Upgradeability and Predictability Analysis for Mesh Topologies in Optical Distribution Networks

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Abstract—The new tendencies for network planning methods are evolving to more mechanized and automated systems that can determine the best way to deploy networks. In the case of fiber optic networks, aspects such as the length of the physical links are critical due to the high cost and high availability requirements of this kind of networks. Regarding the planning of optical networks, FTTH methods are reasonably advanced, but in the case of core or distribution networks there is still room for many improvements. In this paper, two mesh topologies have been studied in order to be deployed as optical networks, Honeycomb and Grid. In order to contribute to the improvement of distribution network design, the benefits of a preventive planning (upgradeability properties) are depicted along the document. In addition, to simplify the process of topology decision, approximations are defined to avoid some costly parameters calculations. For the validity of the results it is important to mention that Genetic algorithms are used in order to decide the interconnection of the networks under study.

I. INTRODUCTION AND MOTIVATIONS

Optical networks are expensive to deploy, the civilian construction cost can be up to 70% of the total investment [1]. Therefore, it is critical to improve the network planning methods. Future networks should be all optical systems, from the core to the last mile and development trend should follow this direction. Distribution networks are traditionally mostly deployed as rings that, currently, are sufficient, but in future, higher degree physical networks might be necessary [2].

Higher degree regular topologies have been proposed as optical networks for future perspectives. The higher the degree of the structure, the lower the possibility of losing connectivity in the network implying, lower unavailability periods of the system. Regular topologies, including mesh topologies, provide high performance due to what is known as high SQoS (Structural Quality of Service). SQoS is the benefit of using the mathematical properties of network topology to improve the global performance [3].

This paper presents the study of two mesh topologies, Honeycomb and Grid in terms of two different but related topics: Upgradeability and Predictability.

The first objective is related to upgradeability, defined as the capability of modifying a network structure to improve the performance in an easy and efficient way. Honeycomb topologies can be upgraded to Grids or the opposite, Honeycombs can be formed by removing links from Grids. The study illustrates the difference between designing a network as an optimal Honeycomb and then add some links to form a Grid and the opposite situation, designing it as an optimal Grid and remove links to form a Honeycomb. Optimal is referred to as the shortest physical network in this case. This kind of way of planning can be defined as Preventive Design.

Optical networks implementation time is relatively long, up to 15 years, and their expected life time can go up to 40 [3]. In this period, many unexpected situations, that might require an upgrade of the structure, can take place, such as killer applications or high availability demand services. Thus, the planning and design of optical network should consider these possibilities in order to avoid bottlenecks, highly expensive network modifications or complex and low-efficient node degree increment.

The second objective consists on proving and illustrating that the physical length of networks is predictable when they are implemented as one of these two topologies. This property is defined as Predictability in this study. Network length approximation formulas are presented along the document together with their accuracy to the real values. The key property of these approximations is that the length of the network can be predicted just by the information of the area covered by the nodes. The specific interconnection between the nodes is not necessary.

The planning process is a combination of long and complex tasks, topology decision, layout of the lines using GIS (Geographical information System) or facility location are examples. Hence, more automated and systematic methods will improve the time consumption, resources usage and investment on the planning stage. Ideally, the goal of previous, current and future research on this topic is to provide shortcuts to be able to identify the potential structures to deploy a network in a given region, being able to discard or accept solutions expending as few resources as possible on this stage. For example, to calculate the length of the links of a network, without having any methodology, it would be necessary to make the whole planning for each of the potential solutions. This would require a lot of effort in solutions finally not used, so this planning method is not efficient. Instead, by using regular topologies and finding approximations, the length of a network can be predicted in a very simple way. Examples of this kind of studies are the ratio between Euclidean and real road distance between two points in a map [4] or the approximation of regular topologies length based on the area covered by a set of nodes to be connected [5].

The rest of the paper is as follows: Section II briefly introduces the topologies under study. Section III depicts the
concepts related to the Genetic Algorithms and methodology to perform the study. In Section IV the results are presented. Finally Section V exposes the conclusions of the work.

II. BACKGROUND

Honeycomb structures (HC for the rest of the document, see Fig. 1.a) are planar mesh networks with the capability of being upgraded to Grids (GR for the rest of the document, see Fig. 1.b) just by adding a few links. For more information about this topology it is strongly recommended to see [6].

This fact is highly applicable to the optical networks since in early stages of the deployment, a network can be implemented as a HC but always having in mind the possibility of an organized upgrade in case of being required. This option of being able to implement an organized structure even when the network is not completely finished allows an early advantage of the regular distribution and interconnection of the nodes. But only a set of the HC configurations are suitable to be upgraded as GR, as explained in depth in Subsection III-A, this work is limited to this specific group.

GR are planar mesh topologies that have been proposed for many different scenarios such as fibre optic backbone structures [1], wireless networks such as ORBIT [7] or for energy saving on sensor networks [8] (wireless). Part of this work is motivated by [5], where several network topologies were used to find approximations of the total length of a network deployment. In this study the Grid topology approximation was not as satisfactory as expected, the error on the approximation was rather high. Therefore, approximations must be improved. This problem is solved, or at least the average error is lowered including a second approximation based on the dimensions ratio (vertical/horizontal dimensions) of the coverage area. This comparison is treated in Section IV.

III. METHODOLOGY

A. Topologies Upgrade

Preventive planning might simplify and lower the cost of upgrading a network. The design of networks using the two topologies under study should consider their upgradeability characteristics. For example, for a given region, it is required to build a network forming a HC. If at some point the network needs an upgrade it should be ready to be transformed, in this case it would be into a GR structure.

The same idea is applicable to the opposite situation, a network for a region is meant to form a GR. In order to be able to use and provide a reasonable performance network, it would be beneficial to implement first the links that will form an organized structure, in this case it can be HC and while the last links are being installed the network is perfectly operable.

Not all the HC configurations are suitable to be easily upgraded into GR, only specific HC configurations are studied in order to provide a Preventive Design. The following examples illustrates the proper HC configuration in order to satisfy the upgradeability requirements, the HC figures are presented in “Brick” format [6] for a better understanding.

Fig.2.a presents a perfectly correct HC configuration, but when upgraded to a GR some difficulties are found. Fig.2.b illustrates one of the ways of upgrading the network, the dotted lines are the additional links to form a GR. The main constraint when upgrading an optical network is the maintenance of the planar characteristics so there is no lines crossing to each other. The most clear consequence of this simple upgrade is that the resulting GR is not square, directly implying that the number of degree two nodes is not minimized. Non square GR are not necessarily a bad option only in scenarios where the location of the nodes requires to implement it due to budget constrains with the consequence of lowering the availability of the system by having more than four degree two nodes. In this study the goal is to study the upgradeability of HC into square GR, but in future it might be interesting to include any type of GR structure in the analysis.

It is possible to solve the degree two nodes problem but it is not efficient. Fig.2.c presents the solution, some of the links used for the HC will not be part of the GR (double-lined links). Hence, more new links must be implemented (dotted lines) and the investment for those not forming part of the final topology is misused. It would not be economically optimized.

The proposed configuration is presented in Fig.3.a, it can be defined as any HC with odd number of rows (mandatory for any HC) and even number of columns or vice versa. All the rows should have the same number of nodes and the same for the columns. In future, other possible HC distributions or degree three structures might be interesting to discuss in order to be used with other values of number of nodes. The opposite situation might be interesting as well, what kind of degree four topologies are feasible to be upgraded to form HC configurations that are not suitable for GR. Fig.3.b illustrates how this network can be easily upgraded.
B. Upgradeability Analysis

The next step is to explain the upgradeability concept of the HC and GR structures. The problem is treated from two different perspectives of optimization. The first option is to optimize the resulting HC for the given sets of nodes, and then, the links to upgrade the network to a GR optimize the resulting different perspectives of optimization. The second option is to optimize the topology as a GR, and then remove the proper links to form a HC, this is referred to along the document as HC optimization. Let LHb and LGb be the total length of the HC and GR optimized a HC respectively and LHg and LGg the opposite situation, the same topologies optimized as a GR. Exactly the same location of the nodes are studied for both optimizations. Thus, it is possible to define the first theoretical comparison given by Eq. (1).

\[ LH_b \leq LH_g \quad \& \quad LG_b \leq LG_g \quad (1) \]

The decision of which option to use depends on the previous relations and a risk factor analysis. In this document, only the difference between the options is provided, not the decision criteria. But just as a small interpretation, the decision procedure should consider that to build an optimal GR, it requires in early stages a more expensive HC. There is always a possibility that the final GR will not be implemented, hence, the difference in the investment between the optimal HC and the preventive HC has to be measured as a risk factor.

C. Approximation procedure

The point corresponds to the predictability of HC and GR using the two optimization criteria introduced above. The procedure to approximate the four topologies follows similar baseline as the proposed in [5] and is explained in the next Subsection III-D. It basically consists of calculating the total length of several different scenarios following the two optimizations and the two commented topologies.

The results of these experiments provide enough samples to determine a trend line of the length of the network as a function of the area covered by the nodes. This trend line can be translated into approximation formulas that can provide, with a reasonable error, the total length of the network depending on the case without any specific link calculation.

The area covered is the square formed by the edges of the region and is given by Eq. (2) being \( x \) and \( y \) the coordinates of the nodes. \( \text{Max}(\Delta) \) stands for maximum variation.

\[ \text{Area} = \text{Max}(\Delta x) \times \text{Max}(\Delta y) \quad (2) \]

Let \( L \) be the number of links of any topology, it is possible to relate \( \text{Area}/L \) and \( \text{Length}/L \) using an approximation. In the mentioned previous work [5], the approximation was relating the \( \text{Length}/L \) with \( \text{Area}/N \) (linear relation). In the current study it has been decided to use a new relation with better results for average error but worse maximum error.

Due to this maximum error an improvement has been introduced to the approximations including the ratio between the dimensions of the covered area, see Eq (3). This improvement will reduce around 10% of the maximum error in all the cases.

\[ \text{Ratio} = \frac{\text{Max}(\Delta x)}{\text{Max}(\Delta y)} \quad \text{if} \quad \text{Max}(\Delta x) < \text{Max}(\Delta y) \]

\[ \text{Ratio} = \frac{\text{Max}(\Delta y)}{\text{Max}(\Delta x)} \quad \text{or} \quad \text{Max}(\Delta x) > \text{Max}(\Delta y) \quad (3) \]

The number of links in both HC and GR can be calculated based on the \( X \) (rows) and \( Y \) (columns) dimensions. Equation (4) presents the number of links for each topology (\( X \) should correspond to the even dimension).

\[ L_{HC} = (Y - 1) + X + \frac{X \times Y + (Y-1) \times 1}{2} \]

\[ L_{GR} = (Y - 1) \times X + (X - 1) \times Y \quad (4) \]

The physical length of the links is calculated as a Euclidean distance between the end points. There are already several studies that provide a ratio factor Euclidean-real distance of the road [4] and [9]. Thus, the use of this kind of factors combined with the Euclidean distance gives an acceptable approximation to the real length of a link between two nodes avoiding the road layout design.

D. Experiments and Genetic Algorithms (GA)

In order to obtain numerical values for the previously commented concepts, scenarios are created randomly placing the nodes. The area covered by the nodes is also random and varying form 1 to 4X10^6km^2. For each of the scenarios, the best way to interconnect the nodes following the proposed topologies and using both optimizations is calculated using the mentioned GA. The number of nodes is varied from 20 to 40, to be able to convert a HC into a GR, (4X5, 4X7, 4X9, 6X5, 8X5) and the number of scenarios is 70.

GA are the proper tool for this purpose since it has been clearly described for network topology decision in [10] and practically used for this kind of problems [5] or [11].

The parameters used in GA such as population, number of iterations or fitness operators are critical for the performance of the method. It is not the intention of this paper to get into all these details since GA are just a tool to obtain the results. But it is worthy to mention that for each of the cases (different N and topologies), these variables are tested on regular locations of the nodes where the solution is easily known. The GA is executed under these conditions several times and it is assumed that if the results are always the optimal, the parameters values...
should be correct for random or real location scenarios. For example, for each of the cases, the same process is repeated 100 times and the shortest option is considered as valid.

IV. EXPERIMENTAL RESULTS

After performing the experiments, the values for all the scenarios of LH$_h$, LH$_g$, LG$_h$ and LG$_g$ are calculated. Then, the difference between the two optimizations is presented and finally the approximation formulas and their accuracy are represented.

But first, it is worthy to illustrate one of these scenarios and the solutions for both optimizations. Fig. 4.a presents the result as of the HC optimization and Fig. 4.b the GR optimization, of course both using exact same node locations. The black lines represent the links forming the HC and the green (or clear in b/w version), the rest of the links forming the GR. Notice the values of the distances are fulfilling the theoretical statement of Eq. (1).

A. Upgradeability

The following step is to present the results on the Upgradeability analysis. Table I presents the relations between the length of the networks depending on the topologies and optimizations proposed. The values are the difference between the corresponding values as a percentage of the optimized parameter in each of the cases. The average difference and the range is provided.

<table>
<thead>
<tr>
<th>Topology Difference</th>
<th>Avg. Diff</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{LH_{g} - LH_{h}}{LH_{h}}$ X100</td>
<td>11%</td>
<td>1-34.8%</td>
</tr>
<tr>
<td>$\frac{LG_{h} - LG_{g}}{LG_{g}}$ X100</td>
<td>61%</td>
<td>32-98%</td>
</tr>
<tr>
<td>$\frac{LG_{h} - LH_{g}}{LG_{g}}$ X100</td>
<td>13%</td>
<td>1-46%</td>
</tr>
<tr>
<td>$\frac{LG_{g} - LH_{g}}{LG_{h}}$ X100</td>
<td>21%</td>
<td>9-41%</td>
</tr>
</tbody>
</table>

The results lead to some interesting conclusions related to the investment to deploy the fibre lines. An optimized HC is in average 11% cheaper than a preventive HC. On the other hand, if the network is upgraded to a GD the final budget will be 13% lower. Based on the risk factor concept commented in Subsection III-B, a decision should be made about the optimization used. In case of optimizing the HC (cost $C_o$), if the upgrade is required it can be expected to invest around 61% more of $C_o$ in average. On the other hand, if the preventive HC is implemented (cost $C_p$), around 21% more of $C_p$ would be required to complete the GR. Of course $C_o$ is always lower than $C_p$. In further studies it might be interesting to precise where in the provided range each specific scenarios.

B. Approximations

Fig. 5 presents the results of relating Area/L and Length/L for the case of LH$_h$, including the approximation curve (power approximation). Fig. 6 represent the correction function based on the dimension ratio of the area for the same topology, in this case the most suitable approximation is polynomial. This procedure is repeated for the other three cases, LH$_g$, LG$_h$ and LG$_g$, the figures are not presented to avoid repetition.

Fig. 5. Approximation for LH$_h$

Table II presents the four cases approximations and error correction formulas being $A_L$ the area by link, $R_{xy}$ the ratio between the area dimension and $C_f$ correcting factor. $C_f$ is applied as presented in Eq. (5). Table III illustrates the errors,
The main conclusion extracted from the results in Table III is that there is no need for calculating the topologies and for both optimizations. The key aspect of these approximations is that there is no need for calculating the interconnection configuration, which, in some case can be a long procedure. Therefore, just by using information about the area covered and the number of nodes, it is possible to make budget estimations.

A second approximation can be included to lower the maximum error provided by the equations by 10%. The average error on predicting the length of the network for the four cases, HC and GR for both optimizations, is around 10%.

Two different types of optimizations are covered, HC and GR. In the case of optimizing the HC, the investment is 11% lower than the preventive HC. But in the case of requiring an upgrade the final GR would require 13% more than if the preventive HC was implemented in the first term. Both options are planned and designed allow a simple upgrade, the difference on which one to take depends on a risk factor.

In terms of predictability, the length for deploying HC and GR can be, to some extend, approximated to formulas relating the Area/links and the Length/links of each of the topologies and for both optimizations. The main conclusion is that there is no need for calculating the interconnection configuration, which, in some case can be a long procedure. Therefore, just by using information about the area covered and the number of nodes, it is possible to make budget estimations.

A second approximation can be included to lower the maximum error provided by the equations by 10%. The average error on predicting the length of the network for the four cases, HC and GR for both optimizations, is around 10%.

V. CONCLUSION

The study of Upgradeability and Predictability has provided some interesting conclusions. In terms of upgradeability, the possibilities of what is known as Preventive Design based on planar mesh topologies, Honeycomb and Grid topologies have been defined. Specific rules and interconnection schemes have been established in order to be able to upgrade HC into GR.

### TABLE II
**APPROXIMATION FORMULAS**

<table>
<thead>
<tr>
<th>Topology</th>
<th>Approx. Formula</th>
<th>Corrected Formula (C_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH_{H_L}</td>
<td>5.8285 * A_{L}^{0.2777}</td>
<td>0.9975 * R_{L}^{2} + 1.1896 * R_{XY} - 0.2571</td>
</tr>
<tr>
<td>LG_{H_L}</td>
<td>10.747 * A_{L}^{0.2777}</td>
<td>0.9734 * R_{L}^{2} + 1.2777 * R_{XY} - 0.3389</td>
</tr>
<tr>
<td>LH_{L_L}</td>
<td>7.051 * A_{L}^{0.2777}</td>
<td>1.0509 * R_{L}^{2} + 1.2859 * R_{XY} - 0.283</td>
</tr>
<tr>
<td>LG_{L_L}</td>
<td>6.3194 * A_{L}^{0.2777}</td>
<td>0.9244 * R_{L}^{2} + 1.1248 * R_{XY} - 0.249</td>
</tr>
</tbody>
</table>

### REFERENCES


[5] Jose Gutierrez, Tahir Riaz, Jens Pedersen and Ole Madsen, “Influence of approximatedLength on predicting the length of the network for the four cases, HC and GR for both optimizations, is around 10%.


