DELAMINATION OF COMPRESSED THIN LAYERS AT CORNERS

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Summary An analysis of delamination for a thin elastic film, attached to a substrate a corner, is carried out. The film is in compression and the analysis is performed by combining results from fracture mechanics and the theory of thin shells. The results show a very strong dependency of the angle of the corner. In contrast with earlier results for delamination on a flat substrate, the present problem is not a bifurcation problem.

INTRODUCTION

Effects of system curvature on buckling-driven delamination have been studied by Hutchinson [1] and Sørensen and Jensen [2]. The motivations for these studies have typically been metals with coatings used in e.g. high temperature turbine blades and hip joint implants to mention a few. In a number of technically important applications, local curvatures are so high that they are better modelled as sharp corners. One example is the main spar of a wind turbine wing shown in Fig. 1, which is taken from Sørensen et al. [3]. In Fig. 1(c) a post failure cross section is seen. Buckling driven delaminations in the layered glass fibre main spar are seen at the corners both on the inside and outside. The present work focuses on studying buckling driven delamination at corners and the effects on the energy release rate and mode mixity.

In the following, the concept of buckling-driven delamination is summarized. Consider a system consisting of a thin layer bonded to a substrate, with a delamination close to the sharp corner defined by the angle φ, see Fig. 2(a). The case of $\varphi = 0$ corresponds to a flat substrate. The thin layer has the thickness t and is subject to a uniform, biaxial compressive stress, σ_0. For $\varphi \neq 0$ the film will deflect away from the substrate for any value of σ_0, as opposed to the situation with $\varphi = 0$, where σ_0 will have to exceed a critical value, σ_c, before a displacement, i.e. buckling, takes place. In either case the displacement will have the effect, that the crack at the interface will be loaded by normal stresses and moments, as shown in Fig. 2(b). In case the energy release rate exceeds the mode adjusted fracture toughness of the interface, the bonding between the delaminated layer and the substrate will break and the size of the delaminated region may grow.

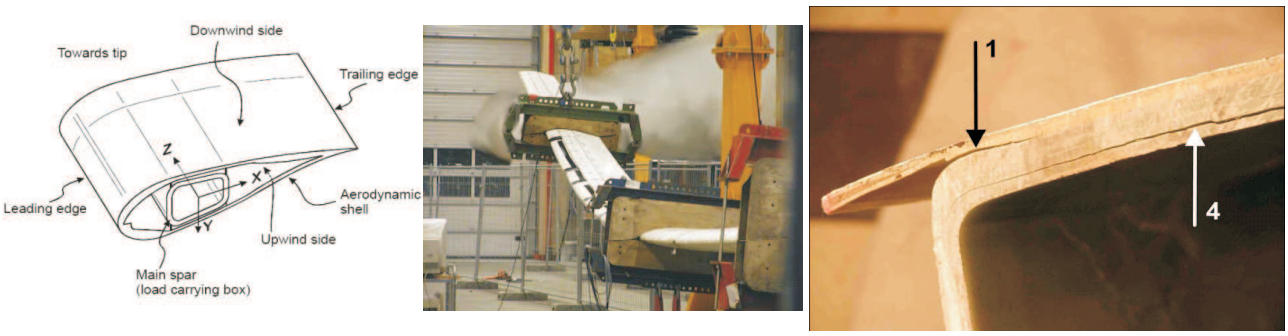
One of the issues which must be taken into account when designing a system as the one shown in Fig. 2(a) is whether the crack is most likely to grow at its front or at the sides. In this paper it will be shown that even though the energy release rate at the front, G_{ss} is considerable lower than at the side, G_l, crack growth along the front is likely. This is due to the fact that the fracture toughness of the interface depends strongly on the so-called *mode mixity*, ψ, which again depends on the loads on the interface crack. Interface toughness laws, which describe this dependency, are formulated in Ref. [4].

THEORY

In the following, steady-state conditions will be assumed. It is also assumed that the boundary between the film and the bonded region can be treated as an interface crack and the stresses in the delaminated region can be obtained from shell theory. The following expressions are found for the energy release rate at the side and front, G_l and G_{ss}, respectively, along with the mode mixity angle, ψ

$$\frac{G_l}{G_0} = \left(\frac{\Delta N}{\sigma_0 t}\right)^2 + 12\left(\frac{M}{\sigma_0 t^2}\right)^2, \qquad G_{ss} = \frac{1}{b}\int_0^b (W_2 - W_1)dy \quad \text{and} \quad \tan\psi = \frac{\Delta N t \sin\omega + \sqrt{12}M\cos\omega}{\Delta N t \sin\omega - \sqrt{12}M\cos\omega}. \tag{1}$$

Here ΔN is the change in resultant stress, M is the bending moment, b is half the crack width, W_1 and W_2 are the energy densities in the deflected film behind the crack front and in the non-deflected film in front of the crack front, respectively. The term ω is a measure of the elastic mismatch between the film and the substrate, which can be found in, e.g., Ref. [5]



(a) (b) (c)

Figure 1. Wind turbine wing. (a) Main elements of the wing. (b) Load test setup. (c) Post failure cross sections.

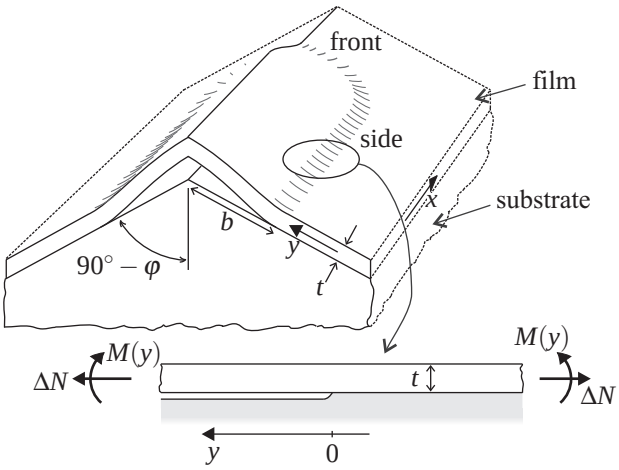


Figure 2. Geometry definitions, forces and moments.

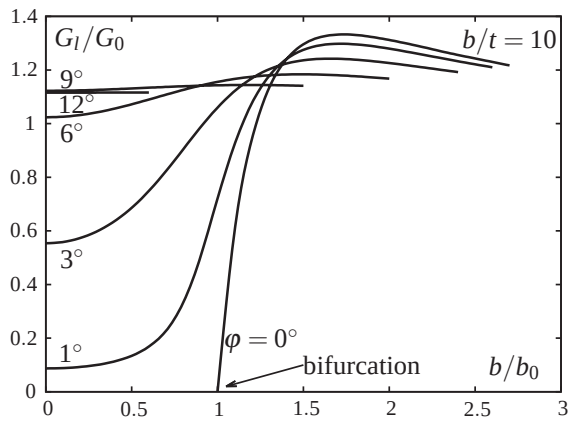


Figure 3. Normalised energy release rate at the side.

along with G_0, which is the energy release rate at the side of the blister at $\varphi = 0$ for large values of σ_0/σ_c . The energy densities W_1 and W_2 are obtained from the theory of thin shells, see, for example, Ref. [6].

RESULTS

Based on the geometry shown on Fig. 2 the solution for M and ΔN is expressed in the term $\eta = b/t \tan\varphi$ as

$$\frac{M}{\sigma_0 t^2} = \frac{\eta}{\pi^2 s}\frac{\pi\sqrt{n}(1-\cos(\pi\sqrt{n}))}{\sin(\pi\sqrt{n})}, \qquad \frac{\Delta N}{\sigma_0 t} = 1 - \frac{n}{s} \qquad \text{with} \quad \frac{n}{s} = 1 - \frac{6}{\pi^2 s}\eta^2 \frac{\sin(\pi\sqrt{n}) - \pi\sqrt{n}\cos(\pi\sqrt{n})}{\pi\sqrt{n}(1+\cos(\pi\sqrt{n}))}. \tag{2}$$

Here $s = (b/b_0)^2 = \sigma_0/\sigma_c$ where b_0 denotes the half-width of the blister when buckling initially occurs at $\varphi = 0$. Results obtained with Eqs. (1a) and (2) are shown in Fig. 3 for $b/t = 10$.

Contacting crack faces

The curves in Fig. 3 are terminated when the mode mixity reaches $\psi = -90°$ indicating contact between the crack faces, as the solutions given above presupposes no contact between the film and the substrate crack. It is seen from the results, that the range of stress values in which solutions exist becomes strongly limited already at small values of φ. In cases where there is no friction between the crack faces, the expressions for G_l and G_{ss} are unaffected as long as it is realised that the crack is under pure mode 2 conditions. For cases with frictional sliding with constant friction between the crack faces, the energy release rate is reduced as analysed by Thouless et al. [7]. This, together with a mixed mode fracture criterion prior to crack closure by Jensen [8], gives the values of G_l after crack closure, i.e. at $\psi = -90°$. An example is plotted in Fig. 4. From the figure it can be seen that when a frictional stress, τ, exist between the crack faces the energy release rate for the side can drop below that of the crack front.

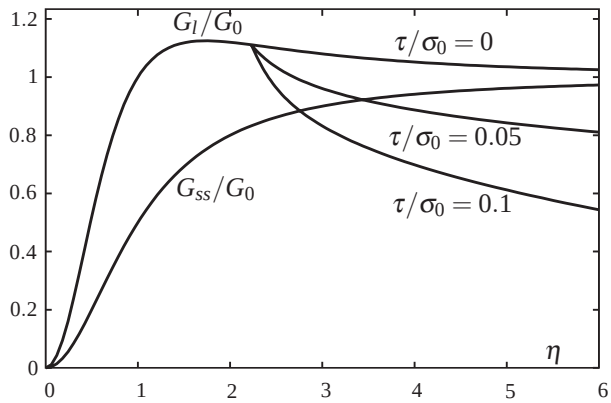


Figure 4. Energy release rates incorporating the effect of crack closure. τ denotes the frictional stress between the crack faces.

CONCLUSIONS

A model for predicting delamination of a thin film attached to a substrate containing a sharp corner was presented. It is shown that if there is no contact between the crack faces the crack will most likely propagate at the sides, but if there is frictional contact, the crack can propagate at the crack front.

References

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