAMME

<table>
<thead>
<tr>
<th>Mortar:</th>
<th>Solid brick</th>
<th>Brick with holes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lime/sand:</td>
<td>L-S</td>
<td>L-H</td>
</tr>
<tr>
<td>100/1200 (by weight)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lime/cement/sand:</td>
<td>LC-S</td>
<td>LC-H</td>
</tr>
<tr>
<td>50/50/750 (by weight)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2 - VALUES OF FAILURE FOR THE ECCENTRICALLY ACTING AXIAL LOAD \( K \). MEAN VALUES AND STANDARD DEVIATIONS OF FIVE EXPERIMENTS

<table>
<thead>
<tr>
<th>Combinations of material</th>
<th>Load distribution ( K_c )</th>
<th>type 1</th>
<th>type 2</th>
<th>type 3</th>
<th>type 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 Mean value (kN)</td>
<td>584.</td>
<td>211.</td>
<td>36.6</td>
<td>7.09</td>
<td></td>
</tr>
<tr>
<td>C.V. (%)</td>
<td>8.1</td>
<td>9.4</td>
<td>20.</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>12 Mean value (kN)</td>
<td>187.</td>
<td>87.7</td>
<td>27.2</td>
<td>6.89</td>
<td></td>
</tr>
<tr>
<td>21 Mean value (kN)</td>
<td>494.</td>
<td>231.</td>
<td>48.7</td>
<td>18.4</td>
<td></td>
</tr>
<tr>
<td>C.V. (%)</td>
<td>2.8</td>
<td>12.</td>
<td>24.</td>
<td>16.</td>
<td></td>
</tr>
<tr>
<td>22 Mean value (kN)</td>
<td>220.</td>
<td>92.9</td>
<td>49.7</td>
<td>21.0</td>
<td></td>
</tr>
<tr>
<td>C.V. (%)</td>
<td>4.7</td>
<td>7.3</td>
<td>6.0</td>
<td>21.</td>
<td></td>
</tr>
</tbody>
</table>

Corresponding mean values of the bending moment \( M \) and the failure load \( K_c \) are plotted in Figure 8. For each material the four points are fitted with a second degree polynomial. Thus, the figure shows the failure criteria found in experiments relating the axial load \( K \) and the bending moment \( M \). For small axial loads \( K \), where second order effects can be neglected,
Figures 3 - 7 Stress relationship for eccentric compression tests.
The failure criterion will be seen to be of the Coulomb type:

$$M = \mu K$$

(4)

with zero cohesion and a friction of $\mu_0$. The values of $\mu_0$ found in the experiments are given in Table 8.

**TESTS WITH COMBINED TORSION AND BENDING (OBLIQUE YIELD LINES)**

The failure properties of the material in an oblique yield line have been studied by means of a specially designed test on small piers subjected to lateral loading, bending and torsional moments. A special testing machine was built for this purpose. The principle of the test-bed is illustrated in Figure 9.

During the tests, which were performed with constant axial load and continuously increasing lateral load, measurements were taken of the bending moments $M_{11}$ and $M_{21}$ in joint 1 and 2, respectively, see Figure 10, and of the torsional moments $M_{12}$ and $M_{22}$, together with other quantities such as the corresponding angular strains $\phi_{11}$, $\phi_{21}$, $\phi_{12}$ and $\phi_{22}$, see figure 10.

The angular strains $\phi_{11}$, $\phi_{21}$, $\phi_{12}$ and $\phi_{22}$ were measured by means of specially designed extensometers placed across the joints. Two extensometers were placed at each joint, glued in place on either side of the specimen, see Figure 11. Each extensometer measured two displacement components: one perpendicular to the joint, and one parallel to it.

The bending-torsion specimen was fastened to the arms of the test-bed by means of friction jaws acting on opposite sides of each brick. The mounting and starting procedure was planned and carried out to ensure minimum unintentional stress in the specimen. Unintentional prestress in the
specimen was also reduced to a minimum by means of hinges, which could be eliminated by locking, and by very accurate shaping of the test specimens.

The main geometry of the test-bed is shown in Figure 12. It was designed so that the quantity

\[ \alpha = \frac{\xi_1}{\xi_2} = \tan \theta \]  

(5)

corresponding to different slopes of the yield line, see Figure 9, could be adjusted by moving the crosspiece on arm 2, shown in Figure 12.

Both the horizontal load \( K_2 \) and the vertical load \( K_1 \) were imposed on the structure by hydraulic presses. The axial load \( K_2 \) was kept constant by means of an oil pressure control unit, and the lateral load \( K_1 \) was controlled by a servo pacer unit, in order to achieve a ramp-shaped progress for the lateral displacement \( \delta_1 \) corresponding to the load \( K_1 \).

In addition to the moments and the corresponding angular strains, all of which were measured as functions of the time \( t \), \( M_{11} = M_{11}(t) \), \( \phi_{11} = \phi_{11}(t) \), etc., the lateral load \( K_1 \) and the corresponding lateral displacement \( \delta_1 \) were also measured.

The following parameters were varied in the test programme:

- The material. Four combinations of materials, L-S, L-H, LC-S and LC-H, see Table 1.
- The axial load. The load \( K_2 \) was varied through five values from zero up to about 20\% of \( K_{2c} \), where \( K_{2c} \) is the compression failure load of the test specimen.
- The slope of the yield line. The quantity \( \alpha \) was varied through the values \( \alpha = 1.149, 0.940, 0.795 \) and \( 0.689 \).

As a principal rule, each test was repeated three times. The chosen values of the axial load \( K_2 \), together with measured values of the compression strength are given in Table 3.

<table>
<thead>
<tr>
<th>Axial load ( K_2 ) (kN)</th>
<th>Compression failure loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>load 1</td>
<td>load 2</td>
</tr>
<tr>
<td>1.25</td>
<td>7.39</td>
</tr>
<tr>
<td>Material 11</td>
<td></td>
</tr>
<tr>
<td>1.24</td>
<td>2.47</td>
</tr>
<tr>
<td>Material 12</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>7.39</td>
</tr>
<tr>
<td>Material 21</td>
<td></td>
</tr>
<tr>
<td>1.24</td>
<td>3.08</td>
</tr>
<tr>
<td>Material 22</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3 - COMPRESSION STRENGTH AND IMPOSED VALUES OF AXIAL LOADS FOR THE DIFFERENT TORSION-BENDING SPECIMENS.**

The data collected were dealt with as described below with a view to determining whether the masonry material possesses any ductile properties against the effects considered here. The work done by the forces acting on one of the joints, say joint one, can, with good approximation, be written as:

\[ W_1(t) = \int_{0}^{t} (M_{11}(t) \frac{d\phi_{11}(t)}{dt} + M_{12}(t) \frac{d\phi_{12}(t)}{dt}) dt \]

(6)

or, if we choose the strain \( \phi_{11} \) as integration parameter instead of the time, as:

\[ W_1(\phi_{11}) = \int_{0}^{\phi_{11}} (M_{11} + M_{12} \frac{d\phi_{12}}{d\phi_{11}}) d\phi_{11} \]

(7)

On the basis of this result it proves convenient to define a fictive stress parameter:

\[ \sigma_{1}^{*} = \frac{1}{t} \frac{dW_{1}}{d\phi_{11}} = \frac{1}{t} (M_{11} + M_{12} \frac{d\phi_{12}}{d\phi_{11}}) \]

(8)

or by division with the mortar compression strength \( f_m \) (The mortar strength was measured by means of cylindrical test specimens made from the same batch of mortar as the considered torsion-bending specimen. The mortar strength is shown in Table 4)

\[ S_{1}^{*} = \frac{1}{f_m} \frac{dW_{1}}{d\phi_{11}} = \frac{1}{f_m} (M_{11} + M_{12} \frac{d\phi_{12}}{d\phi_{11}}) \]

(9)

Similarly, for joint two:

\[ S_{2}^{*} = \frac{1}{f_m} \frac{dW_{2}}{d\phi_{11}} = \frac{1}{f_m} (M_{21} + M_{22} \frac{d\phi_{22}}{d\phi_{21}}) \]

(10)

The stress-strain relationships \( S_{1}^{*} \) and \( S_{2}^{*} \), \( \phi_{21} \) can be regarded as usual stress-strain relationships and provide a direct illustration of the behaviour of the material, as if we were talking about a compression test, for example. A linear relationship indicates elastic behaviour, and a curve which changes from constant slope to zero slope indicates plastic behaviour, etc.
If we could impose on the structure precisely the displacement field wanted, we would have:

\[
\frac{\phi_{12}}{\phi_{11}} = \frac{\phi_{22}}{\phi_{21}} = \alpha
\]

(11)

which means that the relationship between the torsional and the bending angular strain for both joints should be linear.

In practice, of course, the imposed displacement field will always show a certain deviation from the intended field. In order to see how good agreement there was between the theoretical and the imposed displacement field, the measured bending angular strain was for each pair plotted against the measured torsional angular strain, together with the intended relationship indicated by a straight line. A typical result is shown in Figure 13. As will be seen, the agreement between the theoretical and the imposed displacements was best in the latter part of the test.

The stress-strain relationships \( \pi^{*} \), \( \phi_{11}^{*} \) and \( \pi^{*} \), \( \phi_{11}^{*} \) for the same test are shown in Figure 14, together with the contributions \( \Delta \pi_{1}^{*} \) and \( \Delta \pi_{2}^{*} \) due to bending alone.

\[
\Delta \pi_{1}^{*} = \frac{1}{f_{11}^{*}} M_{11} \quad \Delta \pi_{2}^{*} = \frac{1}{f_{22}^{*}} M_{21}
\]

(12)

As will be seen from the figure, the material showed distinctly plastic behaviour.

Finally, for the same test, Figure 15 shows the lateral load \( K_{1} \), plotted against the corresponding displacement \( \delta_{1} \).

The results shown here can generally be considered as representative of all the tests, although there was a tendency for tests with lower values of the axial load \( K_{2} \) to result in more irregular curves (data showing greater deviations from the mean trend), and vice versa in cases of higher values, where the curves are even more regular. However, the general tendency is clear enough: all stress-strain relationships show distinctly plastic behaviour, as illustrated by the results shown in Figure 14.

With respect to the imposed deformations, these seem to be in agreement with the intended deformations only in the latter part of the test, when fracture had completely developed, because the test specimens possessed considerable resistance to torsional strain in the early stages of fracture. On the basis of the relationships obtained, failure values for the lateral load \( K_{1} \), and for the moments \( M_{11c}, M_{12c}, M_{21c} \) and \( M_{22c} \), together with the failure bending angular strain \( \phi_{c}^{*} \), were determined for all tests.

The failure strain \( \phi_{c}^{*} \) was determined as the mean value of the bending angular strains for the two joints, corresponding to the kink-point in the elastoplastic approximation of the measured stress-strain relationships. No dependence on the geometrical parameter \( \alpha \) could be traced; therefore, all the failure strains corresponding to the same material and the same axial load have been pooled. The results obtained are shown in Figure 16. Here, the failure strains are plotted against the stress level \( K_{2}/K_{2c} \), where \( K_{2} \) is the actual axial load, and \( K_{2c} \) is the compression failure load. It will be seen that with the data depicted in this way, there seems to be no dependence on the choice of material, but — with good approximation — linear dependence on the axial load. However, the most important aspects of the tests reported here are the valuation of the measured failure...
all for the values of $\alpha$ considered here, it is at any rate weak and of minor significance. It can be neglected, and we have therefore pooled all data corresponding to the same material and the same axial load. Readers who are interested in further investigations regarding this assumption are referred to the test report Brincker [12]. The reasonableness of the assumption can be checked by applying the pooled failure values for the bending and torsional moments to obtain a plastic solution for the dependence of the lateral load $K_1$ on the geometrical parameter $\alpha$, and comparing this result with the measured failure values for the lateral load. All the data required can be obtained from Tables 5, 6, 7, giving the results of the measured failure values for the lateral load and the moments. This investigation - which serves to support the assumption - is also performed in the report Brincker [12].

In Figures 17 and 18 the failure values for the bending moments and the torsional moments are each plotted against the axial load. The figures show the failure criteria for the different materials for bending and torsion measured by the tests. These curves are one of the most important results of the test programme.

The measured failure criteria for the bending moments can be described in the following way. For small values of the axial load, $K_2$, the criteria seem to be the same, independent of the choice of material, and the criteria are all of the Coulomb type with zero cohesion. For higher values of $K_2$ (values of $K_2$ greater than, say, 10% of the compression strength), second order effects reduce the failure values for the bending moments compared with the extrapolated linear behaviour found for small values of $K_2$. 

It is not difficult to see that the masonry material can be ascribed two yield moments based on the ideas and results given here characterising the ability of the material to resist lateral loading - and this is essential if we wish to apply the yield line theory in the usual way - only if the failure values for the bending and torsional moments measured by the tests can be assumed to be independent of the geometrical parameter $\alpha$.

In fact, this seems to be the case. The tests show that if there is any dependence on $\alpha$ at

![Figure 15. Simultaneous plot of the lateral load $K_1$ v. lateral displacement $\delta_1$ for the test shown in Figures 13 and 14.](image)

![Figure 16. The dependence of the failure strain upon the stress level $K_2/K_2c$.](image)
Similar conclusions apply to the failure criteria for the torsional moments, except that in this case, the choice of material plays an important role, even for small values of $K_2$, since here, the failure criteria, which are also of the Coulomb type, are not equal, but differ as regards both cohesion and friction parameters. It is particularly interesting to see that the cohesion appears to differ significantly from zero.

If second order effects are neglected, then the failure values for the bending moments can be written

$$M_1 = \mu_1 K_2$$

(13)

and similarly for the torsional moments

$$M_2 = c + \mu_2 K_2$$

(14)

The values measured for the cohesion and friction parameters are given in Table 8.

![Figure 18. Failure criteria found for the different materials by experiments relating the torsional moment $M_2$ and the axial load $K_2$.](image)

**CONCLUSIONS**

On the basis of the tests described in the foregoing, which were carried out to investigate the fracture conditions in horizontal and oblique yield lines, the following conclusions can be drawn:

- That the masonry material in a horizontal yield line is able to resist a bending moment independent of the choice of material, which - for small values of axial loads - can be written as an expression of the Coulomb type with the cohesion $c = 0$ and the friction $\mu = 0.90 \cdot t/2$;

- That the material at each of the horizontal segments of the staircase-shaped oblique yield line is able to resist a bending moment independent of the choice of material, which - for small values of the axial load - can be written as an expression of the Coulomb type, with the constant $c = 0$ and the friction $\mu = 0.75 \cdot t/2$;

- That the material at each of the horizontal segments of the staircase-shaped oblique yield line is able to resist a torsional moment, which - for small values of axial loads - can be written as an expression of the Coulomb type, where the cohesion and friction parameters depend on the choice of material;

- That, for the slopes of the yield line considered here, the above-mentioned failure values for the bending and torsional moments in an oblique yield line can be assumed to be constant, independent of the slopes;

- That the above-mentioned failure values for the bending and torsional moments can be regarded as yield moments since they are present in a wide range of strain values;

- That the failure bending strain in an oblique yield line can be assumed to be a linear function of the compression stress level, and independent of the choice of material;

<table>
<thead>
<tr>
<th>Material</th>
<th>$\alpha = 1.149$</th>
<th>$\alpha = 0.940$</th>
<th>$\alpha = 0.795$</th>
<th>$\alpha = 0.689$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_m$ (MN/m²)</td>
<td>1.19</td>
<td>3.86</td>
<td>1.03</td>
<td>3.23</td>
</tr>
<tr>
<td>$f_m$ (MN/m³)</td>
<td>1.03</td>
<td>4.06</td>
<td>1.33</td>
<td>3.88</td>
</tr>
<tr>
<td>$f_m$ (MN/m²)</td>
<td>3.33</td>
<td>4.53</td>
<td>1.30</td>
<td>4.08</td>
</tr>
<tr>
<td>$f_m$ (MN/m³)</td>
<td>1.71</td>
<td>4.06</td>
<td>1.51</td>
<td>3.43</td>
</tr>
<tr>
<td>$f_m$ (MN/m²)</td>
<td>1.05</td>
<td>3.72</td>
<td>1.51</td>
<td>3.64</td>
</tr>
</tbody>
</table>

The standard deviation is of the order of 10% of the mean values.
That second order effects, which reduce the failure values for the moments in both horizontal and oblique yield lines, should be taken into account in cases of greater values of the axial loads.

All things considered, it can be concluded that the results of the investigation support the application of yield-line theory as a design method for laterally loaded masonry walls.

Nevertheless, it must be admitted that the investigation and its results are more of a qualitative than a quantitative nature, and even though we have supported the assumption that masonry walls do possess some ductile properties against lateral loads that may justify the use of yield line theory, the results are limited by simplifying points of view, as for instance the fact that we ignore the contributions to the lateral strength due to the cross joints.

However, this does not weaken the main results of the investigation, which prove that laterally loaded masonry walls do have ductile properties, and prove that — and explain why — the oblique yield lines greatly increase the lateral strength of such walls, especially in cases of small values of the in-plane forces.

ACKNOWLEDGEMENTS

Financial support from the Danish Council for Scientific and Industrial Research is gratefully acknowledged.

REFERENCES


### Table 5 - Values of Failure $\mu_{IC}$ for the Lateral Load $E_1$ for the Different Combinations of Materials, Axial Loads, and Values for the Geometric Parameter $\gamma$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_1$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
<th>$E_{LC}$ (kN)</th>
<th>$E_{LC}$ (kN)</th>
<th>$E_{LC}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>0.30</td>
<td>0.32*</td>
<td>0.30</td>
<td>0.25*</td>
<td></td>
</tr>
<tr>
<td>7.30</td>
<td>1.56*</td>
<td>1.47**</td>
<td>1.33</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>11 14.78</td>
<td>2.42</td>
<td>2.56</td>
<td>1.84</td>
<td>2.38</td>
<td></td>
</tr>
<tr>
<td>20.53</td>
<td>4.38*</td>
<td>3.68*</td>
<td>3.55</td>
<td>5.28*</td>
<td></td>
</tr>
<tr>
<td>49.24</td>
<td>5.85</td>
<td>4.85</td>
<td>5.28*</td>
<td>5.88*</td>
<td></td>
</tr>
</tbody>
</table>

The standard deviations are of the order of 12% of the mean values.

* only two tests  
** only one test  
*** no tests

### Table 6 - Values of Failure $\mu_{LC}$ for the Bending Moment for the Different Combinations of Materials, Axial Loads, and Values for the Geometric Parameter $\gamma$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_3$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>0.030</td>
<td>0.030</td>
<td>0.041</td>
<td>0.025</td>
</tr>
<tr>
<td>7.20</td>
<td>0.094</td>
<td>0.072</td>
<td>0.092</td>
<td>0.080</td>
</tr>
<tr>
<td>14.78</td>
<td>0.135</td>
<td>0.130</td>
<td>0.109</td>
<td>0.115</td>
</tr>
<tr>
<td>29.53</td>
<td>0.231</td>
<td>0.195</td>
<td>0.178</td>
<td>-</td>
</tr>
<tr>
<td>49.24</td>
<td>0.342</td>
<td>0.305</td>
<td>0.261</td>
<td>0.331</td>
</tr>
</tbody>
</table>

The standard deviations are of the order of 20% of the mean values.  
* All tests with same material and same axial load are pooled.

### Table 7 - Values of Failure $\mu_{LC}$, for the Torsional Moment for the Different Combinations of Materials, Axial Loads, and Values for the Geometric Parameter $\gamma$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_3$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
<th>$E_{IC}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24</td>
<td>0.055</td>
<td>0.043</td>
<td>0.048</td>
<td>0.038</td>
</tr>
<tr>
<td>3.47</td>
<td>0.063</td>
<td>0.057</td>
<td>0.050</td>
<td>0.048</td>
</tr>
<tr>
<td>4.93</td>
<td>0.091</td>
<td>0.070</td>
<td>0.063</td>
<td>0.071</td>
</tr>
<tr>
<td>9.86</td>
<td>0.135</td>
<td>0.124</td>
<td>0.119</td>
<td>-</td>
</tr>
<tr>
<td>19.72</td>
<td>0.230</td>
<td>0.205</td>
<td>0.143</td>
<td>0.151</td>
</tr>
</tbody>
</table>

The standard deviations are of the order of 19% of the mean values, including the standard deviations for the pooled data.  
* All tests with same material and same axial load are pooled.

### Table 8 - Coherence and Friction Parameters for the Different Test Situations and Materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cohesion</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c$</td>
<td>$\mu$</td>
</tr>
<tr>
<td></td>
<td>(Hm)</td>
<td>(mm)</td>
</tr>
<tr>
<td>Horizontal</td>
<td>11</td>
<td>44.8</td>
</tr>
<tr>
<td>yield-line</td>
<td>12</td>
<td>54.1</td>
</tr>
<tr>
<td>Bending</td>
<td>21</td>
<td>0*</td>
</tr>
<tr>
<td>moment</td>
<td>22</td>
<td>0*</td>
</tr>
<tr>
<td>Oblique</td>
<td>11</td>
<td>0*</td>
</tr>
<tr>
<td>yield-line</td>
<td>12</td>
<td>0*</td>
</tr>
<tr>
<td>Bending</td>
<td>21</td>
<td>0*</td>
</tr>
<tr>
<td>moment</td>
<td>22</td>
<td>0*</td>
</tr>
</tbody>
</table>

The cohesion is put equal to zero in agreement with the results of the investigation.

* All tests with same material and same axial load are pooled.