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IMPACT OF SCHEDULING POLICIES ON CONTROL SYSTEM PERFORMANCE

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ABSTRACT

It is well known that jitter has an impact on control system performance, and this is often used as an argument for static scheduling policies, e.g. a time triggered architecture. However, it is only completion jitter that seriously disturbs standard linear control algorithms in a way similar to the delay inherent in a time triggered architecture. Thus we propose that standard control algorithms are scheduled dynamically, but without preemption. Analysis of this policy is contrasted with a corresponding time triggered architecture and is shown to have better impulse response performance both in the deterministic case and under white noise disturbances. The conclusion is that under very reasonable assumptions about robustness of control algorithms, they are insensitive to relase jitter, albeit strongly sensitive to completion jitter, thus priority based scheduling without *preemption is may be preferable for such systems.*

1. INTRODUCTION

Control systems are almost exclusively implemented on multitasked computers, where task scheduling is determined by a real-time scheduling policy. The overall performance of a system is therefore dependent on cooperation between the disciplines of control engineering and software engineering of hard real-time systems. Highly optimized solutions may be found by merging the disciplines [16, 17]; but the solutions are usually highly specialized, and requires a development effort by highly skilled people with a knowledge spanning both areas. In most development organizations, control and scheduling are done by different specialists, thus a smooth cooperation depends on a clear division of responsibilities - a contract. The basis of a standard contract may be phrased as follows:

> The control engineers will deliver a collection of tasks to be executed periodically. Each task is characterized by its period T and its worst case computation time C.
>
> The software engineer promises to execute each task periodically with the given period and with sufficient computation resources for the worst case computation time.

When both parties are conservative, i.e. the control engineers use very robust algorithms and the software engineers load the computers moderately, the cooperation works well, and there is no need to elaborate the contract. However, when we want efficient solutions, the fine print of the contract becomes important. What does it mean to execute a task periodically?.

The interpretation of the control engineer is that the task is released and reads its inputs from sensors at times r_k for $k = 0, 1, \ldots$, where the release times are equidistant $r_{k+1} - r_k = T$, and where the first release r_0 happens within the first period of time. Furthermore, the tasks are assumed to be completed at times $c_k = r_k + C$, when outputs to actuators has been done. In some cases, controls are developed assuming that computations take no time, i.e. $C = 0$; but this assumption is clearly the responsiblity of the control engineer and does not influence the common contract.

The software engineer, who applies a scheduling policy, has another interpretation. A periodic task shall be executed once within each of the periods $[0, T], [T, 2T], [2T, 3T], \ldots$ (Here we make the usual assumption that the deadline for a task is the same as its period.) Tasks may thus actually be released and completed at times $r'_i, c'_i \in [T \cdot i, T \cdot (i+1)]$. The differences $J^r_i = r_i - r'_i$ and $J^c_i = c_i - c'_i$ are respectively the *release* and the *completion jitter*. Generally a maximum release jitter may be guaranteed, i.e. $J^r_i \leq \delta$ as well as a maximum delay D between release and completion, i.e. $D_i = c_i - r_i \leq D$.

The fine print of an extended contract will explain, how jitter is handled. For the control engineer, several compensation mechanisms are possible [11]. They are typically based on per sample modification of the applied control law, which requires knowledge of the actual jitter. The software engineer may also choose scheduling disciplines that makes jitter predictable. When we consider popular scheduling policies, we have the following characteristics:

Static scheduling: With a preplanned schedule, there is no jitter, or at most a small, amount due to asynchronous interrupts.

A special case is the time triggered architecture [19, 3], where, similar to PLC-controllers, input is read at the start of a period and output is produced at the end. Both operations are assumed to take no time, which in general is reasonable. One can say that in some sense, the release jitter is minimized at the cost of maximized delay.

Dynamic scheduling: Here tasks are assigned priorities according to some criteria, see e.g. [7, 4, 5] for deadline montonic, rate monotonic or other policies. When there are only periodic tasks and when computation times do no vary, there is no more jitter than in a static schedule; but in most cases, where dynamic scheduling is used, because it adapts better to situations with aperiodic tasks or server tasks, jitter may become difficult to control. However, if scheduling is done without preemption, delay almost is eliminated, at the cost of introducing blocking and in turn increasing release jitter for higher priority tasks.

When a system is developed, there are thus the following options:

1. Use static scheduling or the time triggered approach and pay the price in the form of complex analysis of the concrete configurations.
2. Use dynamic scheduling and complicate real time control by jitter and or delay compensation.

The thesis of this paper is that we can avoid the complexities for standard proportional control systems by using dynamic scheduling without preemption. In this very common case, our analysis shows that control performance is almost unaffected by release jitter, and that the control performance is better than the one that can be expected from a comparable time triggered scheduling policy.

Overview

The following section surveys related work, while Section 3 analyses the effect of release jitter on simple first order proportional control systems, as well as the effect of delay, as introduced in time triggered architectures. Jitter and delay are analysed from deterministic and stochastic viewpoints. Section 4 provides a generalization to higher order systems of the approach presented in the previous section. Section 5 concludes and indicates directions for future research.

2. RELATED WORK

The study of sampled control systems is by no means new. However the special case for irregular sampling due to real time scheduling seem to gain much interest recently. The paper [8] and the report [9] provide overviews of the problem and surveys of related work within real time systems and control engineering. The article [10] considers sampling delay and jitter and a rational model for varying delay is presented along with an accompagning robust controller design based on μ-synthesis [15]. The effect of limited sampling jitter is illustrated by an example of a double integrator. In [11] an online jitter compensation is introduced; it uses per sample recalculation of control law parameters based on timestamps. Optimal ressource distribution to control tasks is investigated in [12] and [1]. In [1] the time triggered approach is adopted as a basis for an optimization scheme yielding optimal sampling periods under various preemptive scheduling disciplines. Optimality is defined on the basis of an appropriate objective function reflecting system robustness w.r.t. stability margins. The paper [18] presents a technique to bound release jitter, based on the modification of task temporal parameters in DM and EDF scheduling. In [13] we find stability analysis for sampled systems with non-ideal sampling, where both jitter and delay is considered. The sampling proces is considered for 3 cases: constant, periodic and general. In the general case a simple common one sample criterion is assumed for the transition matrix.

In comparison, our work provides analytical results for the impact of jitter on linear control system comprising first and higher order continuous time systems equipped with state proportional discrete time controllers. We advocate a simplistic approach allowing release jitter whereas delay is considered tightly bounded since non preemptive scheduling is assumed. As in [1] we consider neither redesign nor on line adaptation to accomodate for sampling irregularities. We choose the time triggered approach from [1] as a paradigm for comparison and provide robustness analysis for jittered sampling similar to [13], however, our analysis of the sample process is more general than presented in [13].

3. FIRST ORDER CONTROL SYSTEMS.

Consider a first order continous time dynamic system evolving in time according to differential equation (1)

$$\dot{x} = A \cdot x + B \cdot u \tag{1}$$

A ZOH-equivalent continuous to discrete transformation [6] produces the following discrete time system

$$x_{k+1} = F_T \cdot x_k + G_T \cdot u_k \tag{2}$$

wher F_T and G_T are given by

$$F_T = exp(AT) \tag{3}$$

$$G_T = \frac{B}{A} \cdot (exp(AT) - 1) \tag{4}$$

and T is the nominal time between samplings k and $k+1$. Assume that a discrete time state space control law K is derived yielding a closed loop system

$$x_{k+1} = F_T \cdot x_k - G_T \cdot u_k = (F_T - G_T \cdot K) \cdot x_k = Q_T \cdot x_k \tag{5}$$

where Q_T is given by

$$Q_T = exp(AT)(1 - \frac{KB}{A}) + \frac{KB}{A} \tag{6}$$

As an example let $A = -0.02, B = 1, T = 1$ and the nominal pole placement be $Q_T = 0.8$ then $K = 0.18$. Thus by proportional feedback the bandwidth of a stable first order system is increased approximately be a factor 10 which seems reasonable.

3.1. Deterministic analysis of jitter

Suppose sampling times vary over time, so that the time between samplings k and $k+1$ is now T_k, then the following solution to (5) is found

$$x_k = \Pi_{j=0}^{k-1} Q_{T_j} \cdot x_0 \tag{7}$$

where $T_j = r'_{j+1} - r'_j$. Assuming $J_i^r \leq \delta$ for some positive real number δ, we obtain

$$m\ T - \delta \leq \sum_{i=k}^{k+m-1} T_i \leq m\ T + \delta \quad \text{for all } k, m \geq 0 \tag{8}$$

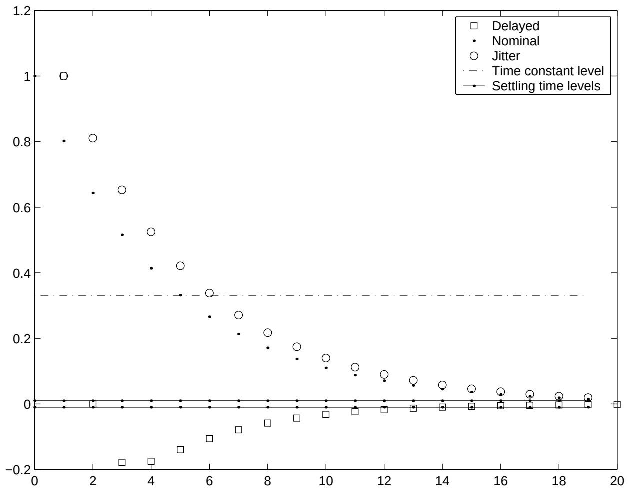

Figure 1: Nominal, delayed and jittered step error responses.

We consider error convergence in response to a reference value change at $t_0 > 0$. The first sampling instant following the change may ultimately be delayed δ. After the first sampling, convergence may be bounded as in inequality (9). Assuming $Q_{T_j} > 0$, Q_{T_j} is decreasing in T_j, so for the constraints (8) there is a symmetric interior maximum $T_j = T - \frac{\delta}{k}$ for x_k on the hyperplane $\sum_{j=0}^{k-1} T_j = kT - \delta$, i.e.

$$\begin{aligned} x_k &= x_0 \ \Pi_{j=0}^{k-1} Q_{T_j} \\ &= x_0 \ \Pi_{j=0}^{k-1}(exp(AT_j)(1 - \frac{KB}{A}) + \frac{KB}{A}) \\ &\leq x_0 \ \bar{x}_k \\ &= x_0 \ (exp(A(T - \frac{\delta}{k}))(1 - \frac{KB}{A}) + \frac{KB}{A})^k \end{aligned} \quad (9)$$

The above reasoning is based on the assumption that $Q_{T_j} > 0$ which however, is considered to be a rather mild restriction. For the simple example above, the bound for T_j ensuring $Q_{T_j} > 0$ would be $T_j \leq 5.26\ T$, which is equivalent to occasional vacations of the sampling process lasting more than 5 times the nominal sampling period. The latter is clearly quite significant.

An overall upper bound for error convergence is found by the pairs $(t_0 + \delta + kT, x_0\bar{x}_k)$ as shown in Figure 1, where $\delta = T$ is assumed for the above example to make it comparable with the delayed situation.

3.2. Deterministic analysis of time triggered control

Fixed priority scheduling along with the celebrated RMA priority assignment [2] as well as EDF schdeduling [2] provide conditions for completion jitter below task periods, i.e. T. When control dynamics are slow compared to the nominal sampling rate such an approach may work satisfactory, but it may also deteriorate even reasonably robust nominal designs significantly.

Introducing a one period in loop delay to the example from the previous section results in the second order system of equation (10)

$$\begin{aligned} x_{n+1}^1 &= x_n^2 \\ x_{n+1}^2 &= -K\ G_T\ x_n^1 + F_T\ x_n^2 \end{aligned} \quad (10)$$

with a resulting pole pair $(0.23, 0.73)$. Impulse responses for the nominal system and its delayed counterpart in Figure 1 show a significant overshoot of about 20 % introduced by delay.

Delay compensation can for fixed delay of one single or an integer number of sample periods be performed through the well known Smith predictor governed by the dynamics in equation [14].

$$\begin{aligned} \hat{x}_{k+1} &= F_T \cdot \hat{x}_k + G_T \cdot u_k \\ u_k &= -K(x_{k-1} - \hat{x}_{k-1} + \hat{x}_k) \end{aligned} \quad (11)$$

The result of compensation is also shown in Figure 1 revealing the expected closed loop response with an additional delay of 1 sample period. Ideally more advanced control designs should accompany application of the Smith predictor as pointed out in [14]. However we believe the rather simplistic approach above is justified for comparative reasons since we compare approaches of similar complexities.

3.3. Comparison for deterministic analysis

When $Q_{T_P} > 0$, no overshoot is introduced by release jitter. Thus convergence speed is simply stated in time constant or settling time terms, i.e. $t^* = \min\{\delta + kT \mid \bar{x}_k \leq \alpha\}$ where α assumes values 0.33 or 0.01 for time constant or settling time respectively. As seen in Figure 1 jitter increases the time constant from 5 to 6 sampling periods and settling time is approximately unchanged from the nominal case. It must be noted that the delayed impulse response is superior to both the nominal and the jittered ones when observing time constant and settling time alone. However in many cases overshoot is undesirable or even hazardous, as in positional controllers for mechanical systems, and in general overshoot may indicate robustness problems, i.e. the ability to maintain stability under system uncertainties. All together the jittered response is by most standards far closer to the nominal design than the delayed one. Application of the Smith predictor produces a result almost identical to the worst case jittered response.

3.4. Stochastic analysis.

Disturbances are incorporated into the state space model by restating equation (1) as a stochastic differential equation, i.e.

$$dx = A \cdot x + B \cdot u + dw \quad (12)$$

where w is a standard brownian motion. Let t_k and t_{k+1} be separated by T_k in time and define x_k by

$$x_k = x(t_k) \quad (13)$$

then the following stochastic discrete time model is obtained from a ZOH transformation

$$x_{k+1} = F_T \cdot x_k + G_T \cdot u_k + \int_0^{T_k} e^{A\cdot(T_k - \tau)} dw \quad (14)$$

The last term is readily shown to be an independent Gaussian random variable w_k with a variance $C_w(T_k)$ given by

$$C_w(T_k) = \int_0^{T_k} e^{2A\cdot(T_k-\tau)} d\tau \tag{15}$$

Introducing the discrete time state space control law K the following approximative closed loop model is obtained

$$x_{k+1} = Q_{T_k} x_k + \sqrt{C_w(T_k)} \cdot w_k \tag{16}$$

where w_k is a standard Gaussian variable. Computing variances yields

$$\sigma_{k+1}^2 = Q_{T_k}^2 \sigma_k^2 + C_w(T_k) \tag{17}$$

In general, it is difficult to see which pattern of $\{T_j\}$ maximizes (17). We shall proceed less general. We assume $\delta = T$ and deduce results valid for the above example. In that case all sequences $\{T_1, .., T_n\}$ reside within the sets $D_n \subset R^n$ defined by

$$\begin{aligned} D_n \quad &= \quad \{(T_1, .., T_n) \mid (k-1)T \\ &\leq \quad \sum_{j=i}^{i+k-1} T_j \leq (k+1)T \ \forall (i,k) \mid i,k \geq 1, i+k-1 \leq n\} \end{aligned}$$

The variance $\sigma_n^2 = \phi(T_1, .., T_n) + \Pi_{j=0}^{n-1} Q_{T_j}^2 \cdot \sigma_0^2$, where ϕ is independent of σ_0^2. Thus under stability assumptions for sequences $\{T_j\}$ where $(T_1, .., T_k) \in D_k$, $\lim_{n\to\infty} \sigma_n^2 = \phi(T_1, .., T_n)$, i.e. the effect of initial conditions disappear. Next we define $\sigma_n^2(T_1, .., T_n)$ by

$$\sigma_n^2(T_1, .., T_n) = \phi(T_1, .., T_n) + \Pi_{j=0}^{n-1} Q_{T_j}^2 \cdot M \tag{18}$$

where

$$M = \frac{C_w(2T)}{1 - Q_{2T}^2} \tag{19}$$

i.e. $\sigma_n^2 = \sigma_n^2(T_1, .., T_n)$ for the case $\sigma_0^2 = M$. We define M_n by

$$M_n = \max_{(T_1, .., T_{2n}) \in D_{2n}} \sigma_{2n}^2(T_1, .., T_{2n}) \tag{20}$$

and likewise

$$g(x,y) = C_w(x) + Q_x^2 \ C_w(y) + Q_x^2 \ Q_y^2 \ M \tag{21}$$

Assume $M_{n-1} = M$ and let the real sequences S_k be defined by $S_k \in R_+^{2k}$, $S_k(i) = 0$ for $i = 0, 2, .., 2(k-1)$ and $S_k(i) = 2T$ for $i = 1, 3, .., 2k-1$

Since $S_n \in D_{2n}$

$$M_n \quad \geq \quad \sigma_{2n}^2(S_n) = g(2T, 0) \tag{22}$$

By definition of g in (21)

$$M_n \leq \max_{(x,y) \in D_2} g(x,y) \tag{23}$$

It can be verified for the above example that

$$\max_{(x,y) \in D_2} g(x,y) \quad = \quad g(2T, 0) \tag{24}$$

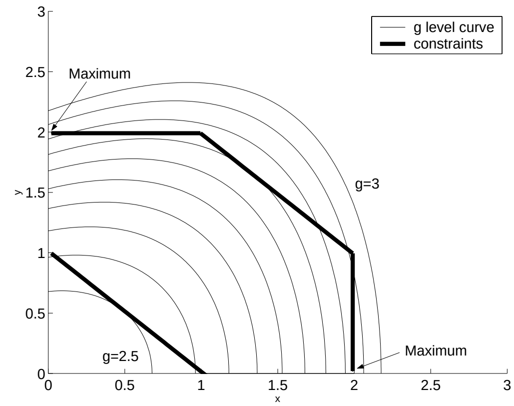

Figure 2: Maximizing the function g within D_2

as illustrated in figure (2). So by inequalities (24) and (22) $M_n = g(2T, 0) = \sigma_{2n}^2(S_n) = M$.

Since

$$\lim_{n\to\infty} \sigma_n^2 = \lim_{n\to\infty} \sigma_n^2(T_1, .., T_n) = \phi(T_1, .., T_n) \tag{25}$$

M asymptotically bounds σ_n^2 independent of the initial variance.

For the example above, the variance in the nominal case would amount to $\frac{C_w(T)}{1-Q_T^2} = 2.74$. The above analysis gives an assymptotic variance bound of $M = 3.05$ assumed for sampling period sequences $\{0, 2T, .., 0, 2T\}$ equivalent to the case where each second sample is released exactly one sample period to late or to a periodic sampling with a double period.

With time triggered feed back, analysis is carried out through standard matrix analysis of the second order system (26)

$$\begin{aligned} x_{n+1}^1 \quad &= \quad x_n^2 \\ x_{n+1}^2 \quad &= \quad -K \ G_T \ x_n^1 + F_T \ x_n^2 + w_n \end{aligned} \tag{26}$$

where w_n is a normally distributed random variable with zero mean and variance $C_w(T)$ it can be found that the error variance amounts to 3.28 for the above example, which is a significant increase from the nominal case. In this example, jitter of one sample period increases error variance only half as much as a time triggered delay of one sample period. Performing similar computations for an equivalent system with Smith predictor, as described in equation (11), yields 3.7 for the stationary output variance. A simplistic use of the Smith predictor in conjuction with a time triggered sampling approach should therefore be strongly discouraged, whereas more advanced control designs may yield highly improved performance as pointed out in [14].

4. GENERALIZATION TO HIGHER ORDER SYSTEMS.

Analysing simple systems may yield insight and give guidelines for a more general approach. In this section we shall provide a generalization to higher order systems as well as a general approach to stochastic analysis with arbitrary jitter bound δ.

Consider a continous time dynamic system evolving in time according to differential equation (27)

$$\dot{x} = A \cdot x + B \cdot u \tag{27}$$

A ZOH-equivalent continuous to discrete transformation produces the following discrete time system

$$x_{k+1} = F_T \cdot x_k + G_T \cdot u_k \tag{28}$$

wher F_T and G_T are given by

$$F_T = exp(AT) \tag{29}$$

$$G_T = \int_0^T exp(A(T-t))dtB \tag{30}$$

T is the nominal time between samplings k and $k+1$ and $exp(\cdot)$ is the matrix exponential. Assume that a discrete time state space control law K is derived giving a nominal closed loop system

$$x_{k+1} = F_T \cdot x_k + G_T \cdot u_k = (F_T - G_T K) \cdot x_k = Q_T \cdot x_k \tag{31}$$

for $Q_T = F_T - G_T K$

4.1. Deterministic Analysis.

We shall proceed with the aid of the following first order approximation for Q_T

$$Q_T = I + A \cdot T - B \cdot K \cdot T = I + T \cdot (A - BK) \tag{32}$$

which is valid whenever $|AT| << 1$. Then the approximate eigenvalues of Q_T are found to be

$$\lambda_T = (1 + T \cdot \lambda) \tag{33}$$

where λ is a corresponding eigenvalue of $A - BK$. Up to a first order approximation eigenvalues of Q_T match those of $A - BK$, i.e. they are constant. Assuming K to be chosen so that $A - BK$ has distinct eigenvalues, a basis of eigenvectors $V = [v_1, v_2, .., v_n]$ exists. Defining principal outputs z by $z_k = V^{-1} x_k$ we obtain in correspondence to equation (7)

$$z_k^i = \Pi_{j=0}^{k-1} \lambda_{T_j}^i \cdot z_0^i \tag{34}$$

where $\lambda_{T_j}^i = (1 + T \cdot \lambda^i)$, i.e. a 1. st order approximation of the i th. eigenvector of Q_{T_j}. Thus a modulus bound on z_k is

$$|z_k^i| = \Pi_{j=0}^{k-1} |\lambda_{T_j}^i| \cdot |z_0^i| \tag{35}$$

For a well damped nominal design and low values of $T \cdot |\lambda^i|$, $|\lambda_T^i|$ is decreasing approximately affinely with T, so as for the one dimensional case, there is a symmetric interior

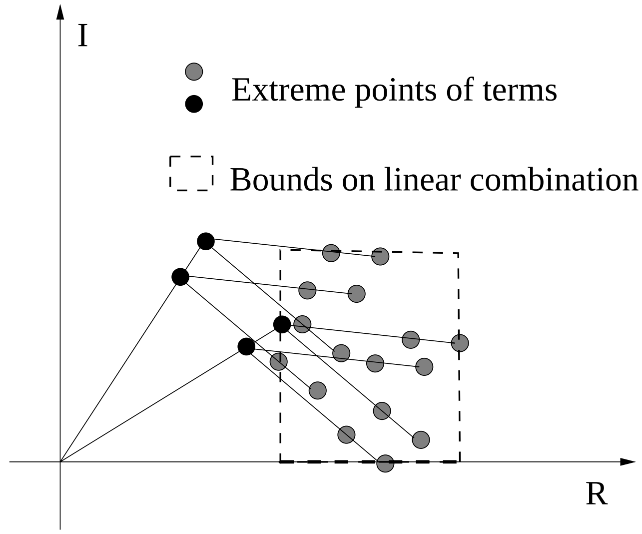

Figure 3: Computing bounds for complex linear combination.

maximum $T_j = T - \frac{\delta}{k}$ for $|z_k^i|$ on the hyperplane $\sum_{j=0}^{k-1} T_j = kT - \delta$ for the constraints (8). Thus a closed form modulus upper bound is given by

$$|z_k^i| \leq |1 + (T - \frac{\delta}{k}) \cdot \lambda^i| \cdot |z_0^i| = \bar{z}_k^i \tag{36}$$

Lower modulus bound are found by realizing that products attain minimum values in extreme points of the constraint set. So generally we have

$$|z_k^i| \geq |z^i(m\ T)|^{\lceil \frac{k}{m} \rceil} \tag{37}$$

for $(m-1)\,T \leq \delta \leq m\,T$. Inequality (37) expresses modulus bounds $m\ T, 0, .., 0, m\ T, 0, .., 0, m\ T, 0, ..$
For the phase ϕ_k^i of z_i^k we have

$$\phi_k^i = \sum_{j=0}^{k-1} \angle \lambda_{T_j}^i + \angle z_0^i \tag{38}$$

and a corresponding first order approximative bound

$$k \angle \lambda_T^i + \Lambda_T^i \delta + \angle z_0^i \leq \phi_k^i \leq k \angle \lambda_T^i - \Lambda_T^i \delta + \angle z_0^i \tag{39}$$

where Λ_T^i denotes the first order derivative of λ_T^i w.r.t. T, computed at the nominal design values. Upper and lower bounds for system states x_k are found by inspecting $x_k = V z_k$ for maximal and minimal values, as illustrated in figure (3). The required linear combination of complex numbers is performed every combination of extreme values of modulus and phase.

4.2. Example: Mechanical system.

Working cycles of mechanical automata are frequently defined by stepwise positional changes. Transitions should most often comply to specifications on rise time, settling

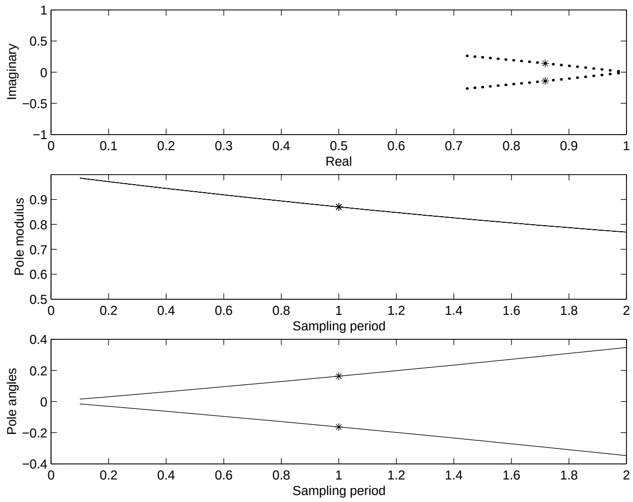

Figure 4: Root locus and corresponding moduli and angles for varying sample period in mechanical system.

time and overshoot. Between transitions reference positions should be maintained in the presence of mechanical disturbances in the shape of random forces acting on the system.

Our example system is defined in state space form by

$$\dot{x} = A \cdot x + B \cdot u + C \cdot w \tag{40}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.2 \end{bmatrix} \tag{41}$$

and $B = C = [0\ 1]^T$. State vector components are position and velocity respectively. For nominal pole placements $(1 + 0.2 \cdot exp(-i \cdot 3/4 \cdot \pi), 1 + 0.2 \cdot exp(i \cdot 3/4 \cdot \pi))$ a feedback vector $K = [0.05\ 0.1]$ is obtained. Root (pole) locus and corresponding moduli and angles for sample periods varying over $[0, 2T]$ are shown in Figure 4, to validate the first order approximation conducted. In Figure 4 the nominal design is indicated with "*".

Step response error results for the nominal design, the time triggered approach as well as upper and lower bound for the jittered approach and $\delta = T$ are found in Figure 5.

As seen in Figure 5 jitter may increase the time constant and overshoot. Overshoot from one time delay is however even higher, though not significantly. Allthough the example is carried out for $\delta = T$ the approach presented remains generally valid within the validity domain of the first order approximations conducted.

4.3. Stochastic analysis.

We obtain the following recursion for the covariance matrices of a jittered system

$$\begin{aligned} \Sigma_{k+1} &= \Sigma^{(1)}(\Sigma_k, T_k) \\ &= Q_{T_k} \Sigma_k Q_{T_k}^T + N(T_k) \end{aligned} \tag{42}$$

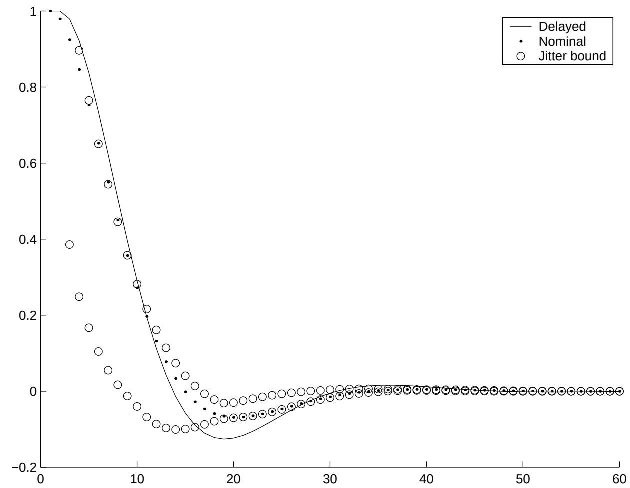

Figure 5: Nominal, delayed and jittered step responses of mechanical system.

where

$$N(T) = \int_0^T exp(A(T-t))CC^T(exp(A(T-t)))^T dt \tag{43}$$

for which a close form expression is available by realizing the the eigenvectors of $exp(At)$ remain constantly identical to the eigenvectors of A and the eigenvalues of the matrix exponential are found by taking exponentials of the eigenvalues of A. We let M denote the set of, real symmetric, positive definite matrices of appropriate dimension, i.e. $\Sigma_k \in M$. Higher powers of $\Sigma^{(1)}(,)$ may be recursively defined by

$$\Sigma^{(n+1)}(\Sigma_k, T_{k+n}, .., T_k) = \Sigma^{(1)}(\Sigma^{(n)}(\Sigma_k, T_{k+n-1}, .., T_k), T_{k+n}) \tag{44}$$

A recursive expression for release jitter by the sampling intervals T_k is given by a nondeterministic automaton in the shape of a double *token bucket* filter. We define the bucket heigth $\bar{c}_k$ recursively by

$$\bar{c}_{k+1} = min\{\delta, max\{0, \bar{c}_k + T_k - T\}\} \tag{45}$$

and T_k nondeterministically by $\bar{c}_k + T_k - T \leq \delta$, then for $\bar{c}_0 \in [0, \delta]$ the automaton generates exactly all sequences $\{T_k\}$, where $\sum_{i=k}^{k+m-1} T_i \leq m\ T + \delta$. Conversely a bucket depth $\underline{c}_k$ is defined recursively by

$$\underline{c}_{k+1} = max\{-\delta, min\{0, \underline{c}_k + T_k - T\}\} \tag{46}$$

and $\underline{c}_k + T_k - T \geq -\delta$. In this case for $\underline{c}_0 \in [-\delta, 0]$ the automaton generates all sequences $\{T_k\}$, where $\sum_{i=k}^{k+m-1} T_i \geq m\ T - \delta$. Combining (45) and (46) and requiring

$$-\delta + T - \underline{c}_k \leq T_k \leq \delta + T - \bar{c}_k \tag{47}$$

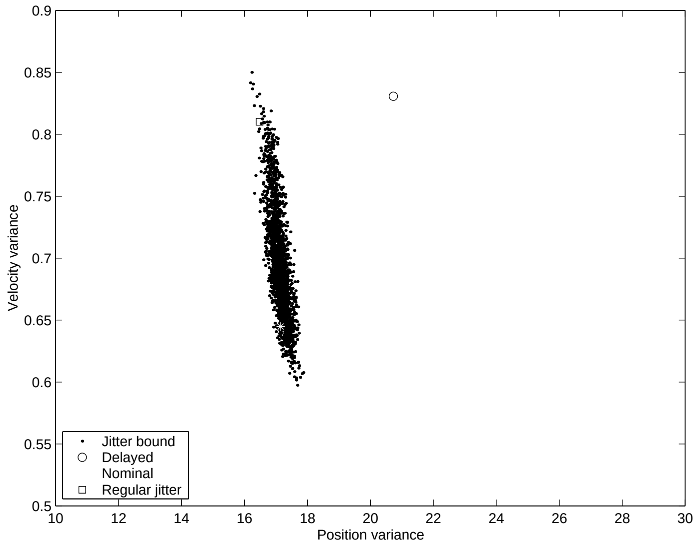

Figure 6: Stationary variances of nominal, jittered, delayed and regularly jittered systems.

our automaton generates exactly sequences where $m\,T-\delta \leq \sum_{i=k}^{k+m-1} T_i \leq m\,T+\delta$, i.e. the sampling sequences fulfilling (8). Including the covariance matrix defined in (42) into the state of our automaton, we have a state vector $[\underline{c}_k, \bar{c}_k, \Sigma_k]$ of a nondeterministic automaton defined by (42),(45),(46) and (47).

We define the state $S_0 = [0, \delta, \Sigma]$ where Σ uniquely solves $\Sigma = \Sigma^{(1)}(\Sigma, T)$. When for some $K > 0$ $\Sigma_K(\cdot, T_{k+K-1}, .., T_k)$ is a contraction for all sequences fulfilling (8), then the reachable set from S_0 is bounded. For stable nominal designs it is readily verified that S_0 is globally assymptotically reachable.

Now $\Sigma^{(1)}(\cdot, T)$ is continous for all $T \in [0, \delta]$. Consider some state S reachable from S_0 and an ϵ-neighbourhood of S. Then there exists a μ-neighbourhood B_μ of S_0, so from all states in B_μ some state within B_ϵ is reachable. Alltogether define I as the set of all states assymptotically reachable from S_0, then all states in I are mutually assymptotically reachable. Thus I is the closure of a unique smallest invariant set of the defined non deterministic automata.

Obtaining feasible hard bound estimates of I consitutes a challenge left for further research. However random explorations where T_k is drawn uniformly within the limits of (47) yields a finite dimensional irreducible, ergodic Markov chain Γ with a stationary distribution concentrated on I. Additonally for every interior point $r \in I$, there is a neighbourhood with positive measure. Thus empirical distributions of Γ may constitute feasible I estimates. Results of such an approach for the mechanical system defined above for $\delta = T$ are shown in Figure 6

Along with results from nominal, jittered and delayed systems, results from a regular jitter pattern is also shown in Figure 6. Jitter and delaying produces approximately the same increase in velocity variance, whereas delaying yields a significantly higher positional variance.

5. DISCUSSION AND FUTURE WORK

In the above we considered different approaches to the implementation of discrete time controllers on multitasking platforms. We argue that the time triggered approach and pre emptive scheduling may introduce undesirable performance drawbacks such as decreased robustness w.r.t. stability, overshoot and additional control error variance. An alternative approach based on dynamic *non preemptive* scheduling and thus allowing significant release jitter but bounding delay in the feedback loop is proposed. A typical proportional control system is analyzed under both schemes, and results strongly support the initial thesis that the non preemptive priority based scheduling performs better than a time triggered approach for standard control algorithms. The nonpreemptive approach alters the deterministic system response only insignificantly and overshoot is hardly possible as opposed to the time triggered approach. The effect of white noise disturbances is investigated and the proposed approach again performs significantly better than its time triggered counterpart. Deterministic and stochastic analysis of the effect of jitter in higher order systems is presented and an example mechanical system is presented. Both deterministic and stochastic results point in favour of the non preemptive approach.

Altogether we find evidence that nonpreemptive priority based scheduling is well suited for control algorithms. In contrast to a static schedule, well known priority assignment schemes can be used. A slight difficulty is the default preemptive implementation of control tasks found e.g. in POSIX compliant operating systems like RT-Linux. Non preemptive kernels are typically lighter than their preemptive counterparts and thus more suitable for embedded applications. From a performance, view point, context switching is typically lighter in non preemptive systems. Preemptivenes typically generates platform dependence, so migrating preemptive kernels to alternative embedded environments may by unessesarily tedious.
The example systems presented are simple though representative relevant control system existing. The results presented in this work call for significant generalization higher order controllers, e.g. with integral action or oberver schemes, coloured noise disturbances, measurement noise, open loop unstable systems and even non linear systems. The immediate direction of our future work point towards the analysis of higher order state controlled systems with observers and controllers with integral action.

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