PFS-based Resource Allocation Algorithms for an OFDMA System with Multiple Relays

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Abstract—We address the problem of radio resource allocation in the Downlink (DL) of a cellular system with multiple Relay Stations (RS), based on Orthogonal Frequency Division Multiple Access (OFDMA) transmission technology. The design of efficient resource allocation algorithms for such a system is crucial, in order to exploit the potential capacity and coverage increase offered by the relays. We propose different resource allocation algorithms which provide excellent throughput/outage trade-off with low complexity and minimized amount of Channel State Information (CSI). The simulation results show the benefits of our algorithms, for various number of relays and for different user distributions.

I. INTRODUCTION

One of the major goals of the 4th Generation (4G) wireless system is to provide higher data rates over larger areas in a cellular system. Orthogonal Frequency Division Multiplexing (OFDM) transmission technology is a very promising candidate for the physical layer of the 4G system, due to its high achievable spectral efficiency by bit loading and inherent robustness against inter-symbol interference caused by frequency-selective fading. There has also been an increasing focus on the relaying technology which has the potential to enhance the data rates in remote areas of the cell while keeping a low cost of infrastructure. However, to fully exploit this potential gain in capacity and coverage, many challenges are left to the radio resource allocation.

Hence, the problem of resource allocation and scheduling for relay-aided cellular systems has been a flourishing topic for some time. Some works on resource allocation for OFDMA relay systems can be found in the literature [4] [5], but to the best of the authors’ knowledge, there has been little work on specific resource allocation schemes for multiple access in a relay-aided cellular system based on OFDMA technology. Thus, our goal is to design allocation algorithms for such a system, which can enhance both capacity and coverage, but with reasonable degree of complexity. We have proposed such algorithms for a system with one relay in [6], and for multiple relays in [7]. One of the key points of these algorithms is that they are designed in such a way that reduces the algorithm complexity and required amount of Channel State Information (CSI). However, the algorithms in [6] are only designed to maximize the system throughput and [7] considers only a specific case of multiple relays in the cell. It was shown in [6] [7] that the proposed algorithms outperformed the conventional Max CSI algorithm without relays in terms of both throughput and outage. In this work, we further improve these algorithms to provide proportional fairness and compare them with the conventional Proportional Fair Scheduling (PFS) algorithm without relays. Furthermore, we evaluate these algorithms under more general conditions by varying the number of relays and user distributions. The simulation results show the efficiency of our algorithms under various conditions and the excellent trade-off realized between throughput and outage probability.

II. SYSTEM MODEL

We focus on the Downlink (DL) transmissions from a Base Station (BS) to Mobile Stations (MS) or Relay Stations (RS) in a single cell, where users feed back to the BS their CSI on every subchannel, defined as a group of adjacent subcarriers. We use a Discrete Adaptive Modulation (DAM) model, applied per subchannel, for each user, where the rates [1, 2, 4, 6, 8][b/s/Hz] corresponding to the modulations [BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM] are supported when the link Signal-to-Noise-Ratio (SNR) is above their respective predefined SNR thresholds [−5, 13.6, 20.6, 26.8, 32.9][dB]. The SNR thresholds from QPSK to 256-QAM are determined for a target Bit Error Rate (BER) of 10⁻⁶ for uncoded M-QAM symbols in flat fading channels with a known fixed gain, based on [8] [9]. For SNRs below 13.6dB, BPSK is used, and there is no transmission below −5dB. The BS is surrounded by multiple RS, as depicted in Fig. 1. The RS are placed at a distance of 0.8km from the BS, where the cell radius is 1km. As in [6], we consider the frame structure depicted in Fig. 2, where the BS and each RS transmissions are divided in time. Within the frame of length \( TS \), \( T_{BS} \) denotes the time allocated to the BS-subframe where transmissions from the BS occur, and \( T_{RS} \) the time allocated to each RS-subframe where transmissions from the RS occur. Since the interference between opposite RS can be assumed low, they are allocated the same subframe in a parallel manner, as shown in Fig. 2. When there are \( I = 6 \) relays, there are 3 groups of opposite RS: \((RS_1, RS_4), (RS_2, RS_5)\) and \((RS_3, RS_6)\). We assumed that in the BS-subframe, each subchannel can be allocated to a BS-MS link or a BS-RS link, and \( T_{BS}, T_{RS} \) can be adapted frame to frame.
recall the main steps of these algorithms, but with a slight modification in the allocation of the BS–subframe (Step 3) which provides a huge improvement compared to the usual Proportional Fair Scheduling (PFS) algorithm, as shown in the simulation results.

1) Allocation of each $RS_j$-subframe by each RS: Each subchannel is allocated to the user with the best $\phi_{k,n}$, but with queued packets, to ensure that the allocated capacity will be used, thereby increasing throughput. $\phi_{k,n}$ is defined as

$$\phi_{k,n} = \frac{r_{k,n}}{\beta_k(i-1)},$$

where $R$ is the minimum data rate requirement and $r_{k,n}$, the instantaneous rate of user $k$ on subchannel $n$, $\beta_k(i-1)$ is the past average rate allocated to user $k$ at frame $i$ over a window of $p$ frames and is updated after every frame allocation (as in PFS)

$$\beta_k(i) = \frac{p-1}{p} \times \beta_k(i-1) + \frac{1}{p} \times \sum_{n=1}^{N} c_{k,n} r_{k,n}(i),$$

where $N$ is the total number of subchannels, $c_{k,n}$ is equal to one if subchannel $n$ was allocated to user $k$ and zero otherwise, so $\sum_{n=1}^{N} c_{k,n} r_{k,n}(i)$ is the sum of allocated rates to user $k$ in the current frame $i$. With the metric $\phi$, users whose achieved throughput is far from the rate requirement and with higher instantaneous CSI are prioritized. The users having a higher $\phi_{k,n}$ than the allocated one but without packets queued at RS are represented by the set $U_{Req}$.

2) RSs send request message to BS: This message contains

- the IDs of the users in $U_{Req}$ for which packets are requested,
- the order of these users in terms of $\phi$–metric, e. g., the maximum $\phi_{k,n}$ over all $n$ for user $k$ in $U_{Req}$ (needed to determine the priority of the packets sent on the BS–RS link, see Step 3)
- the value of $\phi_{max}$, defined as the maximum value of the $\phi$–metric for the relayed users in $U_{Req}$, averaged over the subchannels (see Step 3).

3) Allocation of BS–subframe made by BS, based on the requests by RSs: BS allocates (tentatively) each subchannel to the best direct user. The BS–RS subchannels are allocated as follows. If $U_{Req}$ is non empty, the scheduler calculates the number of subchannels $n_{BR}$ required to send all the packets queued at the BS of the users in $U_{Req}$. As mentioned in section II, all the BS–RS subchannels are allocated with the same rate, corresponding to their average SNR level. Thus, any $n_{BR}$ subchannels among all $N$ subchannels can be chosen for the BS–RS transmission. Whereas in [6], the subchannel allocation between the direct users and the BS–RS links was based on the rate metric, a new criterion based on the $\phi$–metric is introduced to ensure a fair distribution of subchannels. For each user in $U_{Req}$, the average $\phi$–metric over the BS–MS subchannels is determined, and the maximum average $\phi$–metric is denoted $\phi_{max}$. In each subchannel,
the $\phi$-metric of the initially allocated direct user is compared with $\phi_{\text{max}}$, and the subchannel is allocated to the link with the highest value. This gives $y$ subchannels allocated to the BS–RS link. But not all $y$ subchannels may be required, so we compare $y$ with $n_{BR}$.

a) If $y < n_{BR}$, the $y$ subchannels are not enough for all the packets, e.g., some remain at the BS queue. To decide which packets to send on the BS–RS link, the RS–MS users for which packets were required are ordered by $\phi$ (only this order needs to be fed back to BS, not the $\phi$ values nor the CSI since these are not needed at the BS). Packets are allocated from the best RS–MS users, until all $y$ subchannels are filled.

b) If $y > n_{BR}$, all $y$ subchannels are not needed for the BS–RS link since there are less queued packets. Only the $n_{BR}$ worst subchannels for direct users are allocated to the BS–RS link, and the remaining $y - n_{BR}$ subchannels to the best direct users.

**B. Multiple Relay Allocation Algorithms**

We propose two algorithms which both use the initial algorithm presented above, namely the the Multiple-RS Parallel with Activation (MRPA) algorithm and the Multiple-RS Adaptive Activation (MRAA) algorithm. In the MRPA algorithm, the initial frame is as shown in Fig. 2. The difference of the present algorithms compared to those in [6] mainly arises from step 3), where the subchannel allocation between the BS–RS and BS–MS links is based on the $\phi$ metric instead of the rate metrics, allowing for an increased level of fairness. The concept of Long-Term RS-Activation is introduced, where, if after the path selection, 2 opposite RS in the same group are not activated (for example, $RS_1$ and $RS_4$), their corresponding subframe is removed and the frame is redivided in time among the remaining RS groups. After this initial phase fixing the frame, the algorithm above is applied. For MRAA algorithm we have in the initial frame $T_{BS} = T_F/2$ and the RS-subframe is equally divided in time between all the relays, without frequency reuse, resulting in a $RS_j$ subframe length of $T_F/2/1$. Here the Per-Frame RS-Activation is performed on top of the Long-Term RS-Activation. Basically, the Long-Term RS-Activation phase removes all $RS_j$ subframes for relays which were not selected by any user during path selection (based on long-term channel qualities), and the Per-Frame RS-Activation phase removes all the relays which decrease the overall throughput for the current frame, hence based on the instantaneous channel qualities (see [6] for more details).

Thus, compared to a completely fixed allocation, the MRPA algorithm improves the resource utilization by removing the RS-subframes of relays without any user, and redistributing them to the "activated" relays. The aim of MRAA algorithm is to further improve the achieved throughput, by discarding in each frame the relays which do not enhance the overall throughput.

**IV. NUMERICAL RESULTS**

The simulations are made over 150000 sets of channel realizations, where user locations are kept constant for a fixed number of channel realizations, then regenerated. The path loss model in [10] is used for the three links: BS–MS, BS-RS and RS-MS. Log-normal shadowing with 0dB mean and 8dB standard deviation is assumed for the BS–MS links, and 6dB standard deviation for the RS-MS links. The multipath fading channel model in [11] is used. The BS power and RS power are fixed to 20 Watts and 5 Watts. Given the subframe, there is equal power distribution in each subcarrier. There are 48 subcarriers and 12 subchannels, each composed of 4 contiguous subcarriers. The frame duration is fixed to 12ms. Packets arrive at the BS queue following a Poisson process. The reference algorithms described in [6] are used: the RS-Max Full Buffer algorithm for the throughput upper bound, and the Optimized Outage algorithm for the outage lower bound. In these algorithms, we take the ideal assumption that the time division between BS/Rs transmissions is optimized per subchannel, to ensure the bounds. In the RS-Max Full Buffer algorithm, the user with the best achievable rate is allocated among all choices of users and links, in each subchannel. Similarly, Optimized Outage algorithm allocates in each subchannel the user with the best $\phi$ metric among all choices of users and links. As our algorithms are PFS-based, they are also compared with the PFS algorithm without relays, where the user with the best $\phi$ metric is allocated in each subchannel.

**A. User Generation in the cell**

In this work, we have considered two different user distributions: uniform distribution and distribution towards cell edge. Since one of the major goals is to support more users located in the cell edge, it is of great interest to consider such a distribution. The edge distribution model is derived from [12], e.g., clustering to the edge of the cell. The cell is divided into square bins as shown in Fig. 3, which are each attributed a part of a uniformly distributed random number between 0 and 1. To generate more users towards the edge compared to the center, the bin probability increases as the distance from the bin center to the BS increases. The bin sharing the same center to BS distance are characterized by the same bin probability. In this case, 3 regions can be defined where each region corresponds to bins of equal probability, namely:

- the central area $C$ regroupings bins $6, 7, 10$ and $11$,
- the closer edge area $E_1$ regrouping bins $2, 3, 5, 8, 9, 12, 14$ and $15$,
- the further edge area $E_2$ regrouping bins $1, 4, 13$ and $16$.

Users are allowed to be generated anywhere in the whole square grouping the 16 bins, including outside the hexagonal cell. The user generation is carried out similarly as described in [12]. First, a bin is selected by sampling the CDF of the bin probabilities with a uniformly distributed random number between 0 and 1. Then, a user is generated randomly within the selected bin area.

Once users are generated either uniformly or towards the edge, each user is attached to the BS or one of the RS, following the path selection mentioned in section II. After that the resource allocation is performed. The path selection is renewed each time the average user SNR changes.
B. Performance Metrics

Two performance metrics are used: system goodput and outage probability. The goodput $\gamma$ in [b/s/Hz], where the overhead for CSI feedback is included, is defined as

$$\gamma = \tau \times \frac{n_{\text{data}}}{n_{\text{data}} + n_{\text{OH}}},$$

(3)

where $\tau$ is the throughput, $n_{\text{data}}$ the number of OFDM symbols in the frame carrying data and $n_{\text{OH}}$ the number of symbols carrying the CSI, assuming QPSK modulation.

The system outage is defined as the probability that the allocated user rates $\overline{\tau_k}$ are lower than a reference rate $\overline{\tau}$, where $\overline{\tau}$ is averaged over $p = 100$ frames. The system outage probability $P_{\text{out}}$ is expressed as

$$P_{\text{out}} = \frac{\sum_{s=1}^{S} K_s}{K \times S},$$

(4)

where $K_s$ denotes the number of users in outage for the sample $s$ and $S$ is the total number of samples, $K_s = \text{Card}\{ k, \overline{\tau_k} < \overline{\tau} \}$, where Card denotes the number of elements in the set.

With the frequency reuse carried out in MRAA, some sub-channels are used at the same time, so the opposite relays transmitting in parallel interfere with each other. The effect of this interference is taken into account in the simulations.

Results for goodput and outage probability are plotted for varying number of relays, $I = 3, 6, 8, 12$, with $K = 20$ users. For the outage probability, the target rate is fixed to 100 kbps. In these evaluations, the amount of resource used for the preambles and the user mapping information vary with the number of relays. The impact of these control fields is taken into account in the overall throughput. Simply, there is one preamble per relay which occupies one OFDM symbol, $n_{\text{prec}} = 1$. In case of frequency reuse by opposite relays, their corresponding preambles also reuse the frequencies. The DL mapping information consists of the ID of the allocated user (or relay for the BS–RS link), for each subchannel. Denoting $n_{\text{MAP}}$ the number of OFDM symbols used for the DL mapping, the goodput is determined as

$$\gamma = \tau \times \frac{n_{\text{data}}}{n_{\text{data}} + n_{\text{OH}} + (N_{RS} + 1) \times (n_{\text{prec}} + n_{\text{MAP}})},$$

(5)

where $N_{RS}$ is the number of non–overlapped, activated $RS_j$ subframes.

Figs. 4 and 5 show the system goodput and outage performance for uniform user distribution, respectively. The goodput and outage of PFS algorithm remain constant since it is independent from the number of relays. The goodput decrease of RS-Max Full Buffer in Fig. 4 for $I = 8, 12$ is due to the increased amount of signalling with the higher number of relays. For Optimized Outage, the outage at $I = 6$ is already so low that increasing the number of relays does not provide noticeable improvement. However, note that Optimized Outage is an infeasible scheme so that the difference with the actual optimal scheme is smaller. MRAA achieves the best outage performance for all number of relays and outperforms PFS, but its best goodput occurs for $I = 8$ relays. Its goodput decreases from $I = 8$ to $I = 12$ because the time allocated to each $RS_j$-subframe becomes very small at $I = 12$. The worst performance of MRPA happens at $I = 3$ as there is no frequency reuse. Thus, there is a large improvement for $I \geq 6$. MRAA outperforms PFS in terms of both goodput and outage, for all number of relays. It can be seen from Fig. 4 that the goodput of MRAA increases with the number of relays, even though the amount of signalling increases as shown in (5), which indicates the robustness of the proposed scheme to increasing signalling overhead. At the same time, the outage probability of MRAA increases as the number of relays grows.
for $I \geq 8$, since this algorithm is designed to allocate in each frame, the relays which increase the overall throughput. Still, the outage performance of MRAA at $I = 12$ is considerably lower than the one of PFS. When users are distributed towards the edge, an interesting point is that the performance of all the algorithms is generally degraded for all number of relays, as observed in Figs. 6 and 7. While the outage lower bound of PFS is largely degraded, the outage of the proposed algorithms is only slightly lower than with uniform user distribution, pointing out their robustness against varying user distributions. The gain in goodput of MRAA compared to PFS is even higher than with uniform distribution, and MRPA also outperforms PFS for $I = 6$, while achieving the best outage for all $I$.

The simulations with different number of relays in the cell showed the efficiency of the proposed algorithms, which are robust to varying number of relays and to different user distributions. Under the considered assumptions, the optimal number of relays differ depending on the algorithm. MRPA achieves its best performance for $I = 8$, for both user distributions. For MRAA, $I = 8$ also achieves the best compromise between goodput and outage, but $I = 12$ offers the best goodput level while considerably reducing the outage probability compared to the conventional PFS algorithm. Interestingly, continuously increasing the number of relays may not be the best solution even when there are more users in the cell edge, since, for example, the goodput of MRPA is affected by the increasing amount of signalling overhead as more and more relays are activated and less resource is allocated to each $RS_j$–subframe. As the number of relays increase, the goodput of MRAA improves but not the outage since more $RS_j$–subframes are not allocated.

V. CONCLUSION

This work investigated the problem of resource allocation for a relay–aided cellular system based on OFDMA. The proposed algorithms are designed to provide both throughput and outage probability improvements with low complexity, while the required signalling information is minimized compared to the optimal solution. The simulation results showed that they achieved an excellent outage/throughput trade-off compared to reference algorithms, and outperformed the well known conventional PFS algorithm. Moreover, simulations under different user distributions and various number of relays showed the robustness and efficiency of the proposed algorithms.

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