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Tracking of the time-variant parameters of radio propagation paths using a particle filter

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Abstract—In this contribution we design a particle filtering approach to track the time-evolution of the parameters of propagation paths in radio channel. The time-evolution of the parameters is described using a dynamic state space model, where the state vector of a path contains the delay, direction of arrival, Doppler frequency, the rates of change in these dimensions, and the complex amplitude of the path. The proposed particle filter is designed specifically for the MIMO channel sounding scenario, where the posterior probability density function of the path parameters are highly concentrated. Preliminary simulation results demonstrate the performance of the particle filter.

Index Terms—Radio propagation channel, particle filter, path parameters, extended Kalman filter and maximum-likelihood estimation.

I. INTRODUCTION

The response of the radio propagation channel can be modelled as a superposition of multiple path components. Each component is contributed by an electromagnetic wave propagating along a path between the transmitter (Tx) and the receiver (Rx). Along its path, each wave may interact with objects called scatterers. The path components are characterized by various dispersion parameters, such as delay, direction of arrival (DoA), direction of departure (DoD), polarization, as well as Doppler frequency. In time-variant scenarios, due to long-term/large-scale fluctuations, the above dispersion parameters may vary with time. As an example, as the Rx moves along a certain trajectory, the length of the propagation paths changes correspondingly, and so do the propagation delays of the path components. Temporal fluctuations can also be observed for the other dispersion parameters. Knowing the time-evolution behavior of these parameters is of paramount importance in mobile communications since they heavily affect the overall behaviour of the propagation channel and therefore of modern communication systems operating in this channel.

In recent years, estimation and tracking of the time-variant path parameters for channel sounding have gained a lot of attention \cite{1, 2, 3, 4, 5}. In \cite{1}, a recursive expectation-maximization (EM) and a recursive space alternating generated EM (SAGE)-inspired algorithms are proposed for tracking of the DoA of individual paths. In \cite{2, 3, 4, 5}, the standard extended Kalman filter (EKF) is used to track the delay, DoA, DoD and polarizations of the paths. A common feature of these algorithms is that they employ approximation of the non-linear observation model with a linear model which relies on the Taylor-series expansion at the previous estimates of model parameters. The accuracy of this linear approximation becomes poor when the path parameters vary severely in time. In such cases, the algorithms may lose the tracks of the parameters. Furthermore, in these algorithms parameter updating requires solving the second-order derivative of the received signal with respect to (w.r.t.) the path parameters. This poses the necessity of computing the second-order derivative of the array response w.r.t. the angular parameters. In the channel sounding scenario, array responses usually have no analytical expression. Thus, the derivatives are computed numerically using the measured response. When calibration errors exist, these derivatives can be erroneous and as a consequence, the performance of the algorithms degrades. Furthermore, the EKF and the recursive EM and SAGE-inspired algorithms are only applicable in the case where the driving process in the parameter dynamics is Gaussian. However this condition cannot be fulfilled in the case of distributed scatterers. From \cite{6} it is shown that the dynamics of the parameters of the paths induced by distributed scatterers are driven by a process with a heavy-tail distribution.

In this contribution, we propose to use the particle filter (PF) to track the path parameters. The PF is a Bayesian estimation method based on Monte-Carlo simulations. Different from the EKF and the recursive EM and SAGE-inspired algorithms, the PF is applicable in the case where the parameter transition model and the observation model are nonlinear, and in the case where the driving process in the parameter dynamics is non-Gaussian. These cases are common in the channel sounding scenario, where the received signal is nonlinear w.r.t. the path parameters, and the driving processes in parameter dynamics are not necessarily Gaussian. Therefore, the PF is an appropriate algorithm for tracking the path parameters in channel sounding.

Tracking the parameters of propagation paths using PFs has been investigated in radar applications, such as target tracking and navigation. However, the parameter space considered in these contexts has usually one or two dimensions,
i.e. delay or/and Doppler frequency for each target. In our case of channel sounding, the parameter space can be up to 14-dimensional, i.e. in delay, Doppler frequency, direction (i.e. azimuth and elevation) of arrival and departure, as well as complex polarization matrix. Another feature of channel sounding is that the observation apertures of the sounding equipment can be large, leading to high resolutions in multiple dimensions. As a result, the posterior probability density function (pdf) of the path parameters is highly concentrated in the multi-dimensional parameter space. It is a difficult problem to "steer" the particle sets to the regions where the significant parts of the pdf are located. In this contribution, we propose two techniques to solve this problem. They prove to be effective by means of simulations with synthetic data.

The organization of the paper is as follows. II presents the state space model for the path parameters and the model of observation signals. In Section III, the framework of the proposed PF is formulated. Section IV describes the results of simulation studies for performance evaluation of the PF. Conclusion remarks are made in Section V.

II. SIGNAL MODEL

In this section, we introduce a state space model describing the dynamics of the path parameters. The observation model for the received signal in the Rx of the sounding equipment is provided. For simplicity, the presentation of these models are based on a single-path scenario. However, the extension of the models to multiple-path scenarios is straightforward.

A. State Space Model

We consider a scenario where the time-variant path parameters are delay $\tau$, azimuth of arrival $\phi$, elevation of arrival $\theta$, Doppler frequency $\nu$, the rates of change of these parameters denoted with $\Delta \tau$, $\Delta \phi$, $\Delta \theta$, and $\Delta \nu$ respectively, as well as the complex amplitude $\alpha$. For the $k$th observation, these parameters are written as a state vector $\Omega_k$

$$\Omega_k = [\tau, \Delta \tau, \phi, \Delta \phi, \theta, \Delta \theta, \nu, \Delta \nu, |\alpha|, \arg(\alpha)]^T,$$

where $\alpha$ and $\arg(\alpha)$ represent the magnitude and the angle of the complex amplitude $\alpha$ respectively, and $[\cdot]^T$ denotes the transpose operation. We model the state vector $\Omega_k$ as a Markov process, i.e.

$$p(\Omega_{k+1}|\Omega_{k}) = p(\Omega_{k} | \Omega_{k-1}).$$

The transitions of $\Omega_k$ w.r.t. $k$ can be modelled as

$$\Omega_k = F_k \Omega_{k-1} + w_k, \quad k = 1, \ldots, K,$$

where $K$ denotes the total number of the observations and the transition matrix $F_k$ reads

$$F_k = \begin{bmatrix}
1 & T_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & T_k & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & T_k & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & T_k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}.$$  \hspace{1cm} (4)

In (4), $T_k$ is the interval between the starts of the $(k-1)$th observation period and the $k$th observation period. The driving vector $w_k$ in (3) reads

$$w_k = \begin{bmatrix} 0 \ w_{\Delta \tau, k} \ 0 \ w_{\Delta \phi, k} \ 0 \ w_{\Delta \theta, k} \ 0 \ w_{\Delta \nu, k} \ |\alpha|_k \ \arg(\alpha)_k \end{bmatrix}^T,$$

where the entries $w_{(\cdot),k}$ with $(\cdot)$ replaced by $\Delta \tau$, $\Delta \phi$, $\Delta \theta$, $\Delta \nu$, $|\alpha|$ and $\arg(\alpha)$ are Gaussian random variables $w_{(\cdot),k} \sim N(0, \sigma_{(\cdot),k}^2)$.

In this contribution, we consider the case where $T_k = T$, $k \in [1, \ldots, K]$. The variances $\sigma_{(\cdot),k}^2$ are assumed to be time-invariant. Thus, the subscript $k$ in $F_k$ and $\sigma_{(\cdot),k}^2$ are dropped in the sequel.

B. Observation model

The measurement data received in the $k$th observation period can be written as

$$y_k(t) = x_k(t) + v_k(t),$$

where the signal contribution $x_k(t)$ is

$$x_k(t) = \alpha_k \exp\left(j2\pi f_k t\right) \cos(\phi_k, \theta_k) u(t - \tau_k)$$

with $c(\phi_k, \theta_k)$ being the array response at azimuth-elevation $(\phi_k, \theta_k)$, and the noise vector $v_k(t)$ is a vector-valued zero-mean Gaussian process. The entries of $v_k(t)$ have identical variance denoted with $\sigma_v^2$. The path parameters arising in (7), i.e. $\phi_k$, $\theta_k$, $\tau_k$, are $\alpha_k$, belong to a subset of the state-vector $\Omega_k$. The rate of change parameters in $\Omega_k$ are “invisible” in the observation model. For notational convenience, we use $Y_k = [y_{k1}(t_1), y_{k2}(t_2), \ldots, y_{KN}(t_N)]$ to represent the received signal matrix in the $k$th observation period.

From (6) we see that the radio channel is assumed to be memoryless. So the received signal $y_{kl}(t)$ is independent of $y_{k'}(t)$ and $\Omega_{k'}$, for $k' \neq k$. It is shown in [7] that this feature, together with the assumption that the path state vector is a Markov process, allow to estimate the posterior pdf of the path parameters using recursive Bayesian estimation methods.

III. A PARTICLE FILTER APPROACH

In this section we describe the framework of a PF for tracking the vector $\Omega_k$ based on the models (3) and (6). This framework is formulated for a single-path scenario. But it is straightforward to extend it to the multiple-path scenario.

A. The framework of the PF

We start by discussing the dimensions in which particles are distributed. From (3) we see that the dynamics of the parameters are driven by the random components in the rate of change of the kinematic parameters and the complex amplitude. This means that given the state vector $\Omega_{k-1}$, the new vector $\Omega_k$ can be determined by specifying $w_{\Delta \tau, k}$, $w_{\Delta \phi, k}$, $w_{\Delta \theta, k}$, $w_{\Delta \nu, k}$, $|\alpha|_k$ and $\arg(\alpha)_k$. The observed signal $y_{k}(t)$ is a linear function of the complex amplitude $\alpha_k$. Thus, the maximum likelihood estimate of $\alpha_k$ can be calculated in an analytical expression of $y_{k}(t)$ and the other parameters. Thus, as long as the initial state $\Omega_1$ or its estimate is given, the
where $\Delta \tau$, $\Delta \phi$, $\Delta \theta$, and $\Delta \nu$ following [7, Section 6.1.4], this space is called effective state space. The vector $\Omega_k = [\Delta \tau_k, \Delta \phi_k, \Delta \theta_k, \Delta \nu_k]^T$ is an effective state vector of the path at the $k$th observation period.

The PF using $I$ particles performs the following steps when a new observation, say $Y_k$, arrives. The outputs are the updated state vectors $\hat{\Omega}_k^i$ and their importance weights $w_k^i$, $i = 1, \ldots, I$.

**Step 1, Predict the state vectors of particles.** In this step, we predict the state vectors of all particle in the $k$th observation period. Firstly, the parameters in the effective state space, i.e. $\hat{\Omega}_k^i$ are updated as

$$\hat{\Omega}_k^i = \hat{\Omega}_{k-1}^i + w^i, \quad i = 1, \ldots, I. \quad (8)$$

Here, the vector $w^i \in \mathbb{R}^4$ is a realization of a distribution $N(0, \hat{\Sigma}_w)$. The covariance matrix $\hat{\Sigma}_w$ reads

$$\hat{\Sigma}_w = \text{diag}(\sigma^2_{\Delta \tau}, \sigma^2_{\Delta \phi}, \sigma^2_{\Delta \theta}, \sigma^2_{\Delta \nu}), \quad (9)$$

where diag$(\cdot)$ denotes a diagonal matrix with diagonal elements equal to the given arguments. The values of $\sigma^2_{\Omega}$ with $(\cdot)$ replaced by $\Delta \tau$, $\Delta \phi$, $\Delta \theta$ or $\Delta \nu$, are set to be predetermined values.

Given $\hat{\Omega}_k^i$, the other kinematic parameters in the state vector of a particle are calculated as follows:

$$\phi_k^i = (\phi_k^{i-1} + \Delta \phi_k^i), \quad (10)$$

where $(\cdot)$ refers to $\tau$, $\phi$, $\theta$ or $\nu$. The complex amplitude $\alpha_k^i$ is computed by using the least square method with (6) as

$$\alpha_k^i = \frac{\text{vec}(S_k^i)^* \text{vec}(Y_k)}{||S_k^i||^2}, \quad (11)$$

with $\| \cdot \|$ denoting the norm of the given argument. In (11),

$$S_k^i(t) = [s_k^i(t_1), s_k^i(t_2), \ldots, s_k^i(t_N)]$$

with

$$s_k^i(t) = \exp \{ j 2 \pi \nu_k^i t c(\phi_k^i, \theta_k^i) u(t - \tau_k^i) \}, \quad t = t_1, \ldots, t_N. \quad (12)$$

and vec$(\cdot)$ representing vectorization operation which concatenates the entries of the given matrix in a vector.

**Step 2, Calculate the particle importance weights.** The importance weights of the particles are computed recursively as

$$w_k^i = \frac{w_{k-1}^i p(Y_k|\hat{\Omega}_k^i)}{\sum_{i=1}^I w_{k-1}^i p(Y_k|\hat{\Omega}_k^i)}, \quad i = 1, \ldots, I \quad (13)$$

with

$$p(Y_k|\hat{\Omega}_k^i) = \frac{1}{\sqrt{2\pi \sigma_w}} \frac{1}{NM} \exp \{ - \frac{1}{2\sigma_w^2} || Y_k - \alpha_k^i S_k^i ||^2 \}. \quad (14)$$

In the channel sounding scenario, the number of entries of $Y_k$ is significant. This leads to the problem that the values in the exponent in (14) are so small that computation softwares, such as MatLAB, return zero regardless of the value of $\Omega_k$. To solve this problem, we include constant number $a = -\frac{\Delta \phi_k^i}{2}$ in the exponent. This value coincides with $-||W_k||^2/2\sigma_w^2$ when $N \times M$ is large. Here, $W_k$ denotes the noise matrix $W_k = [w_k(t_1), w_k(t_2), \ldots, w_k]$. Simulations show that introducing this constant can solve this problem. Notice that the particle importance weights are not influenced by introducing the constant.

**Step 3, Resample the particles.** In this step, the particles with significant importance weights are first selected. We use $\{i^p\}$ to denote the index set of the selected particles. Based on the states of those particles, new particles are generated

$$\hat{\Omega}_k^i = p(\Omega_k^i|\Omega_k^{i(i)}) \quad i = 1, \ldots, I, \quad (15)$$

where $j(i)$ denotes a particle index within $\{i^p\}$. The notation $j(i)$ indicates that the state of the $i$th new particle is generated based on the state of the $j(i)$th particle. The importance weights of the new particles are computed as

$$w_k^i = \frac{p(Y_k|\hat{\Omega}_k^i) w_k^j(i)}{\sum_{i=1}^I p(Y_k|\hat{\Omega}_k^i) w_k^j(i)} \quad (16)$$

This step repeats until all new particles have non-negligible importance weights.

**Step 4, Estimate the posterior pdf and its moments.** The estimate of the posterior pdf can be approximated with the particle states and importance weights

$$\hat{p}(\Omega_k|Y_{1:k}) = \sum_{j=1}^I w_k^j \delta(\Omega_k - \hat{\Omega}_k^j), \quad (17)$$

which can be used to compute the estimates of the moments of $\Omega_k$.

**B. Sample Management Technique**

The particle filter described in Subsection III-A is applicable under the condition that, the particles with non-negligible importance weights are found before the resampling step. However, a noticeable problem resulting from the highly concentrated posterior pdf is that, the particles generated in Step 1 are too diffuse to “catch” the significant parts of the pdf. A solution is to increase the number of particles, resulting in high computational complexity unfortunately. We propose a solution which uses a small number of particles.

We call this solution a sample management technique. The basic idea is to control the amount of the observation samples used in calculation of the particle importance weights in such a way that, the posterior pdf becomes less concentrated. This solution is implemented as follows. When Step 2 in Subsection III-A is performed, a partition of $Y_k$, denoted with $Y_k$, is first used to calculate $w_k^i$ in (13). Note that this partition must be selected in such a way that the likelihood $p(Y_k|\Omega_k)$ does not exhibit ambiguity problems. As the number of observation samples is less in $Y_k$ than in $Y_k$, the posterior pdf becomes more dispersive. Consequently the probability to find particles with significant importance weights is enhanced. We perform Steps 2 and 3 in Subsection III-A until all particles have non-negligible importance weights. Then, the partition $Y_k$ is resampled with more observation samples included. This iterative operation is performed until all observation samples in $Y_k$ are considered.
C. A New Approach to Calculate Particle Importance Weights

A drawback of using the proposed sample management technique is that, when the small partition \( \hat{Y}_k \) is selected the resolutions of the measurement equipment become so low that the posterior pdf \( p(\Omega_k | \hat{Y}_k) \) differs significantly from \( p(\Omega_k | Y_k) \). An alternative method to the sample management technique is that when calculating the particle importance weights, the posterior probability \( p(Y_k | \Omega_k^i) \) in (13) is substituted by its log, i.e.

\[
\log w_{k,i}^w = \frac{w_{k-1}^w \log(p(Y_k | \Omega_k^i))}{\sum_{i=1}^I w_{k-1}^w \log(p(Y_k | \Omega_k^i))}, \quad i = 1, \ldots, I. \tag{18}
\]

We call this new important weight as log importance weight. The particles and their log importance weights approximate the posterior pdf in the log scale. Since the pdf in the log scale is much wider than in the linear scale, the negligible importance weights computed using (13) may become non-negligible. Applying Steps 2 and 3 in Subsection III-A, the states of the particles are updated until all the log importance weights are significant. This method can be used as the initialization of the particle states.

IV. SIMULATION STUDIES

In this section, preliminary simulation results are presented for evaluation of the performance of the proposed PF. A channel sounding system with a SIMO configuration is considered. The Rx is equipped with an isotropic 4 \( \times \) 4 planar array. The specification of the sounding system is reported in Table II. The synthetic environment consists of a point scatterer, the Tx and the Rx. The scatterer and the Tx are stationary during the measurement, while the Rx moves along a trajectory with a constant speed. Table I reports the values of the Tx position and the scatterer position, as well as the trajectory that the Rx follows in the measurement. Figure 1 depicts the visual representation of these values.

The sounding system simulated operates in a parallel sounding mode. The time interval between the beginnings of two consecutive observation periods is set to 2.6 ms, i.e. 10 times the individual observation period. We consider totally 100 active observation periods in the simulation. The mobile speed of the Rx is set to 5 m/s. In Figure 1, the positions of the Rx at the beginnings of these observation periods are marked with asterisks.

We assume that there is only one propagation path existing between the Tx and the Rx, which is a one-bounce non-line-of-sight path with its bouncing point located at the position of the scatterer. The kinematic parameters of this path as functions of the observation periods can be calculated based on the geometrical constellation of the Tx, the Rx and the scatterer. The propagation delay \( \tau_k \) is computed as

\[
\tau_k = c^{-1}(||r_{st} - r_{Tx}|| + ||r_{st,k} - r_{Rx,k}||), \tag{19}
\]

where \( r_{st} \) and \( r_{Tx} \) denote the location vector of the scatterer and of the Tx respectively, and \( r_{Rx,k} \) is the Rx location vector at the beginning of the \( k \)th observation period. The elevation and the azimuth are computed as, respectively

\[
\theta_k = \cos^{-1}(\omega_{k,z}) \tag{20}
\]

\[
\phi_k = \cos^{-1}(\omega_{k,x} / \sqrt{\omega_{k,x}^2 + \omega_{k,y}^2}) \tag{21}
\]

where \( \omega_{k,x}, \omega_{k,y}, \omega_{k,z} \) are the entries of the vector \( \omega_k \)

\[
\omega_k = ||r_{st} - r_{Rx,k}||^{-1}(r_{st} - r_{Rx,k}). \tag{22}
\]

The Doppler frequency can be approximately calculated using the delay difference as

\[
\nu_k \approx \frac{(\tau_k - \tau_{k-1}) \cdot c}{\lambda T} \tag{23}
\]

with \( \lambda \) denoting the wavelength. The rate of change parameters \( \Delta(\cdot)_k \) are computed as

\[
\Delta(\cdot)_k = \frac{(\cdot)_k - (\cdot)_{k-1}}{T} \tag{24}
\]

with \( (\cdot) \) being replaced by \( \tau, \phi, \theta \) or \( \nu \). The black curves depicted in Figure 2 demonstrate the trajectories of these parameters of the path during 100 observation periods. In this

<table>
<thead>
<tr>
<th>Item</th>
<th>Horizontal location ((x, y)) [m]</th>
<th>Height (z) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scatterer</td>
<td>(15, 15)</td>
<td>2</td>
</tr>
<tr>
<td>Tx</td>
<td>(20, 20)</td>
<td>1</td>
</tr>
<tr>
<td>Rx initial</td>
<td>(0, 10)</td>
<td>1</td>
</tr>
<tr>
<td>Rx ending</td>
<td>(13, 10)</td>
<td>1</td>
</tr>
<tr>
<td>Rx trajectory</td>
<td>((0 + 13k/K, 5\sin(2\pi k/K) + 10))</td>
<td>1</td>
</tr>
</tbody>
</table>

Table II

<table>
<thead>
<tr>
<th>Item</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx-array</td>
<td>isotropic single antenna</td>
</tr>
<tr>
<td>Rx-array</td>
<td>(4 \times 4) planar array</td>
</tr>
<tr>
<td>Sounding signal</td>
<td>510 chips PN sequence</td>
</tr>
<tr>
<td>Chip rate</td>
<td>2 MHz</td>
</tr>
<tr>
<td>Sampling interval</td>
<td>0.5(\mu)s</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>5.25 GHz</td>
</tr>
</tbody>
</table>
preliminary study, \( \alpha_1 = \exp\{j \pi/4\} \) is applied and the noise component \( w_{\alpha|k}, w_{\text{arg}(\alpha), k} \) \( k = 1, \ldots, K \) are set to zeros. The received signals \( Y_k, k = 1, \ldots, K \) are generated using the observation model (6) based on the true parameters. The signal-to-noise ratio (SNR) is 30 dB.

In the simulations, the maximum-likelihood estimation (MLE) method derived based on the observation model (6) is used to estimate the parameters \( \tau_k, \phi_k, \theta_k \) and \( v_k, k = 1, \ldots, K \). This MLE method does not use the assumption that the state vector \( \Omega_k \) is a Markov process. We call this MLE as “Instantaneous-MLE (IMLE)” in the sequel. The estimates obtained with the IMLE are considered as a benchmark for comparison with those obtained using the proposed PF. However we should point out that this comparison is actually not fair, because the IMLE method estimates the path parameters using the observation samples collected in individual periods.

In the future studies, we will conduct the comparison of the PF with other tracking algorithms, such as the EKF [3] [5], the MLE algorithm derived based on both the state space model (3) and the observation model (6), or the approximation of this MLE algorithm using the recursive EM and SAGE-inspired algorithms [1].

In the simulations, we use the IMLE parameter estimates obtained in the first two observation periods to initialize the particles’ states, i.e. the position parameters in the vectors \( \Omega_k^i \), \( k = 2, i = 1, \ldots, I \) are set to be identical with the IMLE estimates with \( k = 2 \). The initial values of the rate of change parameters in \( \Omega_k^i \), \( k = 2, i = 1, \ldots, I \) are computed using the IMLE estimates obtained at \( k = 1 \) and \( k = 2 \). The proposed PF is applied to track the true state vector \( \Omega_k \) for \( k = 3, \ldots, K \).

Figure 2 depicts the parameter estimates obtained using the PF with different numbers of the particles. The corresponding estimation errors are depicted in Figure 3. It can be observed from Figure 2 that the PFs are capable to track the path parameters in all observation periods. More simulation results demonstrate that the PF succeeds in tracking the true parameters provided the number of particles \( I \) is larger than 15. The PF with larger \( I \) outperforms the PF with smaller \( I \) in terms of lower absolute estimation errors. The complexity of the PF is observed to increase linearly with respect to \( I \).

In Figure 4 the absolute estimation errors obtained using the PF with \( I = 500 \) are compared with those obtained using the IMLE method. It can be observed that the IMLE exhibits estimation “outliers”, i.e. parameter estimates with significant errors, in the observation periods \( k = 87 - 91 \) and 93 - 99. The errors are so significant that the true path is undetected in these periods. Simulations show that this phenomenon does not occur for the PFs with \( I > 15 \).

In Figure 5 the RMSEEs obtained using the PFs with \( I \) ranging from 50 to 900 are depicted. The RMSEEs obtained using the IMLE are also shown, which are computed without taking into account the estimate “outlier”. It can be observed that the RMSEEs of the PF decrease as the number \( I \) of the particles increases. They converge to stable values when \( I \) is large sufficiently. It can also be observed that the PFs using more than 150 particles all outperform the IMLE in the delay and Doppler domains. However, in the azimuth and elevation domains the IMLE outperforms the PFs. The observation that the PF outperforms the IMLE in some of the parameter domains can be due to the different curvature of the marginal posterior pdfs in the parameter domains. In the azimuth domain, the marginal posterior pdf exhibits a wide lobe for the small observation aperture in this domain. Similar for the elevation domain. As a result, more particles are needed in order to approximate the true posterior pdf accurately. While in the delay and the Doppler domains, the observation aperture is relatively large. In these domains, less particles are sufficient to estimate the pdf accurately. So the PF can outperform the IMLE in these domains with relatively small \( I \).
Figure 4. Absolute estimation errors of the PF with $I = 1000$ and the IMLE.

Figure 5. RMSEEs of the PF with the number of particles as a parameter. The RMSEEs of the IMLE are also demonstrated.

V. CONCLUSIONS

In this contribution, a particle filter was designed and used to track the time-variant parameters of propagation paths in the channel sounding scenario. We used a state space model to describe the dynamics of the path parameters. The state vector of a path contains the delay, azimuth and elevation of arrival, Doppler frequency, rates of change of these parameters, as well as complex amplitude of the path. A noticeable challenge for this application is that the multivariate posterior probability density function of the parameters can be highly concentrated in the multi-dimensional parameter space. Two techniques were proposed and proved to be effective by simulations.

Preliminary simulation results demonstrated that the particle filter outperforms the maximum-likelihood estimation method derived based on individual snapshots, in terms of lower root mean square estimation errors and high stability in tracking the path parameters.

REFERENCES