Backstepping Strategy for Induction Motor Control
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Publication date: 2000

Citation for published version (APA):
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ABSTRACT

Using backstepping, which is a recursive nonlinear design method, a novel approach to control of induction motors is developed. The resulting scheme leads to a nonlinear controller for the torque and the amplitude of the field. A combination of nonlinear damping and observer backstepping with a simple flux observer is used in the design. Assuming known motor parameters the design achieves stability with guaranteed region of attraction. It is also shown how a conventional field oriented controller may be obtained by omitting parts of the nonlinear controller.

Keywords: Nonlinear control, backstepping design, induction motors

NOMENCLATURE

- \(a\) complex spatial operator \(e^{j2\pi/3}\)
- \(i_{A,B,C}\) stator phase currents A, B and C
- \(u_{A,B,C}\) stator phase voltages A, B and C
- \(I_s\) stator current complex space vector
- \(u_s\) stator voltages complex space vector
- \(R_s, R_r\) resistances of a stator and rotor phase winding
- \(L_{ss}, L_{sr}\) self-inductance of the stator and the rotor
- \(L_m\) magnetizing inductance
- \(T_r\) rotor time constant (\(T_r = L_m/R_r\))
- \(\sigma\) leakage factor (\(1 - L_m^2/(L_s L_r)\))
- \(R_e\) referred rotor resistance (\(R_e = (L_m/L_r ? R_r\))
- \(L_e\) referred stator inductance (\(L_e = \sigma L_s\))
- \(L_{em}\) referred magnetizing inductance (\(L_{em} = (1 - \sigma) L_s\))
- \(p\) time derivative operator (\(p \equiv d/dt\))
- \(Z_p\) number of pole pair
- \(\omega_{mech}\) angular speed of the rotor
- \(\omega_{r}\) angular speed of the rotor flux
- \(i_{m}\) rotor magnetizing current
- \(\rho\) rotor flux angle
- \(c_m\) torque factor \(c_m = 1.5Z_p L_m\)
- \(N_{ref}\) rotor speed reference

1 INTRODUCTION

The development of the design of high-performance controllers for drives using an induction motor as an actuator is shortly stated by Leonhard [6] as "30 Years Space Vectors, 20 Years Field Orientation and 10 Years Digital Signal Processing with Controlled AC-drives". The relevance of field oriented control is witnessed by a large numbers of investigations carried out both from a theoretical and a practical point of view [5]. The scheme works with a controller which approximately linearizes and decouples the relation between input and output variables by using the simplifying hypothesis that the actual motor flux is kept constant and equal to some desired value.

In the last 10 years with digital signal processing, significant advances have been made in the theory of nonlinear state feedback control [3], and particular feedback linearization and input-output decoupling techniques have been successfully applied for control of induction motor drives [7], [4], [1], [2], [8], [9] and [10].

In Kricic, Kanellakopoulos and Kokotovic [11], a view is opened to a largely unexplored landscape of nonlinear systems with uncertainties. The recursive design methodology developed is called backstepping. With this method the construction of nonlinear feedback control laws and associated Lyapunov functions is systematic and guarantees that the designed system will possess desired properties globally or in a specified region of the state space.

While feedback linearization methods require precise models and often cancel some useful nonlinearities, backstepping designs offer a choice of design tools for accommodation of certain nonlinearities and can avoid wasteful cancellations.

In this paper the method is used for design of a field oriented controller for an induction motor assuming measured rotor speed, stator currents and voltages. The control objectives are tracking of

- the amplitude of the estimated flux
- the estimated electrical torque

A Lyapunov function for the designed system guarantee desired properties in the state space region where the motor is magnetized.

The key items in this paper are

- a model assuming the speed as an input
- rotor field orientation
- a simple flux estimator
- control of torque and rotor field amplitude based on nonlinear feedback
- stability analyzed by a Lyapunov function


The paper also shows how a conventional field-oriented controller may be derived from the developed nonlinear controller by omitting some terms.

2 INDUCTION MOTOR MODEL

The motor model is described in a rotating reference frame $e^{i\omega p}$ with $\omega = \frac{d\theta}{dt}$ having the d-axis in the direction of $e^{i\omega p}$ and the q-axis orthogonal to the d-axis.

For the currents and voltages given in stator coordinates ($i_{sA}, i_{sB}, i_{sC}$) and ($u_{sA}, u_{sB}, u_{sC}$) the following equations based on the angle definition given in Fig. 1 give the transformation from stator coordinates to rotating (d, q)-coordinates:

$$
\begin{align*}
\tau_s &= i_{sd} + j i_{sq} = \frac{2}{3}(i_{sA} + a i_{sB} + a^2 i_{sC}) e^{-j\omega p} \\
\tau_s &= u_{sd} + j u_{sq} = \frac{2}{3}(u_{sA} + a u_{sB} + a^2 u_{sC}) e^{-j\omega p}
\end{align*}
$$

The motor model is then given by:

$$
\frac{d}{dt} \begin{bmatrix} L'_{m} i_{sd} \\ L'_{m} i_{sq} \\ L'_{m} i_{md} \\ L'_{m} i_{mq} \end{bmatrix} = \begin{bmatrix} u_{sd} & 0 \\ 0 & u_{sq} \end{bmatrix} - \begin{bmatrix} R_s i_{sd} - \omega L'_{m} i_{sd} + P'_{L}(i_{sd} - i_{md}) - Z_p\omega_{mech} L'_{m} i_{mq} \\ R_s i_{sq} + \omega L'_{m} i_{sq} + P'_{L}(i_{sq} - i_{mq}) + Z_p\omega_{mech} L'_{m} i_{md} \\ R_s i_{md} - i_{md} + (\omega - Z_p\omega_{mech}) L'_{m} i_{mq} \\ R_s i_{mq} - i_{mq} - (\omega - Z_p\omega_{mech}) L'_{m} i_{md} \end{bmatrix}
$$

and the developed electromagnetic torque is:

$$
m_e = \frac{3}{2} Z_p L'_{m} (i_{md} i_{sq} - i_{mq} i_{sd})
$$

The mechanical equation is:

$$
J \frac{d\omega_{mech}}{dt} = m_e - m_L
$$

Because the rotor magnetizing current $\tau_m = i_{md} + j i_{mq}$ is not measured an estimator has to be constructed. A common method used is based on the current equations. Introducing $T_r = L'_{m}/R'_r$ and $\omega_r = Z_p\omega_{mech}$ in the equations gives

$$
\frac{d}{dt} i_{md} = \frac{1}{T_r} (i_{sd} - \tau_m)
\frac{d}{dt} i_{mq} = \frac{1}{T_r} (i_{sq} - \tau_m) - (\omega - \omega_r) i_{md} = 0
$$
gives for $i_{md} \neq 0$ and $i_{mq} = 0$

$\omega_{slip} = \omega - \omega_r = \frac{i_{sq}}{T_r i_{md}}$

The estimation error $\dot{\tau}_m = \dot{i}_m - \dot{i}_m$ has the dynamics

$$
\frac{d}{dt} \begin{bmatrix} \dot{i}_{md} \\ \dot{i}_{mq} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_r} i_{md} + \omega_{slip} i_{md} \\ -\frac{1}{T_r} i_{mq} - \omega_{slip} i_{mq} \end{bmatrix}
$$

It is now shown that the estimation error dynamic is stable. The convergence is based on a Lyapunov function candidate

$$
V_{obs} = \frac{1}{2} (i_{md}^2 + i_{mq}^2)
$$

whose derivative along the solutions is

$$
\dot{V}_{obs} = \dot{i}_{md} (-\frac{1}{T_r} i_{md} + \omega_{slip} i_{mq}) + \dot{i}_{mq} (-\frac{1}{T_r} i_{mq} - \omega_{slip} i_{md})
$$

$$
\dot{V}_{obs} = -\frac{1}{T_r} (i_{md}^2 + i_{mq}^2) \leq 0
$$

The time derivative of the Lyapunov function candidate is negative so the candidate is indeed a Lyapunov function and the estimator is stable.

3 BACKSTEPPING

The control objectives are tracking of
- the amplitude of the estimated magnetizing current $\dot{i}_{md}$
- the estimated electrical torque $\frac{3}{2} Z_p L'_{m} \dot{i}_{md} \dot{i}_{sq}$

The philosophy in the Back Stepping method is to split up the system into disjoint subsystems, assuming that the complementary dynamics reacts as a disturbance on the subsystem. Proof of the stability for the subsystem can only be fulfilled if some nonlinear decoupling terms are introduced, these terms are called nonlinear damping.

Step 1

We first consider the tracking objective of the magnetizing current. A tracking error $z_1 = \dot{i}_{md} - i_{md,ref}$ is defined and the derivative becomes

$$
\dot{z}_1 = \frac{1}{T_r} (\dot{i}_{sd} - \dot{i}_{md}) - \frac{di_{md,ref}}{dt}
$$

To initiate backstepping, we choose $i_{md}$ as our first virtual control. If the stabilizing function is chosen as

$$
\dot{z}_{i_{md,ref}} = \dot{i}_{md} - c_1 T_r \dot{z}_1 + T_1 \frac{di_{md,ref}}{dt}
$$
we get
\[ z_1 = -c_1 z_1 + \frac{1}{T_r} (i_{sd} - i_{sd,ref}) \]

Due to the fact that \( i_{sd} \) is not a control input an error variable \( z_2 = i_{sd} - i_{sd,ref} \) is defined and we have
\[ z_1 = -c_1 z_1 + \frac{1}{T_r^2} z_2 \]

**Step 2.**
The derivative of the error variable \( z_2 = i_{sd} - i_{sd,ref} \) becomes
\[ \dot{z}_2 = -\frac{1}{T_r^2} (i_{sd} - i_{md}) + c_1 (i_{sd} - i_{md}) - T_r \frac{d}{dt} \text{md}_{\text{ext}} - T_r \frac{d^2 \text{md}_{\text{ext}}}{dt^2} \]
\[ \frac{1}{T_r} \dot{z}_2 = \frac{1}{T_r} R_s i_{sd} - \omega L_e i_{sd} + R_f (i_{sd} - i_{sd}) \]
\[ + c_1 \left( \frac{1}{T_r} (i_{sd} - i_{md}) - \frac{d}{dt} \text{md}_{\text{ext}} \right) - c_2 \left( \frac{1}{T_r^2} + \omega^2 \right) z_2 \]

Viewing \( i_{md} \) and \( i_{mq} \) as unknown disturbances we apply nonlinear damping [11] to design the control function
\[ \frac{1}{T_r} \dot{z}_2 = \frac{1}{T_r} (R_s i_{sd} - \omega L_e i_{sd} + R_f (i_{sd} - i_{md})) \]
\[ + c_1 \left( \frac{1}{T_r} (i_{sd} - i_{md}) - \frac{d}{dt} \text{md}_{\text{ext}} \right) - c_2 \left( \frac{1}{T_r^2} + \omega^2 \right) z_2 \]

Defining \( \phi_1 = \frac{R_f}{L_e} \), \( \phi_2 = \omega \frac{L_e}{T_r^2} \) and \( \phi_3 = \phi_1^2 + \phi_2^3 \)insertion of the control function in the dynamics for the error variable \( z_2 \) gives
\[ \dot{z}_2 = -c_2 \left( \frac{1}{T_r^2} \right) z_2 - d_2 \phi_2^2 z_2 + \phi_1 \dot{i}_{md} + \phi_2 i_{mq} \]

**Step 3.**
We now turn our attention to the torque tracking objective. A tracking error is for \( i_{md} \neq 0 \) defined as
\[ z_3 = \dot{i}_{eq} - m_{r,ref}/(2 \omega Z_{p,m} \text{md}) \]
and the derivative is
\[ \dot{z}_3 = \frac{1}{T_r^2} \dot{u}_{eq} - \frac{1}{T_r} (R_s i_{eq} + \omega L_e i_{eq} + R_f i_{eq} + \omega R_f L_e \text{md}) \]
\[ \frac{1}{T_r} \dot{z}_3 = \frac{1}{T_r} \dot{i}_{eq} - \frac{1}{T_r} (i_{eq} + \omega L_e i_{eq} + R_f i_{eq} + \omega R_f L_e \text{md}) \]
\[ - \phi_2 \frac{1}{T_r^2} \text{md}_{\text{ext}} - \text{md}_{\text{ext}} \frac{d^2}{dt^2} \text{md}_{\text{ext}} \]
\[ - c_3 \frac{1}{T_r} (i_{eq} - i_{md}) \]

Insertion of the control function in the dynamics for the error variable \( z_3 \) then gives
\[ \dot{z}_3 = -c_3 \dot{z}_3 - d_3 \phi_2^2 z_3 + \phi_1 \dot{i}_{md} + \phi_2 i_{mq} \]

The combined controller is shown in figure 2 where we have
\[ u_{sd,ff} = R_s i_{sd} - \omega L_e i_{sd} + R_f (i_{sd} - i_{md}) \]
\[ u_{sq,ff} = R_s i_{sq} + \omega L_e i_{sq} + R_f i_{eq} + \omega R_f L_e \text{md} \]
\[ u_{sd,nl} = L_e \left( \frac{1}{T_r^2} \right) (i_{sd} - i_{md}) - \omega \omega \text{md}_{\text{ext}} \]
\[ u_{sq,nl} = -\frac{m_{r,ref}}{T_r^2 \text{md}_{\text{ext}}} \frac{d}{dt} (i_{eq} - i_{md}) \]

**Figure 3** shows a conventional field oriented controller. Often the current controllers are of the PI type and the feedforward terms \( u_{sd,ff} \) and \( u_{sq,ff} \) are omitted. If we compare figure 2 with 3 it is seen that the conventional Rotor Field Oriented Control does not contain the \( u_{sd,nl} \) and \( u_{sq,nl} \) terms but has a PI type controller instead of a P type as in the nonlinear controller. In equilibrium the effect is the same in the two situations if an exact motor parameter knowledge is present, if not the Rotor Field Oriented Control will perform better. Normally the P type controller will be tune to a level where the stationary error will be negligible.

### 4 STABILITY ANALYSIS

Combining the above transformations give the system dynamics
\[
\begin{pmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3 \\
\dot{i}_{md} \\
\dot{i}_{mq}
\end{pmatrix} =
\begin{pmatrix}
-c_1 \dot{z}_1 + \frac{1}{T_r} \phi_1 \\
-c_2 \dot{z}_2 - d_2 \phi_2^2 \dot{z}_2 + \phi_1 \dot{i}_{md} + \phi_2 \dot{i}_{mq} \\
-c_3 \dot{z}_3 - d_3 \phi_2^2 \dot{z}_3 + \phi_1 \dot{i}_{md} + \phi_2 \dot{i}_{mq} \\
\frac{1}{T_r} \dot{i}_{md} + \omega \phi_2 \dot{i}_{mq} \\
\frac{1}{T_r} \dot{i}_{mq} - \omega \phi_2 \dot{i}_{md}
\end{pmatrix}
\]
This system has an equilibrium at $z_1 = z_2 = z_3 = \frac{i_{md}}{z_{ref}} = \frac{i_{mq}}{z_{ref}} = 0$

Furthermore the derivative of the Lyapunov function candidate

$$V = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2 + T_{f}(\frac{1}{d_2} + \frac{1}{d_3})(i_{md}^2 + i_{mq}^2))$$

along the solution of (2) is nonpositive

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - \frac{3}{4} \frac{1}{d_2} - \frac{1}{d_3}(i_{md}^2 + i_{mq}^2)$$

$-d_2(\phi_1 z_2 - \frac{1}{2d_2} i_{md})^2 - d_2(\phi_2 z_2 - \frac{1}{2d_2} i_{mq})^2$

$-d_3(\phi_1 z_3 - \frac{1}{2d_3} i_{md})^2 - d_3(\phi_2 z_3 - \frac{1}{2d_3} i_{mq})^2$

It is seen too that $\dot{V} \leq -W \leq 0$ with

$$W = c_1 z_1^2 + c_2 z_2^2 + c_3 z_3^2 + \frac{3}{4} \frac{1}{d_2} + \frac{1}{d_3}(i_{md}^2 + i_{mq}^2)$$

Assuming boundedness of the reference values and a stable control for $m_{ref}$ based on $\omega_{mec}$ then we have for $i_{md} > 0$ a solution giving $\lim_{-\infty} W \to 0$. Hence, the torque and field magnitude tracking objectives are achieved for any initial condition $i_{md} > 0$.

5 SIMULATIONS

In this section sensitivity due to temperature variations will be analyzed based on a simulation study. The nonlinear control method will be compared with the traditional rotor flux based Field Oriented Control method with d-q current control loops. Both methods will use the following flux estimator:

$$i_{mR}(t) = \frac{1}{1 + \frac{1}{\rho_{mec}}i_{e}(t)}$$

$$\rho(t) = Z_{p}\theta_{mec} + \int_{0}^{t} \frac{i_{e}(\tau)}{i_{mec}(\tau)} d\tau$$

which is defined for $i_{mR} \neq 0$.

The stator and rotor resistances changes considerably due to variation in temperature. For our test motor, a GRUNDFOS 1.1 kW induction motor, these variations can be calculated. The minimum and maximum values for the resistances are obtained for cold and hot motor respectively. The analysis of the sensitivity against variations are based on a simulation study.

The complexity of the simulation model is the same as assumed in the controller design so saturation effects, iron losses and other second order effects are omitted. In order to get realistic control conditions a 200μs delay is introduced in the voltages applied to the motor.

In the control systems a speed controller is introduced which is not shown in figure 4. In the nonlinear control strategy the pure differentiations in the decouplings are omitted, because differentiations are numerical unrobust operators in a control system due to noise.

In stationarity the operators do not contribute with any corrections.

The load is simulated as

$$J \frac{d\omega_{mec}}{dt} = m_{e} - m_{L}$$

The references in the simulation are changed as follows

$$N_{ref} = \begin{cases} 0 & \text{for } t < 0.1 \\ 2000 & \text{for } 0.1 \leq t \end{cases}$$

with the load torque $m_{L}$ equal to 0. The parameters used for the simulation are $J = 0.00140$ and $Z_{p} = 1$.

$$R_{s}, R_{r}, L_{m}, L_{a} - L_{m}, L_{r} - L_{m}$$

Nominal 6.48 0.335 0.0134 0.0190
Cold motor 4.49 4.70 0.335 0.0134 0.0190

The parameters in the controllers are based on nominal parameters for the motor.

Figures 5 and 6 show a comparison between the Nonlinear Backstepping method and the Rotor Field Oriented Control method for cold and warm motor.

As the figures show the two methods demonstrate nearly the same dynamic behavior. The Nonlinear Backstepping method shows distorted torque response for cold motor compared to the Rotor Flux Oriented Control strategy. This deviation is due to incorrect decoupling in the Nonlinear Backstepping method. In the RFOC strategy the PI controllers compensate for the modeling errors.

6 EXPERIMENTS

The simulations of the Nonlinear Controller is verified by experiments. Figure 7 shows the experimental results for heated motor. It is seen from the figure that the simulated response for the control strategy is close to the experimental result. The speed response
fits exactly, while there is a deviation in the torque estimation response. This deviation is due to the fact that friction losses are not included in the simulation model.

7 CONCLUSION

A new method based on nonlinear control theory has been compared to the traditional Rotor Field Oriented Control method. Compared to other strategies evolving from the nonlinear control theory reported in the literature this method do not need an adaptation of the motor load. Elimination of this need for adaptation implies that servo performance is present even at momentary load torque changes. The new method does not outperform the traditional Rotor Field Oriented Control method, but shows a systematic way of developing the strategy. The Backstepping method opens up for including second order effects into the control strategy, actually it is possible to assume that some parameters are unknown and an adaptive strategy for estimating the parameter will evolve from the theory.

This feature is not shown in this article but will be the topic for future work.

REFERENCES