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Terpager Andersen, K.

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Theory for natural ventilation by thermal buoyancy in one zone with uniform temperature

Karl Terpager Andersen*

Energy and Indoor Climate Division, Danish Building and Urban Research, Dr. Neergaards Vej 15, Hoersholm DK-2970, Denmark

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Abstract

Based on fundamental flow equations describing mass balance, energy conservation and momentum, a consistent solution is derived for natural ventilation by thermal buoyancy in a room with two openings and with uniform temperature.

The solution is a reliable tool for analysing and designing natural ventilation systems where thermal buoyancy is the dominating driving force.

Keywords: Natural ventilation; Thermal buoyancy

1. Introduction

In literature, different models are used for explaining the mechanism of natural ventilation by thermal buoyancy. In the earliest literature a column or a fan model is used. In the column model, the driving force is assumed to be the thermal buoyancy on a column of warm air immersed in colder surrounding air. The column has a height equal to the vertical distance between inlet and outlet and a cross-section equal to the opening area of the outlet (cf. e.g. [1]). The fan model assumes the same driving force as the column model, and this force creates the air movement in the openings and overcomes the friction losses.

In recent literature a third model, the pressure model, is commonly used. It is based on the indoor and outdoor barometric pressure distributions and it introduces the neutral plane, which is the level where indoor and outdoor pressure are equal. The position of this level is determined by using a mass balance equation with velocities determined by the Bernoulli equation (cf. e.g. [2]). In the latest literature, "emptying–filling box models" are introduced. They are based on the pressure model, but include temperature stratification considerations known from displacement ventilation (cf. e.g. [3]).

The above-mentioned models are not or only partly based on fundamental flow equations. Various intuitive interventions or tricks are performed to establish the airflow. Different calculation results occur dependent on which model is used and how it is used, and this creates uncertainties of analysis and design of natural ventilation systems (cf. e.g. [3,4]).

The purpose of this paper is to set up a consistent theory for natural ventilation by thermal buoyancy using the fundamental flow equations. Only the simplest case will be considered in this paper. It is a room with two openings placed at two different vertical levels, with uniform indoor temperature and with steady-state conditions.

2. Fundamental flow equations

It is assumed beforehand that the Reynolds number for the airflow through the openings is so high that the flow takes the shape of jets on entering and leaving the room. Then, the air pressure in the smallest cross-section of the jets, the vena contracta, is equal to the surrounding pressure and the air velocity in these sections can be considered uniform.

The airflow between the vena contracta of the inlet and the vena contracta of the outlet in a room is considered, cf. Fig. 1. This flow does not take place in a stream tube although the flow boundaries fulfil the stream tube requirement

* Tel.: +45-45-7423-88.
E-mail address: kta@dbur.dk (K.T. Andersen).

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Nomenclature

A	opening area (m^2)
A_c	area of vena contracta (m^2)
C_c	contraction coefficient
C_v	velocity coefficient
H	vertical distance between inlet and outlet (m)
H_1	vertical distance from neutral plane to inlet (m)
H_2	vertical distance from neutral plane to outlet (m)
G	production rate of pollution (m^3/s)
P	exchange rate of work (W)
R	gas constant of air (J/(kg K))
T	temperature (K)
$T_{i,i}$	internal temperature by frictionless flow (K)
c	air pollution concentration (m^3/m^3)
c_p	specific heat capacity of air by constant pressure (J/(kg K))
g	gravity acceleration (m/s^2)
p	pressure at the opening (Pa)
u	internal energy per unit mass (J/kg)
y	vertical co-ordinate of the opening (m)
v_c	air velocity in vena contracta (m/s)
$v_{c,theo}$	theoretically obtainable air velocity (i.e. by frictionless flow) (m/s)
w_{fr}	internal friction work per unit mass (J/kg)

Greek symbols

Φ	net heat input (W)
Φ_0	net heat input when indoor and outdoor temperature are equal (W)
β_E	kinetic energy correction factor
ρ	density of the gas (kg/m^3)
ψ	flow coefficient
ζ	resistance coefficient

Subscripts

acc	accepted
c	contracted
d	discharge
i	indoor
o	outdoor
req	required
1	inlet
2	outlet

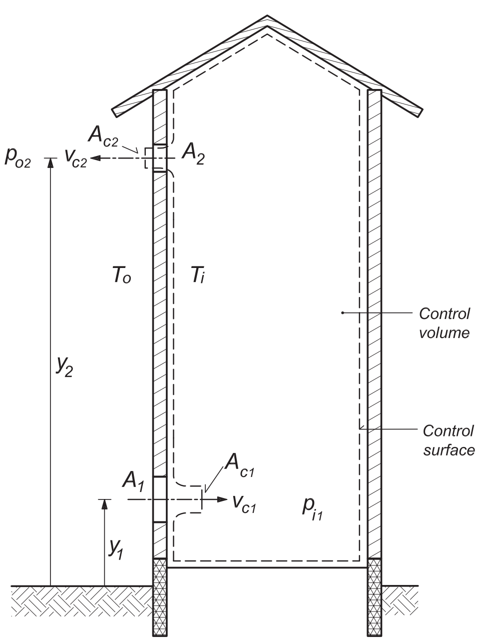

Fig. 1. Control volume and control surface for room with two openings.

of no fluid transfer through the boundaries except through the ends. The fluid does not follow streamlines when flowing from one vena contracta to the next. There is nor any direct relationship between the flow parameters (i.e. velocities, cross-section areas and pressures) in lower and upper part of the room and in the two venas contractas, like what you find for the flow in a stream tube. Therefore, the Bernoulli equation is not valid for the considered airflow. It is only valid when the airflow takes place in a stream tube with no friction, no exchange of heat or work and with uniform air velocity in any cross-section of the flow. Even the modified version of the equation, which take friction and velocity profile into account, is invalid.

2.1. Control volume

In order to ensure that all factors influencing the airflow can be taken into account, a control volume representing the room is defined. The control volume of the case in question is enclosed by the surfaces of the room, the two venas contractas cross-sections and the two short lengths of jet between opening and vena contracta, cf. Fig. 1. In this volume there will be no exchange of mass, energy and momentum through the boundaries except through the two venas contractas.

In a control volume, the total change of an extensive property (i.e. a property dependent on the substance present such as mass, energy and momentum) is considered. According to the so-called Reynolds transport equation, the total change is equal to the change rate of the property of the control volume plus the efflux of the property through the control surfaces (cf. e.g. [5,6]). In connection with the efflux, static pressure, air velocity and cross-section areas are of interest in the venas contractas.

The static pressure in the vena contracta of the inlet jet is equal to the indoor pressure at inlet level, and in the vena contracta of the outlet jet, the static pressure is equal to the outdoor pressure at outlet level.

The air velocity in the venas contractas can be considered uniform and can be determined by

$$v_c = C_c v_{c,theo}, \quad (1)$$

where v_c is the air velocity in vena contracta, C_v the velocity coefficient and $v_{c,theo}$ the theoretically obtainable velocity (i.e. by frictionless flow).

The cross-section area of a vena contracta can be determined by

$$A_c = C_c A, \quad (2)$$

where A_c is the area of the vena contracta (m^2), C_c the contraction coefficient and A is the opening area (m^2).

2.2. Mass balance

For a control volume with steady-state flow, Reynolds transport equation results in the continuity equation, which states that the rate of increase of mass within the control volume plus the net efflux of mass through its control volume is zero. For the case shown in Fig. 1, this can be expressed by

$$\rho_o A_{c1} v_{c1} - \rho_i A_{c2} v_{c2} = 0, \quad (3a)$$

where ρ is the density of the gas (kg/m^3), index 1 is used for inlet, index 2 is used for outlet, index i is used for interior conditions and index o is used for exterior conditions.

By introducing Eqs. (1) and (2) you get

$$\rho_o C_{c1} A_1 C_{v1} v_{theo1} - \rho_i C_{c2} A_2 C_{v2} v_{theo2} = 0$$

or

$$\rho_o C_{d1} A_1 v_{theo1} - \rho_i C_{d2} A_2 v_{theo2} = 0, \quad (3b)$$

where C_d is the discharge coefficient defined by

$$C_d = C_c C_v. \quad (3c)$$

2.3. Energy conservation

Energy conservation expresses the first law of thermodynamics. For a control volume, you get an equation, which states that the rate of change of stored energy (kinetic, potential and internal energy) within the control volume plus the efflux of stored energy across the control surface is equal to the exchange rate of heat and work (i.e. net heat input and exchange of power).

For the case shown in Fig. 1 with steady flow (i.e. no change in stored energy in the control volume), you get [5].

$$\begin{aligned}&(u_{i2} + \tfrac{1}{2}\beta_{E2} v_{c2}^2 + g y_2 + p_{o2}/\rho_i)\rho_i v_{c2} A_{c2} \\ &\quad -(u_{o1} + \tfrac{1}{2}\beta_{E1} v_{c1}^2 + g y_1 + p_{i1}/\rho_o)\rho_o v_{c1} A_{c1} \\ &= \Phi + P, \end{aligned} \quad (4)$$

where u is the internal energy per unit mass (J/kg), β_E the kinetic energy correction factor, g the gravity acceleration (m/s^2), y the vertical co-ordinate of the opening (m), p the pressure at the opening (Pa), Φ the net heat input (W) and P the exchange rate of work (W).

The kinetic energy correction factor takes into account that the velocity profile is not uniform. For a parabolic profile, you have $\beta_E \approx 1.5$, and for the turbulent flow in vena contracta with almost uniform velocity profile, you have $\beta_E \approx 1.02$. In the following a value of unity is assumed.

The net heat input is the heat gain (from persons, electrical equipment, sunshine, heating systems, etc.) minus the heat losses (due to heat transmission through surfaces and to infiltration).

The exchange rate of work is related to work done by, e.g. a fan. In the case in question you have $P=0$. Further, you have the exchange rate of displacement work represented by the p/ρ-terms.

Between internal energy, displacement work and temperature you have the following relationship:

$$\begin{aligned}&(u_{i,2} + p_{o2}/\rho_i) - (u_{o1} + p_{i1}/\rho_o) = c_p(T_i - T_o) \\ &\quad = c_p(T_{i,i} - T_o) + w_{fr},\end{aligned} \quad (5a)$$

where $T_{i,i}$ is the internal temperature by frictionless flow (K), c_p the specific heat capacity of air by constant pressure (J/(kg K)) and w_{fr} the internal friction work per unit mass (J/kg).

Compared to frictionless flow, the friction results in somewhat higher indoor temperature and a smaller air velocity. When the friction loss takes place in an opening, it can be expressed by

$$w_{fr} = \tfrac{1}{2}\zeta v_c^2, \quad (5b)$$

where ζ is the resistance coefficient.

By dividing Eq. (4) with the mass flow $\rho_o v_{c1} A_{c1} = \rho_i v_{c2} A_{c2}$ and by inserting Eq. (5a) into the equation, you get with $P = 0$

$$\begin{aligned}&\tfrac{1}{2}v_{c2}^2 - \tfrac{1}{2}v_{c1}^2 + g(y_2 - y_1) + c_p(T_{ii} - T_o) \\ &\quad + w_{fr} = \frac{\Phi}{\rho_o v_{c1} A_{c1}},\end{aligned} \quad (5c)$$

where the unit for each term of the equation is J/kg.

2.3.1. Net heat input

Some heat gains and heat losses are temperature dependent. For instance for increasing indoor temperature, heat gains from persons (sensible heat) decreases and heat transmission loss and infiltration loss increases. Net heat input can roughly be expressed by

$$\Phi = \Phi_0 - k(T_i - T_o), \quad (6)$$

where Φ_0 is net heat input when indoor and outdoor temperature are equal, and k is a constant, which depends on building type and use of building.

Frequently, a fixed indoor temperature is aimed at, so that a temperature difference can be estimated with reasonable accuracy and so that a constant net heat input can be assumed.

2.3.2. Reduced energy equation

If considering a room with a net heat input of 4000 W, with a vertical distance between two equally large openings of 10 m and with a temperature difference of 4 K, you get an air velocity of about 1.1 m/s in the openings and an airflow rate of about 0.85 m^3/s. In this case, the terms in Eq. (5c) get the following values:

$\frac{1}{2}v^2 \sim 0.6$ J/kg,

$g(y_2 - y_1) \sim 100$ J/kg,

$w_{fr} \sim 0.2$ J/kg,

$c_p(T_{ii} - T_o) \sim 4000$ J/kg,

$\Phi/(\rho_o v_{c1} A_{c1}) \sim 4100$ J/kg.

It is seen that the net heat input and internal energy are the totally dominating terms in Eq. (5c). The energy equation is therefore not suitable for determining any velocities, as it would require exact values for net heat input and temperature difference. The equation is only suited for determining the difference between indoor and outdoor temperatures, and it can, with an error below 1%, be reduced to

$$\rho_o c_p A_{c1} v_{c1}(T_i - T_o) = \Phi. \quad (7)$$

2.4. Momentum equation

Based on Newton's second law for a finite system, an equation for the acting forces in a control volume can be set up. It states that the net internal force acting on the air in the control volume equals the time rate of change of momentum of the air within the control volume plus the net rate of momentum flux or transport out of the control volume through its surface (cf. e.g. [6]). Being a vector equation, the momentum equation can be split into scalar equations.

In the horizontal equation, the velocity terms are small compared to the remaining terms as was the case in the energy equation. In combination with uncertainties on pressure distributions, the equation is unsuitable for determining air velocities in the openings.

In the vertical momentum equation, the vertical velocity component in the room is negligible and the equation results in the well-known equation for linear (hydrostatic) pressure distribution

$$p_{i1} - p_{i2} = \rho_i g H. \quad (8)$$

3. Additional equations

Of the four fundamental flow equations, only three equations are applicable as the horizontal momentum equation is unsuitable. The net heat input (or its constant part) is the independent variable, and there are six dependent variable, ρ_i, T_i, v_{c1}, v_{c2}, p_{i1}, and p_{i2}. Three more independent equations are therefore needed. Two equations can be derived by setting up fundamental flow equations for new control volumes in connection with inlet and outlet airflow, respectively. The third one can be derived from the perfect gas equation.

3.1. Outlet airflow

For the outlet airflow, you can consider a control volume as shown in Fig. 2. The control surface consists of a part of an indoor spherical surface (being entrance of the control volume), the vena contracta section of the outlet (being the exit), and the surface of the jet connecting vena contracta with the spherical surface. For this control volume, you can set up the energy equation, whereas neither the mass balance equation nor the two momentum equations are suitable in this case.

In the entrance of the control volume, the air velocity is very close to zero and the static pressure is p_{i2}. For the exit, the air velocity is v_{c2} and the pressure is p_{o2}. Besides there is no heat supply and height levels for entrance and exit are equal. For this control volume, you get, when comparing with frictionless flow

$$u_{i2} - u_{o1} = w_{fr},$$

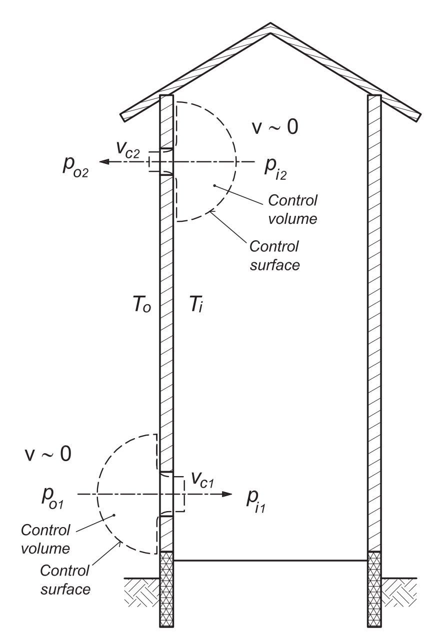

Fig. 2. Control volumes and control surfaces for inlet and outlet.

which inserted into an equation similar to Eq. (4) together with Eq. (5b) results in

$$\tfrac{1}{2}v_{c2}^2 + p_{o2}/\rho_i + \tfrac{1}{2}\zeta_2 v_{c2}^2 - p_{i2}/\rho_i = 0$$

or

$$p_{i2} - p_{o2} = \Delta p_2 = \tfrac{1}{2}\rho_i(1+\zeta_2)v_{c2}^2 = \tfrac{1}{2}\rho_i\psi_2 v_{c2}^2, \tag{9a}$$

where $\psi_2 = 1 + \zeta_2$ is a constant, in the following called the flow coefficient, which is solely introduced to simplify some of the equations in following.

3.2. Inlet airflow

For the inlet airflow, a control volume similar to the one for the outlet can be considered, cf. Fig. 2. In this case, air velocity and static pressure are v_{c1} and p_{o1} at entrance and they are approximately zero and p_{i1} at exit. With no heat supply and entrance and exit at same level, you get in similarity to Eq. (9a)

$$p_{o1} - p_{i1} = \Delta p_1 = \tfrac{1}{2}\rho_o(1+\zeta_1)v_{c1}^2 = \tfrac{1}{2}\rho_o\psi_1 v_{c1}^2. \tag{9b}$$

3.3. Density difference versus temperature difference

The atmosphere can be considered as a perfect gas, where the relationship between pressure, density and temperature is expressed by

$$p = \rho RT, \tag{10a}$$

where R is the gas constant for the air. From this equation, the following relationship between density difference and temperature difference can be derived:

$$\Delta\rho = \rho_o - \rho_i = \frac{1}{R}\left(\frac{p_o}{T_o} - \frac{p_i}{T_i}\right) = \frac{1}{R}\left(\frac{p_o T_i - p_i T_o}{T_o T_i}\right)$$
$$= \frac{p_o}{RT_o}\left(\frac{T_i - (p_i/p_o)T_o}{T_i}\right),$$

which again, after some manipulation, can be rearranged as

$$\Delta\rho = \rho_o \frac{T_i - T_o}{T_i}\left(1 + \frac{T_o}{T_i - T_o}\,\frac{p_o - p_i}{p_o}\right)$$
$$= \rho_o \frac{T_i - T_o}{T_i}(1+K). \tag{10b}$$

In practice, the value of K is smaller than 0.4×10^{-2}. Further, $p_i/p_o \approx 1.0$ with an error smaller than 0.02% so that $\rho_i T_i \approx \rho_o T_2$, cf. Eq. (10a). From Eq. (10b) you then get with an error below 0.5%

$$\Delta\rho = \frac{p_o}{RT_o}\left(\frac{T_i - T_o}{T_i}\right) = \rho_o\frac{\Delta T}{T_i} = \rho_i\frac{\Delta T}{T_o}. \tag{10c}$$

3.4. Outside pressure distribution

In Eqs. (9a) and (9b), the two outside static pressures, p_{o1} at inlet level and p_{o2} at outlet level, are introduced. Assuming one of these pressures to be a known quantity, only one new dependent variable is introduced and only one new independent equation is required. This new equation can be the hydrostatic relationship between the two pressures, i.e.

$$p_{o1} - p_{o2} = \rho_o g H. \tag{11}$$

4. Solutions

A total of seven equations with seven dependent variables are derived, and you can get an unambiguous solution with net heat input as independent variable. Alternatively, the indoor air density or the indoor air temperature can be chosen as independent variable. For instance, the indoor air temperature acts as independent variable, when this temperature is chosen as design criteria. The result is then unambiguous solutions based on air density differences or temperature differences as shown in the following.

4.1. Solution based on density differences

By using Eqs. (3a), (8), (9a), (9b) and (11), a solution with air density difference as independent variable can be derived.

4.1.1. Air velocities

From Eqs. (9a) and (9b), you get

$$p_{i1} = p_{o1} - \tfrac{1}{2}\rho_o\psi_1 v_{c1}^2, \tag{12}$$

$$p_{i2} = p_{o2} + \tfrac{1}{2}\rho_o\psi_2 v_{c2}^2. \tag{13}$$

By inserting these two equations into Eq. (8), you get

$$p_{o1} - \tfrac{1}{2}\rho_o\psi_1 v_{c1}^2 - (p_{o2} + \tfrac{1}{2}\rho_o\psi_2 v_{c2}^2) = \rho_i g H$$

and further, by using Eq. (11)

$$\tfrac{1}{2}\rho_o\psi_1 v_{c1}^2 + \tfrac{1}{2}\rho_o\psi_2 v_{c2}^2$$
$$= -\rho_i g H + (p_{o1} - p_{o2}) = -\rho_i g H + \rho_o g H$$
$$= (\rho_o - \rho_i) g H = \Delta\rho g H. \tag{14}$$

From Eq. (3a) you get

$$v_{c2} = (\rho_o/\rho_i)(A_{c1}/A_{c2})v_{c1} \tag{15}$$

and by inserting this into Eq. (14), you get for determining the inlet velocity

$$\tfrac{1}{2}v_{c1}^2(\rho_o\psi_1 + \rho_i\psi_2(\rho_o/\rho_i)^2(A_{c1}/A_{c2})^2) = gH\Delta\rho$$

or

$$v_{c1} = \left(\frac{2\Delta\rho g H}{\rho_o\psi_1 + \rho_i\psi_2(\rho_o/\rho_i)^2(A_{c1}/A_{c2})^2}\right)^{1/2}$$

$$= \left(\frac{2\Delta\rho g H}{\rho_o\psi_1(1 + (\rho_o/\rho_i)(\psi_2/\psi_1)(A_{c1}/A_{c2})^2)}\right)^{1/2}$$

$$= \left(\frac{2\Delta\rho g H_1}{\rho_o\psi_1}\right)^{1/2}, \tag{16}$$

where

$$H_1 = \frac{H}{1 + (\rho_o/\rho_i)(\psi_2/\psi_1)(A_{c1}/A_{c2})^2}. \tag{17}$$

Similarly, you get for the outlet velocity (or by inserting Eq. (16) into Eq. (15))

$$v_{c2} = \left(\frac{2\Delta\rho g H_2}{\rho_i\psi_2}\right)^{1/2}, \tag{18}$$

where

$$H_2 = \frac{H}{1 + (\rho_i/\rho_o)(\psi_1/\psi_2)(A_{c2}/A_{c1})^2}. \tag{19}$$

By adding Eqs. (17) and (19) you get

$$H_1 + H_2 = \frac{H}{1+n} + \frac{H}{1+1/n} = H. \tag{20}$$

4.1.2. Pressure conditions. Neutral plane

Having determined the two velocities, indoor pressures at inlet and outlet level can be found. From Eqs. (9b) and (16) you get

$$p_{i1} = p_{o1} - \tfrac{1}{2}\rho_i\psi_1 v_{c1}^2 = p_{o1} - \Delta\rho g H_1 \tag{21a}$$

and by using Eq. (8) you get

$$p_{i2} = p_{i1} - \rho_i g H = p_{o1} - (\rho_o - \rho_i) g H_1 - \rho_i g H$$

or by inserting $H_1 = H - H_2$, cf. Eq. (20)

$$p_{i2} = p_{o1} - \rho_o g H + \rho_o g H_2 + \rho_i g H - \rho_i g H_2 - \rho_i g H$$

$$= p_{o1} - \rho_o g H + \rho_o g H_2 - \rho_i g H_2$$

$$= p_{o2} + (\rho_o - \rho_i) H_2. \tag{21b}$$

As to the pressure differences across the openings, you get from Eq. (21a) for the inlet

$$\Delta p_1 = p_{o1} - p_{i1} = \Delta\rho g H_1 \tag{22a}$$

and for the outlet, you get from Eq. (21b)

$$\Delta p_2 = p_{i2} - p_{o2} = \Delta\rho g H_2. \tag{22b}$$

As seen, there is an indoor negative pressure at inlet level and an indoor positive pressure at outlet level. Somewhere in between will be a level with neutral pressure, i.e. where indoor and outdoor pressures are equal. This level is called the neutral plane.

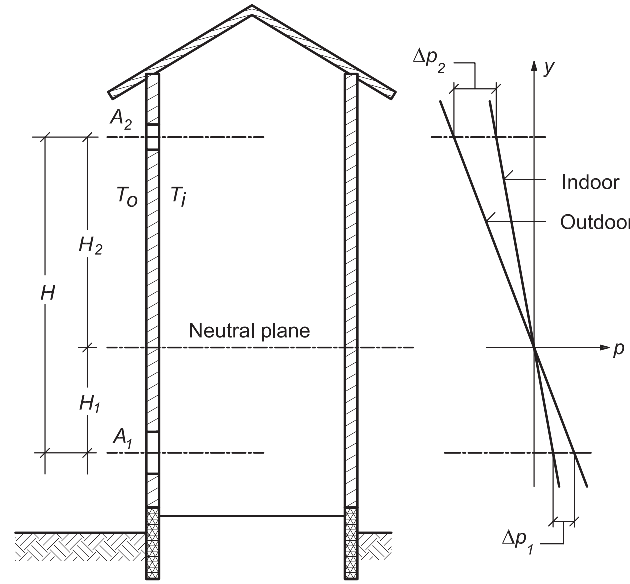

Fig. 3. Indoor and outdoor pressure distributions for room with two openings and with uniform temperature.

The pressure conditions are illustrated in Fig. 3. Outdoor pressure can be expressed by

$$p_{oy} = p_{o1} - \rho_o g y \tag{23a}$$

and indoor pressure can be expressed by, when using Eq. (21a)

$$p_{iy} = p_{i1} - \rho_i g y = p_{o1} + \Delta\rho g H_1 - \rho_i g y. \tag{23b}$$

4.1.3. Neutral plane position

To find the neutral plane position, Eqs. (23a) and (23b) are equated, and you get

$$p_{o1} - \rho_o g y = p_{o1} + (\rho_o - \rho_i) g H_1 - \rho_i g y$$

or

$$(\rho_o - \rho_i) g y = (\rho_o - \rho_i) g H_1$$

or

$$y = H_1.$$

Thus, H_1 is the vertical distance from neutral plane to inlet, and from Eq. (20) it is seen that H_2 is the vertical distance from neutral plane to outlet.

The two distances can be expressed in a shorter form

$$H_1 = \frac{H}{1+n^2}, \tag{24}$$

$$H_2 = \frac{H}{1+(1/n)^2} = \frac{n^2 H}{1+n^2}, \tag{25}$$

where

$$n=(\rho_o/\rho_i)(\psi_2/\psi_1)(A_{c1}/A_{c2})^2$$
$$=(\rho_o/\rho_i)(C_{d2}/C_{d1})^2(A_1/A_2)^2. \quad (26)$$

For instance for $n = 1.0$, you get $H_1 = H_2 = H/2$, and for $n=2.0$ you get $H_1 = 0.2H$ and $H_2 = 0.8H$.

Frequently, it can be assumed that $\rho_{u\approx\rho i}$ and that $C_{d1} \approx C_{d2}$. You then get

$$H_1 \approx \frac{H}{1+(A_1/A_2)^2}, \quad (27)$$

$$H_2 \approx \frac{H}{1+(A_2/A_1)^2}. \quad (28)$$

4.1.4. Airflow rate

The airflow rate through the inlet can be determined by

$$q_{v1} = A_{c1}v_{c1} = C_{c1}A_1C_{v1}v_{\text{theo}1} = C_{d1}A_1\left(\frac{2\Delta\rho g H_1}{\rho_o}\right)^{1/2} \quad (29)$$

and for the airflow rate through the outlet, you get

$$q_{v2} = A_{c2}v_{c2} = C_{c2}A_2C_{v2}v_{\text{theo}2} = C_{d2}A_2\left(\frac{2\Delta\rho g H_2}{\rho_i}\right)^{1/2}. \quad (30)$$

The two airflow rates are different corresponding to the two different air densities. However, the mass flow rates through the two openings are equal.

4.1.5. Stack effect and stack height

It is the quantity $\Delta\rho g H$, which creates the airflow and overcomes the friction in the two openings, cf. Eq. (14). Therefore, this quantity is often called the stack effect, and the vertical distance H between the openings is called the stack height. The unit of the quantity is Pa or J/m^3. It can be interpreted as the pressure difference, which creates the air movement, or the work carried out on 1 m^3 air as to give this amount of air a certain velocity.

The pressure difference across the inlet and the air velocity in the inlet depends on the vertical distance H_1 between inlet and neutral plane. For the outlet, you have a similar dependency on the distance H_2. Therefore, the distances H_1 or H_2 are often called efficient stack heights.

4.1.6. Coefficients

By frictionless flow, i.e. $\zeta=0$, the air velocity in an opening is determined by, cf. for instance Eq. (9a)

$$v_{\text{theo}} = (2\Delta p/\rho)^{1/2} \quad (31)$$

and you get

$$\Delta p = \tfrac{1}{2}\rho v_{\text{theo}}^2.$$

In a more general case with friction, you get from an equation similar to Eq. (9a), and by using Eq. (1), the following equation for determining the velocity coefficient C_v:

$$\Delta p = \tfrac{1}{2}\rho v_{\text{theo}}^2 = \tfrac{1}{2}\rho(1+\zeta)v_c^2 = \tfrac{1}{2}\rho(1+\zeta)C_v^2 v_{\text{theo}}^2$$

or

$$C_v = \frac{1}{(1+\zeta)^{1/2}} = \frac{1}{\psi^{1/2}}. \quad (32)$$

For the discharge coefficient you get

$$C_d = C_v C_d = \frac{C_c}{(1+\zeta)^{1/2}} = \frac{C_c}{\psi^{1/2}}. \quad (33)$$

For a simple sharp-edged opening as a window, you have for instance $\zeta \sim 0.1$ and $C_c \sim 0.7$ so that

$$C_v \approx \frac{1}{(1+0.1)^{1/2}} \approx 0.95$$

and

$$C_d \approx 0.95 \cdot 0.7 \approx 0.65.$$

4.1.7. Air density considerations

So far, only air density differences due to temperature have been considered. However, an air density difference may as well be created by differences in moisture content. Adding moisture to the air results in lower air density as the weight of vapour is smaller than that of air. The relationships between density, moisture content (or relative humidity) and net moisture input are like the relationships between density, temperature and net heat input.

When adding heat as well as moisture to the air, the effect of the heat will be dominating under practical conditions. Therefore, the relationship between air density difference and air temperature, described by Eq. (10c), can still be used. Only in very extreme situations will the error exceed 10%, and you will be on the safe side from a design point of view.

4.2. Solution based on temperature difference

By replacing density differences with temperature differences as expressed by Eq. (10b), you get a solution with temperature difference as independent variable, and with pressure differences, air velocities and airflow rates expressed by

$$\Delta p_1 = \frac{\rho_o g H_1 \Delta T}{T_i} = \frac{\rho_i g H_1 \Delta T}{T_o}, \quad (34)$$

$$\Delta p_2 = \frac{\rho_o g H_2 \Delta T}{T_i} = \frac{\rho_i g H_2 \Delta T}{T_o}, \quad (35)$$

$$v_{c1} = \left(\frac{2gH_1\Delta T}{\psi_1 T_i}\right)^{1/2}, \quad (36)$$

$$v_{c2} = \left(\frac{2gH_2\Delta T}{\psi_2 T_o}\right)^{1/2}, \quad (37)$$

$$q_{v1} = C_{d1}A_1\left(\frac{2gH_1\Delta T}{T_i}\right)^{1/2}, \tag{38}$$

$$q_{v2} = C_{d2}A_2\left(\frac{2gH_2\Delta T}{T_o}\right)^{1/2}, \tag{39}$$

where

$$H_1 = \frac{H}{1+(T_i/T_o)(C_{d1}/C_{d2})(A_1/A_2)^2}, \tag{40}$$

$$H_2 = \frac{H}{1+(T_o/T_i)(C_{d2}/C_{d1})(A_2/A_1)^2}. \tag{41}$$

4.3. Solution based on net heat input

The difference between indoor and outdoor temperature is due to the net heat input, and from Eq. (7) you get following relationship between the two quantities:

$$\Delta T = \frac{\Phi}{\rho_o c_p A_{c1} v_{c1}} = \frac{\Phi}{\rho_o c_p q_{V1}} = \frac{\Phi}{\rho_i c_p q_{V2}}. \tag{42}$$

However, the airflow rate in the denominator is not a fixed quantity. As seen from Eq. (38) it is dependent on the temperature difference, i.e.

$$\Delta T = \frac{\Phi}{\rho_o c_p C_{d1} A_1((2gH_1\Delta T)/T_i)^{1/2}}. \tag{43}$$

For a fixed net heat input, you get a temperature difference that results in an airflow rate, which removes an amount of heat per time unit equal to the net heat input.

4.3.1. Constant net heat input

Assuming constant net heat input, Eq. (43) yields, when solving it with regard to ΔT

$$\Delta T = \left(\frac{\Phi}{\rho_o T_i c_p C_{d1} A_1}\right)^{2/3}\left(\frac{1}{2gH_1}\right)^{1/3} T_i.$$

Outdoor air density can be replaced by outdoor temperature by using Eq. (10a), and after some manipulation you get

$$\Delta T = \left(\frac{R}{p_o c_p}\right)^{2/3}\left(\frac{1}{2g}\right)^{1/3}\left(\frac{T_o}{T_i}\right)^{2/3}\left(\frac{\Phi}{C_{d1}A_1}\right)^{2/3}\left(\frac{1}{H_1}\right)^{1/3} T_i.$$

For the constants, following values can be inserted:

$R = 287$ J/(kg K),

$p_o = 101\,300$ Pa,

$c_p = 1010$ J/(kg K),

$g = 9.82$ m/s^2.

Further, under practical conditions you have with an error below 2%

$$(T_o/T_i)^{2/3} = (1-\Delta T/T_i)^{2/3} \approx 0.97,$$

so that you get

$$\Delta T = 7.1\times 10^{-5}\left(\frac{\Phi}{C_{d1}A_1}\right)^{2/3}\left(\frac{1}{H_1}\right)^{1/3} T_i. \tag{44}$$

When using the outlet conditions, you similarly get

$$\Delta T = 7.5\times 10^{-5}\left(\frac{\Phi}{C_{d2}A_2}\right)^{2/3}\left(\frac{1}{H_2}\right)^{1/3} T_o. \tag{45}$$

By replacing temperature differences with net heat input according to Eqs. (44) and (45), you get a solution with net heat input as independent variable, and with air velocities and airflow rates expressed by

$$v_{c1} = 0.037\left(\frac{\Phi H_1}{C_{d1}A_1}\right)^{1/3}\left(\frac{1}{\psi_1}\right)^{1/2}, \tag{46}$$

$$v_{c2} = 0.038\left(\frac{\Phi H_2}{C_{d2}A_2}\right)^{1/3}\left(\frac{1}{\psi_2}\right)^{1/2}, \tag{47}$$

$$q_{v1} = C_{c1}A_1 v_{c1} = 0.037(\Phi H_1)^{1/3} A_1^{2/3}\left(\frac{1}{C_{d1}}\right)^{1/3} \times \frac{C_{c1}}{\psi_1^{1/2}} = 0.037(\Phi H_1)^{1/3}(C_{d1}A_1)^{2/3} \tag{48}$$

as $C_{c1}/\psi_1^{1/2} = C_{d1}$, cf. Eq. (32).

Similarly, you get the following airflow rate through the outlet:

$$q_{V2} = 0.038(\Phi H_2)^{1/3}(C_{d2}A_2)^{2/3}. \tag{49}$$

4.3.2. Temperature-dependent net heat input

If net heat input is temperature dependent as shown by Eq. (6), you get the following equation instead of Eq. (44):

$$\Delta T = K(\Phi_0 - k\Delta T)^{2/3}$$

or

$$\Delta T^3 = K^3(\Phi_0 - k\Delta T)^2. \tag{50}$$

Solving this equation of third degree, you get a complicated expression for ΔT. It can be solved iteratively.

4.4. Design calculations

In the design situation, the task is to calculate the required opening areas to ensure acceptable indoor air quality in winter and acceptable indoor thermal comfort in summer. In winter as well as in summer, a required airflow rate is determined. In winter, when acceptable indoor air quality is represented by an acceptable CO_2 or moisture level, the required airflow rate is determined by

$$q_{\mathrm{Vreq}} = \frac{G}{(c_{i,acc} - c_o)}, \tag{51}$$

where G is the production rate of pollution (m^3/s), $c_{i,acc}$ the accepted indoor pollution concentration (m^3/m^3) and c_o is the outdoor pollution concentration (m^3/m^3).

In summer, when acceptable thermal comfort is represented by an acceptable air temperature $T_{i,acc}$, the required airflow rate is determined by, cf. Eq. (7)

$$q_{V,req} = \frac{\Phi}{c_p \rho_o (T_{i,acc} - T_0)}. \tag{52}$$

In the design situation, known quantities are building geometry, represented by the vertical opening distance H, and the opening area rate $n = A_1/A_2$.

In cases where it is preferable to work with a solution based on temperature difference, the required inlet area is determined by, cf. Eq. (38)

$$A_{1,req} = \frac{q_{V,req}}{C_{d1}((2gH_1 \Delta T)/T_i)^{1/2}}. \tag{53}$$

If the solution is based on net heat input, you get from Eq. (48)

$$A_1 = 140 \frac{q_{V,req}^{3/2}}{C_{d1}(\Phi H_1)^{1/2}}. \tag{54}$$

By solving Eq. (44), accepted indoor temperature can be included, and you get

$$A_{1,req} = 6.0 \times 10^{-7} \frac{\Phi}{C_{d1}} \left(\frac{1}{H_1}\right)^{1/2} \left(\frac{T_i}{\Delta T_{acc}}\right)^{3/2}. \tag{55}$$

From the required inlet area, the required outlet area can be found as $A_{2,req} = A_{1,req}/n$.

5. Conclusion

By setting up and solving the fundamental flow equations, you get a solution with all parameters taken into account. This gives a consistent explanation of the mechanism of natural ventilation by thermal buoyancy. Further steps can be taken with this fundamental method as to get correct solutions in case of indoor temperature stratification or if the room has more than two openings above each other. Likewise it is possible to analyse analytically such questions of importance for design and control as

- Is the opening orientation (vertical, horizontal or sloped) of any importance?
- What happens, when the neutral plane intersects an opening, and how can this be avoided?
- What is the optimal opening area ratio?
- What is the relationship between airflow rate, opening area and opening area ratio when either temperature difference or net heat input is kept constant?

By using the fundamental method, a reliable tool is obtained for analysing and designing natural ventilation systems in situations where thermal buoyancy is the dominating driving force.

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