Abstract—In this article, a new probabilistic method of assessment of harmonic emission of large offshore wind farms is presented. Based on measurements from the British wind farm Burbo Banks, probability density functions are estimated for the dominating low order harmonic currents injected by a single turbine. The degree and type of dependence between the harmonic emission and the operating point of a single turbine is established. A model of Burbo Banks, suitable for harmonic load flow studies, is created in Digsilent Power Factory along with a DPL-script that deals with the probabilistic issues of the harmonic emission. The simulated harmonic distortion at the PCC is compared to measurement. This reveals some difficulties regarding harmonic load flow studies. The harmonic background distortion in the grid to where the wind farm is connected must be included in the study. Furthermore, a very detailed representation of the frequency dependent short circuit impedance must be used before sufficiently accurate results can be obtained from the model.

Index Terms—Probabilistic harmonic load flow, harmonics, wind farm modeling, harmonic system modeling

I. INTRODUCTION

Due to environmental concerns the amount of wind penetration in traditional power systems is increasing. Because of public resistance towards wind turbines on land, the trend is that the wind turbines are installed offshore in increasingly larger wind farms. As the wind farms increase in size their impact on the existing power grid becomes more important. Because the wind farms are replacing the traditional power plants during operation, the wind farms are required to provide auxiliary services as voltage and frequency support. In order to fulfill these demands, power converters are necessary in the turbines.

The use of power converters introduces new difficulties in regards to the power quality. Switching of the transistors used in the converters causes additional harmonic emission. A popular choice of converter topology in wind turbines is voltage source converters (VSC). The harmonic emission of these converter types is not only linked to the fundamental frequency but is also determined by the switching frequency and modulation type applied by the converter control. Because the controllers operating the turbine converters are not synchronized with each other, the harmonics currents generated may change arbitrary in phase angle. This can lead to harmonic cancelation and should therefore be included when the harmonic distortion at the PCC is evaluated.

According to applicable standards, only the magnitude of the harmonic current is required to be evaluated. Information regarding the phase angle is, despite the fact that it is directly available from the Discrete Fourier transform (DFT), disregarded. The sum of harmonic vectors with time varying angles is either smaller or equal their arithmetic sum. This effect is included in applicable standard (IEC61000-3-6) by using an empirical approach [1]. However, a former study carried out has shown that the method proposed in the IEC standard results in underestimation of the most dominating harmonics [2]. In [3], [4] and [5] it is shown that the harmonic emission from several different types of wind turbines is in fact best described by a probability density function rather than a deterministic value. Therefore a probabilistic approach seems as a feasible idea for evaluating the harmonic distortion in the PCC.

Based on the problems presented with the current method of harmonic evaluation, it is of interest to develop a new method based on a probabilistic approach where appropriate models of the power system components are used. The development of such a method is described in this article.

II. SYSTEM DESCRIPTION

To obtain the necessary probability density functions describing the harmonic emission of a single wind turbine and to validate the newly developed method, real life measurements are needed. A large measuring campaign was ongoing at the British offshore wind farm Burbo Banks. Burbo Banks, owned by DONG Energy, is a 90 MW offshore wind farm situated on the Burbo Flats in Liverpool Bay at in Great Brittan. 25 Siemens SWT-3.6-107 wind turbines with a rated power of 3.6 MW each make up the wind farm. The wind turbine generator is asynchronous and variable speed properties are achieved by use of a full scale two level power converter. Three wind turbine radials are connected a 132/34 kV substation through three submarine cable assembled with three single conductor cables on land. At the 132/34 kV substation, the voltage is transformed to 132 kV, and the power is feed into the main grid. A one step 13 MVAr compensating capacitor is installed at the substation. The layout of the wind farm is seen in Figure 1.

The measurements campaign at Burbo Banks was carried out from the end of November 2007 to the beginning of February 2008. Time series of the three phase voltages and
currents where measured at the high voltage side of wind turbine 38’s transformer (see Figure 1) and at the high voltage side of the 132/34 kV substation using capacitive voltage dividers and Rogowski-coil type of sensors.

The frequency content of the time domain signals was extracted using Fourier analysis. After the time series were processed, 1 min harmonic average values were calculated and 28,830 minutes or 20 days of data are available for the analysis. In this article the focus is on modeling of the low order harmonics ($h<25$). The harmonics related to the switching frequency will be dealt with in a later article, where the same method, as presented for the low order harmonics, is used.

Available for the study is a frequency dependent representation of the short circuit impedance representing the 132 kV grid to where Burbo Banks is connected. The calculation of the short circuit impedance has been done for a limited number of system conditions and loading characteristics.

III. HARMONIC EMISSION AS A FUNCTION OF ACTIVE AND REACTIVE POWER PRODUCTION

Before any stochastic model can be developed for the harmonic emission of a single turbine, it is necessary to determine whether the harmonic emission is dependent on the operating point of the turbine. If this is the case this dependence must be included in the evaluation. To determine which type of dependence is found, if any, a graphical analysis is carried out. The analysis is done for the magnitude and angle of the dominating harmonics in the spectrum. In order to identify the dominating low order harmonic currents, a spectrum representative for the entire measuring campaign is presented. Such a spectrum is seen in Figure 2.

From Figure 2 it is seen that the dominating components in the spectrum are the 5th and 7th harmonic. The 2nd and 3rd harmonics are also present but are theoretically not created by the three phase inverter in the turbines. The cause of these harmonics is most likely other harmonic sources in the 132 kV grid. Because the source of the last two mentioned harmonics cannot be determined from the measurements and because the magnitude is low compared to the 5th and 7th these harmonic are not included in the analysis.

A. Graphical analysis

In order to determine the type of dependence between the active and reactive power and the harmonic current emission, scatter plots are created. As an example the 5th order harmonic magnitude relative to the rated fundamental is plotted against the active power in Figure 3.

From Figure 3 it is seen that the dependence between the active power and the 5th order harmonic magnitude is rather weak with a linear tendency. The harmonic emission is plotted against the reactive power in Figure 4.

From Figure 4 it is seen that the dependence between the reactive power and the harmonic magnitude is very weak as well, with no clear behavior. The same result is obtained for the angle of the harmonic as well as both the magnitude and angle of the 7th harmonic. The graphical analysis shows that assuming a harmonic emission uncorrelated to the operating point of the turbine is fair. A more into depth graphical analysis along with an analytical analysis can be found in [7].
IV. PROBABILITY DISTRIBUTION- AND PARAMETER ESTIMATION

Probability density functions for all dominating harmonic magnitudes and angles must be estimated and their maximum likelihood estimators calculated. The best fits for the probability density functions are determined using the two-sample Kolmogorov-Smirnov test (KS2-test). Generally for the low order harmonic magnitudes and angles, either a normal distribution or a log-normal distribution gives the best fit. As an example, the fit for the 5th and the 7th order harmonic magnitudes are seen in Figure 5 and 6 respectively.

![Fig. 5 Theoretical and empirical probability distribution for the 5th order harmonic current magnitude.](image)

![Fig. 6 Theoretical and empirical probability distribution for the 7th order harmonic current magnitude.](image)

Analyses are carried out for all dominating harmonic magnitudes and angles and the results are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
</tr>
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<tbody>
<tr>
<td>$</td>
<td>I_{h5}</td>
</tr>
<tr>
<td>$</td>
<td>I_{h7}</td>
</tr>
<tr>
<td>$\theta_{h5}$</td>
<td>Normal</td>
</tr>
<tr>
<td>$\theta_{h7}$</td>
<td>Log-normal</td>
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</tbody>
</table>

It should be noted that the estimated probability distributions are likely be affected by the main grid, and the shape of these distributions should therefore be used with care on other systems.

V. METHOD OF STOCASTIC EVALUATION OF HARMONIC DISTORTION AT THE PCC

A probabilistic method for evaluating the systems harmonic distortion must be chosen. The Monte Carlo approach is a good choice for this particular type of study because it requires only a suitable mathematical model of the system and that the model input is expressed by probability density functions. The mathematical system model is in this case a harmonic load flow model constructed in a power system simulation program. DIgSILENT Power Factory including frequency dependent properties of all components.

As the estimated probability distribution functions describes the harmonic emission at the high voltage side of the wind turbine transformers, the idea is to model the wind turbine including the transformer as a simple ideal current source placed directly at the high voltage side of the transformer. The current source will inject stochastic harmonic currents according to the determined probability distribution. The use of an ideal current source can, however, lead to problems because the harmonic emission will not adapt to a change in the short circuit impedance. If for instance, due to a change in system condition, resonance occurs at one of the dominating harmonics, the non-changing flow of current through the short circuit impedance will lead to unrealistic voltage distortion at the PCC. The short circuit impedance must therefore be modeled stochastic as well, so that the unrealistic conditions are limited or prevented from occurring.

The probability density functions estimated based on the measurements are all obtained from a single wind turbine. One of the assumptions of the developed method is that the harmonic emission for all turbines in the park can be described by the same type of probability distribution and using the same maximum likelihood estimators. The cable system interconnecting the wind turbines in the wind park has, according to simulations, a limited effect on the frequency response of the overall system, and the assumption seem therefore fair for this particular study. This should, however, be examined in future work by doing the same measurements at other wind turbines. The proposed method is visualized in Figure 7.

![Fig. 7 Presentation of the wind farm components which must be modelled stochastically and deterministically.](image)

VI. RESULTS

Simulations of the harmonic distortion at the PCC are carried out and the results are presented in the coming sections.
A. 5th harmonic

The calculated probability distributions of the 5th harmonic current and voltage at the PCC are presented in Figure 8 and 9 respectively.

![Figure 8 Probability distribution of the simulated and measured 5th harmonic current.](image)

![Figure 9 Probability distribution of the simulated and measured 5th harmonic voltage.](image)

The figure shows that the simulated harmonic distortion deviated from the measured. From Figure 9 it is seen that the measured observations seems sampled from three different populations with mean values of 0.4 kV, 0.7 kV and 1 kV (marked by 1, 2 and 3 in Figure 9). A reason for this trimodal distribution could be that the loading of the 132 kV grid, and therefore the harmonic short circuit impedance, is relatively constant in three longer time intervals during a day. The limited data available for the short circuit impedance is a direct cause of the mismatch between the shapes of the distributions. If line outage and switching of power factor compensating units are disregarded, the short circuit impedance changes continuously in real life. In the proposed model this is changed in two steps, either 50% to 100%.

Another reason for the mismatch between the measured and simulated harmonic data is using I/V plots. Such a plot is created and the result is seen in Figure 10.

![Figure 10 I-V plot of the simulated and measured 5th harmonic.](image)

The different green clusters in Figure 10 represent the different conditions for the short circuit impedance. The cluster marked by 1 and 2 are obtained at 100% and 50% load with no outage of the lines in the 132 kV grid. The cluster marked 3 is obtained for both 100% and 50% loading with outage on a busbar at a neighboring substation. With this busbar out of service the impedance at the 5th harmonic is more than 50% lower compared to the impedance obtained while all lines are in service. The cluster marked as 4 is obtained during 50% load and outage on busbar at another neighboring substation. Here the resonance point of the short circuit impedance is placed at the 5th harmonic, resulting in a large impedance and therefore a steep slope in the I/V plot. During this system condition both the simulated harmonic currents and voltages are in the range of the measured. It is, however, not likely that there is an outage on one particular busbar during all 21 days of measuring time. The results presented shows that the harmonic short circuit impedance or the part of the main grid included in the model must be represented with a very high detail level. It also shows that it is necessary to have information regarding the condition of the 132 kV grid at the time instance when the measurements are obtained.

B. 7th harmonic

The I/V plot of the 7th harmonic is presented in Figure 11.

![Figure 11 I-V plot of the simulated and measured 7th harmonic.](image)

The figure shows that the measured harmonic current seems uncorrelated to the voltage (no linear behavior). While the short circuit impedance is kept constant, the model can only procure harmonic voltages and currents with linear behavior. Including a continuous representation of the short circuit impedance will cause behavior as seen for the
measured 5th harmonic (Figure 10) where a ‘wider’ line characterized the relationship between the harmonic voltage and current. This is therefore not the solution to the problem. A reason for the uncorrelated measured 7th harmonic voltage and current can be that there is harmonic background distortion in the 132 kV grid.

If a stochastic voltage source is placed at the grid side of the main transformer as seen in Figure 12, the harmonic current and voltage can be uncorrelated as in the measurements. The harmonic content in the 132 kV grid results in deviation between the simulated and measured distributions problems are identified. One of the most important causes of harmonic distortion at the PCC compared to a deterministic solution described by a probability density function rather than a fixed value. Therefore, the model has great potential if the problems stated in the discussion are solved.

![Harmonic equivalent circuit of Burbo Banks wind farm connected to the 132 kV grid with a harmonic voltage source in the 132 kV grid.](image)

Fig. 12 Harmonic equivalent circuit of Burbo Banks wind farm connected to the 132 kV grid with a harmonic voltage source in the 132 kV grid.

To include this phenomenon, probability density functions of the background distortion should therefore be obtained and included in the model. This is not a straightforward task when the wind farm is erected because during times with low wind the wind farm is used for reactive power compensating. As the background distortion is very important these studies should therefore be carried out before a newly constructed wind farm is put into operation.

**VII. CONCLUSIONS**

In this article a new method of assessment of harmonic emission of large offshore wind farms has been developed. The method is based on a probabilistic approach, were the harmonic currents injected by the wind turbines are described by probability density functions instead of the harmonic distortion at the PCC compared to a deterministic because the harmonic current magnitudes and angles are in fact described by a probability density function rather than a fixed value. Therefore, the model has great potential if the problems stated in the discussion are solved.

**REFERENCES**


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