Diversity-Multiplexing Trade-off for Coordinated Relayed Uplink and Direct Downlink Transmissions

Chan Dai Truyen Thai, Petar Popovski, Fan Sun and Elisabeth de Carvalho
Department of Electronic Systems, Aalborg University
Email: {ttc, petarp, fs, edc}@es.aau.dk

Abstract—There are two basic principles used in wireless network coding to design throughput-efficient schemes: (1) aggregation of communication flows and (2) interference is embraced and subsequently cancelled or mitigated. These principles inspire design of Coordinated Direct/Relay (CDR) schemes, where each basic transmission involves two flows to a direct and a relayed user. Considering a scenario with relayed uplink and direct downlink, we analyze the Diversity-Multiplexing Tradeoff (DMT) calculating either the exact value or both upper/lower bounds. The CDR scheme is shown to have a higher diversity gain than the reference scheme at any multiplexing gain.

I. INTRODUCTION

Two-Way Relay (TWR) Network Coding (NC) has recently emerged as one of the key generic techniques that can boost the throughput performance of wireless networks [1]–[3]. There are two basic principles used in designing throughput-efficient schemes with wireless network coding (NC) (1) aggregation of communication flows - NC operates by having the flows sent/processed jointly; (2) intentional cancelable interference: flows are allowed to interfere over the wireless channel, knowing a priori that the interference can be cancelled by the destination.

TWR NC using the principles mentioned above have been extensively discussed, analyzed and evaluated in many different aspects. The TWR scenario in which two users exchange messages over a relay and for which TWR NC has been proposed is actually only one of many scenarios in which the principles of flow aggregation and a priori information can be used. Several schemes are possibly proposed in many different scenarios using the same principles. One of them is a network with a base station (BS), a relay user (RS), a relayed user (U) and a direct user (V). In this scenario, we have considered different traffic sub-scenarios in each of which one user has an uplink or downlink message. In the proposed coordinated direct/relay (CDR) schemes, the transmissions for the relayed user and the direct user can be combined using the principles of flow aggregation and a priori information.

For example, for the traffic scenario of relayed uplink and direct downlink, in the first step of the proposed scheme of the scenario, termed S, user U transmits an uplink message to RS and BS transmits a downlink message to user V simultaneously. It becomes a Multiple Access Channel (MAC) at each receiver of RS and user V. Moreover, in the second step, RS decodes and relays the uplink message to BS. If user V can decode this message, its contribution in the first received signal can be cancelled. Therefore, the transmitted rates and durations are selected and optimized according to several conditions in several options. This will benefit the performance. Enabling such simultaneous transmissions improves the spectral efficiency and communication reliability.

The proposed schemes for other traffic scenarios are described in details in [4] (AF) and [6] (DF). The conventional scheme for each traffic scenario are defined as orthogonal transmissions for the two users. Definitively, TWR NC cannot be used here because there is only one message, uplink or downlink, requested for the relayed user. In the full traffic scenario, each user has both uplink and downlink messages, the state-of-the-art conventional scheme is defined a combination of orthogonal transmissions for the two users and TWR NC scheme. In all full and non-full traffic scenarios, we have analyzed and compared the proposed scheme and the conventional scheme. The proposed scheme was shown to be better in terms of sum-rate and rate region.

The schemes therefore have been so far considered through the prism of spectrum efficiency [4]–[6], in this paper, we will show that the schemes also enhance the communication reliability by calculating/bounding and analyzing diversity-multiplexing trade-off (DMT) functions for a conventional scheme and the CDR scheme using DF in traffic scenario of relayed uplink and direct downlink. From now on we simple say “the scenario” to refer to this scenario.

Because DF is used, the rate transmitted by the relay is not necessary equal to the rate it receives, the durations of the all hop transmissions are therefore different and subject to optimization of rates and outage probabilities. Enabling such simultaneous transmissions improves the spectral efficiency and communication reliability.

The rest of the paper is organized as follows. Section II presents the system model used. We describe and calculate rates of the reference and CDR schemes in Section III. Section IV calculates or bounds the DMT functions of the schemes. Section V presents and discusses the numerical results and Section VI concludes the paper.

II. SYSTEM MODEL

We consider a scenario with one base station (BS), one relay (RS), and two users (U and V), see Fig. 1. All stations are half-duplex, single-antenna and all transmissions have a normalized bandwidth of 1 Hz. Each of the complex channels $h_{i,j}, i \in \{1, 2, 3, 4, 5\}$, is reciprocal. Only the transmitter and the receiver of a transmission know the channel of that transmission.
BS has to receive message $s_1$ from user U via relay RS and send message $s_2$ to user V directly. Because we have a relayed uplink and a direct downlink, there are 3 hop transmissions which are U-RS, RS-BS, BS-V hop transmissions through channels $h_2$, $h_1$ and $h_3$. We assume that the transmit powers of the corresponding transmitters in those 3 hop transmissions are $P_U$, $P_R$ and $P_B$. The noise at all stations is Additive White Gaussian Noise (AWGN) with $CN(0, \sigma^2)$, $\sigma^2 = 1$, distribution. Denote $\gamma_i = \frac{|h_i|^2}{\sigma^2}, i \in \{1, 2, 3\}$.

In the reference scheme denoted as $E$, all transmissions are orthogonally multiplexed in time. In the CDR scheme denoted as $S$, transmissions of the two messages are combined in such a way that as much data as possible sent to the desiring stations in the same total time duration. $D_{1i}^E$ and $D_{1i}^S$, $i \in \{E, S\}$ are maximal average rates over time for U and V respectively in the whole scheme $i$. The sum-rate is therefore estimated as $D_S^S = D_{1i}^S + D_{1i}^U$.

Each hop transmission period may consist of one transmission or two transmissions with different rates. $\lambda$ and $\mu$, $0 < \lambda, \mu < 1$, characterize different transmission durations and are defined as follows. In both schemes, there are $\lambda \mu N$ symbols in the BS-V hop transmission where $N$ is the total number of symbols in the whole scheme as in Fig. 1. In scheme $E$, there are totally $\mu \lambda N$ symbols in U-RS and BS-V hop transmissions; if $\mu < \lambda$, there is no symbol in U-RS hop transmission and thus no data delivered from user U to BS. This case is therefore not considered. In scheme $S$, there are $\lambda \mu N$ symbols in U-RS hop transmission. We have different definitions for $\mu$ in different schemes as above so that we can easily compare the rates of two schemes as shown later on. $R_{1i}^E[j]$ and $R_{1i}^S[j]$ are instantaneous rates in time slot $j, j \in \{1, 2, 3\}$ for user U and V respectively in scheme $i, i \in \{E, S\}$.

We denote $C_1 = \log_2(1 + P_R\gamma_1)$, $C_2 = \log_2(1 + P_U\gamma_2)$, $C_3 = \log_2(1 + P_B\gamma_3)$ and $C_4 = \log_2(1 + P_U\gamma_4)$ which are the maximal rates in RS-BS, U-RS, BS-V and U-V hop transmission if there is no interference. Denote $C_{2-1} = \log_2 \left(1 + \frac{P_B\gamma_2}{P_U\gamma_2 + 1}\right)$ which is the maximal rate for decoding a desired signal from user U over $h_2$ treating the interference from BS over channel $h_3$ as noise at RS. Similarly, we denote $C_{1-2} = \log_2 \left(1 + \frac{P_U\gamma_2}{P_U\gamma_1 + 1}\right)$, $C_{3-4} = \log_2 \left(1 + \frac{P_B\gamma_3}{P_U\gamma_3 + 1}\right)$ and $C_{4-3} = \log_2 \left(1 + \frac{P_U\gamma_4}{P_B\gamma_4 + 1}\right)$. Denote $C_{1,2} = \log_2 \left(1 + P_B\gamma_1 + P_U\gamma_2\right)$ which is the maximal total rate for multiplexing in time between two modes: the first mode decodes the signal from user U treating the signal from BS as noise, cancels its contribution and decode the other signal while the second mode decode in the opposite order because $C_{1,2} = C_{2,1} = C_{1-2} + C_{2} = C_{2-1} + C_{1}[7]$. $|x|$ is the number of symbols in symbol stream $x$ and $\log$ means base 2 logarithm.

**III. SCHEME DESCRIPTION**

In scheme $E$, the sum-rate will be calculated and optimized based on parameter $\mu$. In scheme $S$ we have two more parameters, $R_{U}^S[1]$ and $R_{V}^S[1]$, to optimize.

**A. Reference Scheme**

First, user U encodes $s_1$ to $x_1$ with rate $R_{U}^E[1]$ and transmits it to RS as seen in Fig. 1, RS receives $y_{R1}[1] = h_2 x_1 + z_{R1}[1]$. Second, BS encodes $s_2$ to $x_2$ with rate $R_{V}^E[1]$ and transmits it to user V, V receives $y_{V1}[1] = h_3 x_2 + z_{V1}[1]$. Third, RS decodes $x_1$ to $s_1$, re-encodes it to $x_1^R$ with rate $R_{U}^E[2]$ and transmits it to BS, BS receives $y_{R2}[1] = h_1 x_1^R + z_{B1}[2]$. Since $x_1$, $x_2$ and $x_1^R$ are transmitted with power $P_U$, $P_R$ and $P_B$, the rates $R_{U}^E[1]$, $R_{V}^E[1]$ and $R_{U}^E[2]$ are selected as the maximal rates over the corresponding channels $R_{U}^E[1] = C_2, R_{V}^E[1] = C_3$ and $R_{U}^E[2] = C_1$.

Since all transmissions are performed separately, the duration for RS-BS transmission is $(1 - \mu)N$. The U-RS, RS-BS and BS-V transmissions therefore have durations of $(\mu - \lambda)N, (1 - \mu)N$ and $\lambda N$ symbols respectively. On the other hand, the corresponding maximal rates are $C_1, C_2$ and $C_3$ respectively thus the maximal rate transmitted through them are respectively

$$D_S^E = (\mu - \lambda)C_2, \quad D_S^E = (1 - \mu)C_1, \quad D_S^E = \lambda C_3.$$  (1)

Because the rate transmitted from user U to BS end-to-end is the minimum of the rate transmitted in BS-RS and RS-U transmissions, the sum-rate transmitted for two users is $D_S^E = \min(D_S^E, D_S^E) + D_S^E$. Because the information of all channels is not available, $\mu$ is optimized to have the highest sum-rate based on the distributions of the channels $D_S^E = \max_{\lambda, \mu} D_S^E(\lambda, \mu)$, where $D_S^E$ is the average of $D_S^E$ over all channel realizations, is the maximum sum-rate.

**B. CDR Scheme**

First, user U transmits $x_{1,1}$ with rate $R_{U1}[1]$ to RS and BS transmits $x_{2,1}$ with rate $R_{V1}[1]$ in $\min(\mu, \lambda)N$ symbols simultaneously. RS and V therefore respectively receive $y_{R1}[1] = P_{U}^\gamma_1 + P_{B}^\gamma_3 x_{2,1} + z_{R1}[1]$.
Throughout the rest of the paper, we use the symbol $\lim_{\rho \to \infty}$ to denote exponential equality, i.e. we write $f(\rho) \equiv \rho^\beta$ to denote \( \lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = \beta \) and $\beta \geq \gamma$ are similarly defined. Therefore, equation (2) can be written as in [8]

\[ P_r(\rho) \equiv \rho^{-d}. \] (3)

We also denote $(x)_+ = \max(0, x)$ and log refers to log$_2$ if not stated otherwise. If $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = 1$, we write $\rho \sim \rho_1$ when $\rho \to \infty$. Because $\rho^\beta + \rho^\alpha \sim \rho^{\alpha \max(a,b)}$ when $\rho \to \infty$, we often use $\rho^\beta + \rho^\alpha \equiv \rho^{\alpha \max(a,b)}$ or $\rho^\alpha + \rho^\beta \equiv \rho^{\alpha \max(a,0)}$. Moreover, 

\[ \Pr[\gamma_i < x] = \int_x^\infty e^{-t} \, dt = 1 - e^{-x} \sim x \quad \text{when } x \to 0. \] (4)

Therefore, 

\[ \Pr[\gamma_i < r - a] \equiv \rho^{-a - \beta} \quad \text{when } \rho \to \infty. \] (5)

We investigate on the outage probability when the system tries to achieve a certain target rate pair $(D_U', D_V')$. Here to consider the diversity-multiplexing trade-off we examine how fast the outage probability decays when the rate increases as a multiple of $\log \rho_1$, where $i \in \{ B, R, U \}$, $\rho_1 = \frac{P_i}{\sigma_1^2}$ when station $i$ transmits with power $P_i$ and $\sigma^2 = 1$ is noise power at a receiver. We assume $P_B = 1$ and $P_U = P_V = \rho^\beta$ as in [9]. We assume that $\beta$ is always known at all stations.

Since the stations have different transmit powers, the target rates have different expressions. We calculate the DMT functions for the scheme when the target rate for user $i$ is given by $D_i^0 = r_i \log P_i$ where $r_i$ is the corresponding multiplexing gain and $P_i$ is the transmit power of the transmitter i.e. for direct downlink $P_1 = P_B = \rho$ and for relayed uplink $P_2 = P_U = P_R = \rho^\beta$. The target rate for user $U$ is $D_U' = r_U \log \rho^\beta$ and the target rate for user $V$ is $D_V' = r_V \log \rho^\beta$ where $r_U$ and $r_V$ are multiplexing gains for user $U$ and $V$ respectively. The scheme is in outage when the maximal achievable rate for user $U$ or user $V$ is smaller than the respective target rate. An outage of a certain pair $(\lambda, \mu)$ is defined as when that pair cannot support a target rate pair $(R_U, R_V)$. Therefore $\lambda$ and $\mu$ can be selected such that the average outage probabilities over all channel realizations are the smallest ones \( \min_{\lambda, \mu} \Pr[O] \) where $O$ is the event of outage of the scheme.

### B. Reference Scheme

As described in section III, the rates for U-RS, BS-V and RS-BS hop transmissions are respectively

\[
\begin{align*}
D_U^0 &= (\mu - \lambda) \log(1 + \gamma_2 \rho^\beta) \\
D_V^0 &= \lambda \log(1 + \gamma_2 \rho^\beta) \\
D_U^1 &= (1 - \mu) \log(1 + \gamma_1 \rho^\beta).
\end{align*}
\] (6)

The delivery of $s_1$ from user $U$ to BS is in outage when one of these two conditions occurs $D_U^0 < D_U^1$ and $D_V^0 < D_V^1$. We call these two events $O_U^1$ and $O_V^1$ respectively. The probability of the first event is 

\[ \Pr[O_U^1] = \Pr[(1 + \gamma_2 \rho^\beta)^{\mu - \lambda} < \rho^{\beta \gamma_2}] \equiv \Pr[(\gamma_2 \rho^\beta)^{\mu - \lambda} < \rho^{\beta \gamma_2}] = \Pr[\gamma_2 < \rho^\beta \gamma_2]. \] (7)
Because $\rho \to \infty$, this probability decays only when $\frac{r_i}{\mu - \lambda} < 1 < 0$. It means that we have a positive diversity gain corresponding to this transmission only when $1 - \frac{r_i}{\mu - \lambda} > 0$. In the other case, the diversity gain is zero therefore we can write the diversity gain as a function of $\left(1 - \frac{r_i}{\mu - \lambda}\right)^+$. On the other hand, because of (5), we can write $\Pr[O^E_{U}] = \rho^{-\left(1 - \frac{r_i}{\mu - \lambda}\right)^+}$. Similarly, the probability of the second event is $\Pr[O^E_{V}] = \rho^{-\left(1 - \frac{r_i}{\mu - \lambda}\right)^+}$.

According to the scheme description, the rate of user $V$ is $D^E_{U} = \lambda \log(1 + r_{U})$. It is in outage when $D^E_{U} < D^E_{V}$. We call this event $O^E_{V}$. With some derivations, we have $\Pr[O^E_{V}] = \rho\left(\frac{\gamma}{\lambda}\right)^+$.

The scheme is in outage when one of the conditions $O^E_{U_1}$, $O^E_{U_2}$ and $O^E_{U_3}$ occurs i.e. $O^E = O^E_{U_1} \cup O^E_{U_2} \cup O^E_{U_3}$. Denoting $d_1 = \beta \left(1 - \frac{r_i}{\mu - \lambda}\right)^+$, $d_2 = (1 - \frac{r_i}{\mu})$, $d_3 = \beta \left(1 - \frac{r_i}{\mu - \lambda}\right)^+$, $\Pr(O^E_{U_1}) = \Pr(O^E_{U_2}) + \Pr(O^E_{U_3}) + \Pr(O^E_{V}) = \rho^{-\min(d_1,d_2,d_3)}$.

The first equation comes from the fact that the events $O^E_{U_1}$, $O^E_{U_2}$ and $O^E_{U_3}$ are independent due to the independence of $\gamma_1$, $\gamma_2$ and $\gamma_3$ and that the product of any two or three of the probabilities $\Pr(O^E_{U_1})$, $\Pr(O^E_{U_2})$ and $\Pr(O^E_{U_3})$ decays faster than each of them. The second equation is due to the smaller terms are negligible and the probability is determined by the largest term in the sum when $\rho \to \infty$. According to the definition in (3), the diversity gain is therefore $d^E = \min(d_1,d_2,d_3)$.

Each element in the min function above is actually the diversity gain of each hop transmission (RS, BS-V and RS-V) in the reference scheme. It increases when the corresponding time assigned to it increases ($\mu - \lambda$, $\lambda$ and $1 - \mu$). However, the diversity gain of the scheme is the minimum of them, therefore the time durations assigned to 3 hop transmissions should be balanced such that the minimum diversity gain is maximal. Obviously, the diversity gain can be improved by selecting the right values of $\lambda$ and $\mu$. This is how we improve the diversity gain when the transmitter and the receiver of a transmission do not know the channels not related to them, $d^E = \max_{\lambda,\mu} d^E(\lambda,\mu)$ is the maximum DMT function scheme $E$ can achieve. It is obvious that $d^E$ is maximized when $d_1 = d_2 = d_3$ roots of which are optimal $\mu$ and $\lambda$.

C. CDR Scheme

In the first time slot, there are two decoding options at RS: (Option 1) decode $x_{1,1}$ or $x_1$ treating the interference $x_2$ or $x_{2,1}$ from BS as noise, (Option 2) decode the interference from BS first, cancel its contribution and decode the desired signal. At user $V$, there are also two similar decoding options: (Option a) decode the signal from user U first and (Option b) decode the signal from BS first. In total, we have four options (1a, 1b, 2a, 2b) however multiplexing of more than one of those options in time with any time ratio also give an achievable rate pair ($D^E_{U_1}$, $D^E_{U_2}$).

On the other hand when $\rho \to \infty$, $C_1$, $C_2$, $C_3$ and $C_4 \to \infty$. If $\beta \rho < 1$, $\lim_{\mu \to \infty} C_{1-2} = \lim_{\mu \to \infty} C_{3-4} = \infty$ and $\lim_{\mu \to \infty} C_{2-1} = \lim_{\mu \to \infty} C_{4-3} = 0$. Therefore in case of $\rho \to \infty$ only two decoding options are relevant: 1a (both RS and user V decode the signal from user U first, for larger $\beta$) and 2b (both RS and user V decode the signal from BS first, for larger $\beta$). In two cases below ($\mu \geq \lambda$ and $\mu < \lambda$), we consider these two options when calculating the DMT function for scheme $S$. The subscript $i/j - k, i,j,k \in \{B, R, U, V\}$ means decoding the signal for user $j$ treating the signal for user $k$ as noise at station $i$ and subscript $i/j$ means similarly but decoding without interference. Similarly to the derivations for Reference Scheme we have the following results.

- $\mu \geq \lambda$
  - Option 1a: the rates of three hop transmissions U-RS, BS-V and RS-BS
    $\begin{align*}
    D^S_{U_1} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right) \\
    D^S_{U_2} &= \mu \log\left(1 + \gamma-\rho\right) \\
    D^S_{U_3} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right)
    \end{align*}$
  - Option 2b: the rates of three hop transmissions U-RS, BS-V and RS-BS
    $\begin{align*}
    D^S_{U_1} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right) \\
    D^S_{U_2} &= \mu \log\left(1 + \gamma-\rho\right) \\
    D^S_{U_3} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right)
    \end{align*}$

- $\mu < \lambda$
  - Option 1a: the rates of three hop transmissions U-RS, BS-V and RS-BS
    $\begin{align*}
    D^S_{U_1} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right) \\
    D^S_{U_2} &= \lambda \log\left(1 + \gamma-\rho\right) \\
    D^S_{U_3} &= \lambda \log\left(1 + \gamma-\rho\right)
    \end{align*}$
  - Option 1b: the rates of three hop transmissions U-RS, BS-V and RS-BS
    $\begin{align*}
    D^S_{U_1} &= \lambda \log\left(1 + \frac{\gamma-\rho}{\gamma-\rho+1}\right) \\
    D^S_{U_2} &= \lambda \log\left(1 + \gamma-\rho\right) \\
    D^S_{U_3} &= \lambda \log\left(1 + \gamma-\rho\right)
    \end{align*}$

73
the higher maximum multiplexing gain than scheme $E$.

We describe and calculate the user rates and sum-rate of the reference and CDR schemes for relayed uplink and direct downlink transmission. We calculate the Diversity-Multiplexing Trade-off functions either the exact value or both upper/lower bounds of the schemes. The CDR scheme is shown to have a higher DMT function at any multiplexing gain as well as the higher maximum multiplexing gain than the reference scheme.

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