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Mathe, Laszlo; Rasmussen, Peter Omand; Pedersen, John Kim

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Shaping the spectra of the line-to-line voltage using signal injection in the common mode voltage

Laszlo Mathe, Peter Omand Rasmussen, John K. Pedersen
Institute of Energy Technology; Aalborg University; Aalborg, Denmark
lam@iet.aau.dk

Abstract — A drawback of Pulse Width Modulation in electrical drives is the high harmonic content of the line to line voltages, which gives rise to Electro-Magnetic Interference and acoustic noise. By injection of a signal into the common mode voltage, the fundamental is not affected, but new frequency components are introduced in the line to line voltage spectrum. This paper introduces a new analytical method for estimation of the line to line voltage spectrum, where sinusoidal signal is injected into the common mode voltage. Based on the analytical calculations, also a new fixed carrier frequency quasi-random PWM method is proposed. The advantage of the proposed random PWM method is that the acoustical noise generated by the drive can be improved, without exciting the electrical and mechanical resonances of the full drive system. A graphical representation is also presented, where during a fundamental period the distribution of the zero vectors can be tracked.

I. Introduction

In most of the variable speed drives, a pulse width modulated (PWM) converter is used for generation of three phase voltages for an electrical motor. The advantage of power converters based on PWM is that, it can generate controllable magnitude and frequency voltage with high efficiency. A drawback of PWM based converters is the high harmonic content of the generated voltages, which gives rise to electro-magnetic interference (EMI) and acoustic noise in a motor.

The carrier based sine-triangle PWM method generates a train of pulses for the power switches from an inverter, based on the comparison between a high frequency triangular and a sinusoidal reference signal. By adding the 3rd harmonic to the phase voltages the linear range of the modulation index can be extended by 15% [1]. The modulation index is the normalized fundamental voltage. Another method to generate the pulses for the power switches is called Space Vector Modulation (SVM) [2]. The difference between SVM and sine-triangle method with 3rd harmonic added is that, in case of SVM, the added signal is triangular. Injection of the same signals into the phase voltages does not affect the fundamental of the line-to-line (*l-l*) voltage [3], but it will introduce some new high frequency components into its spectrum. From vector generation point of view, the injection of a signal into the Common Mode Voltage (CMV), changes the time distribution between the zero-state vectors V_{000} and V_{111}. A study, regarding to the distribution of the time length of the applied zero vectors can be found in [4]. The replacement of the zero-state vectors in a random manner gives rise to several fixed carrier frequency (FCF) random PWM (RPWM) methods [5], which are very popular because of their easy implementation in drives with closed loop control. The advantage of spreading the spectrum of the voltages and currents of the motor is that the EMI and acoustical performance of the drive [6] can be improved. The uniform distribution of the spectrum achieved using RPWM has the drawback of exciting the electrical or mechanical resonances of the drive system [7]. The new FCF quasi RPWM method proposed in this paper has the advantage to control the composition of the spectrum, in a way to avoid those frequency components which may excite the electrical or mechanical resonances on the drive system.

So far, general analytical description of the harmonic contents in the *l-l* voltage, caused by the upper harmonic of the fundamental signal injected in the CMV has not been found in the literature, except for the third-harmonic injection. This paper introduces a new analytical method to determine the amplitude and frequency components of the *l-l* voltage spectrum using sinusoidal signal injection in the CMV. A new graphical representation of the three reference voltages, where the distribution of the zero vectors during a fundamental period can be tracked, is also presented.

II. Analogy between representation of one period of fundamental signal in polar coordinates and SVM representation

A two level voltage source converter, (Fig. 1) is built using power transistors. The binary switching functions (q_a, q_b, q_c) describe the state of each switch in time. The converter presented in Fig. 1 can operate on eight different states; on each state one voltage vector is generated, called basic voltage vectors.

During a modulation period using SVM, seven basic voltage vectors are generated as it can be seen in Fig. 2 (a). These basic vectors can be classified as active vectors (AV), when the load takes the energy from the DC-link, and zero vectors (ZV) when the output legs of converter are connected together to plus or minus of U_{dc} (no energy is taken from the DC-link). The six active vectors can be represented in α-β reference frame as in Fig. 2 (b).

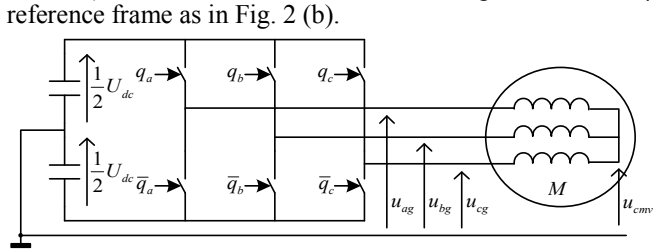

Fig. 1 Three phase voltage-source converter

In Fig. 2 (a) the regions where active vectors were generated, are highlighted.

A graphical interpretation of the voltage space vector in α-β plane can be seen in Fig. 2 (b). The generated $\overline{U}_s$ voltage space vector can be decomposed in two adjoin basic vector with the following equations:

$$\begin{cases} \overline{U}_S = d_{av1} \cdot \overline{V}_x + d_{av2} \cdot \overline{V}_y \\ d_{av1} = \sqrt{3}\dfrac{|U_s|}{U_{dc}}\sin(\gamma) \\ d_{av2} = \sqrt{3}\dfrac{|U_s|}{U_{dc}}\sin(\dfrac{\pi}{3}-\gamma) \end{cases} \quad (1)$$

where the duty ratios d_{av1} and d_{av2} are the ratio between the time-length of the applied vector and the modulation period T, V_x and V_y are adjoin basic vectors. The relation between the time interval of the AV's and duty ratios can be written as:

$$\begin{cases} t_{av1} = d_{av1}T \\ t_{av2} = d_{av2}T \end{cases} \quad (2)$$

From the generated fundamental *l-l* voltage point of view, the time distributions of zero-state vectors between V_{000} and V_{111} are not relevant; they are not represented in SVM representation in α-β plane (Fig. 2 (b)).

Distribution of the zero vectors using sine-triangle comparison and SVM during a period of the fundamental reference signal is shown in Fig. 3. The power transistors are switched when one from the three target reference signals (Fig. 3) is intersecting the triangular carrier waveform. The amplitude of the reference sinusoidal signal (modulation index) was set to 0.7.

Fig. 3 (b) presents the reference signals, which were obtained by adding a triangular signal to the reference signals from Fig. 3 (a). Similar reference signals can be obtained using SVM, where during a fundamental period the t_{z0} and t_{z1}

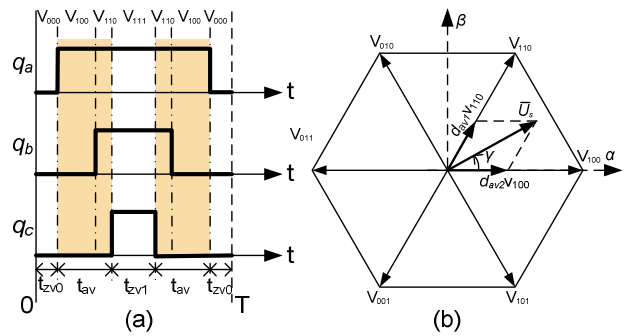

Fig. 2 Representation of a generated vector (a) as switching function for the power switches, (b) SVM representation of the generated voltage vector in α-β plane

are always set to be equal.

The advantage of this kind of reference signals is that the linear range of the modulation index can be extended by 15%. From Fig. 3 (a) and (b) it can be seen that having the same modulation index, the time t_{z1min} is shorter in case of sine-triangular method than in case of SVM. This shows that the maximum modulation index in linear range using SVM is extended 15% compared to sine-triangular method.

The time coordinate representation of the fundamental period from Fig. 3 (b) is presented in Fig. 4 (a) in polar coordinates. In polar coordinates representation, the intersection of the radius at an angle γ with the curves drawn by the three phase voltages gives the individual time components (t_{z0}, t_{av1}, t_{av2}, t_{z1}). The relation between time components obtained by the intersection of the radius with the reference signals and the resultant voltage vector is given by (1) and (2). This relation between radius and voltage vector shows analogy using polar coordinates and SVM representation in α-β plane like in Fig. 4. The benefit of using representation in polar coordinates, is to make the time distribution between the two zero-state vectors visible.

The maximum amplitude which can be reached by the active vector (eliminating the zero-state vectors) in polar

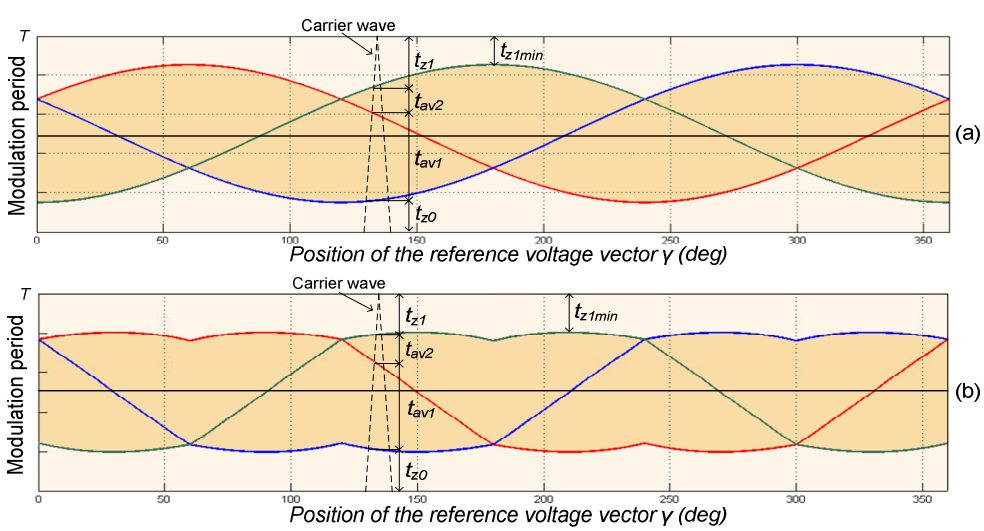

Fig. 3 Representation of phase voltages using (a) sine-triangular modulation, (b) SVM during a period of the fundamental signal

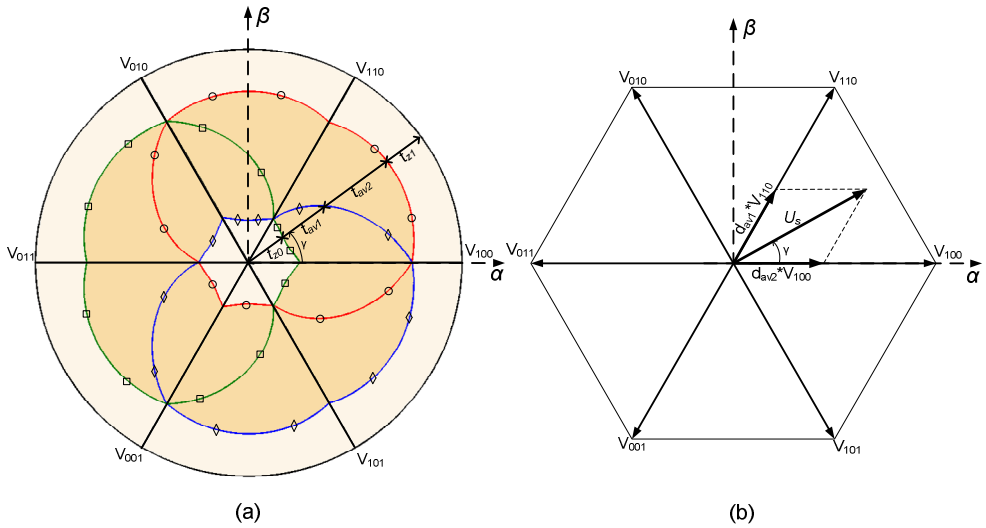

Fig. 4 (a) representation of the phase voltages using polar coordinates (b) representation using voltage space vector

representation, is a circle. Transformation of the time length of the active vector to duty ratios using (2), and representation as vectors like in Fig. 4 (b) transforms the circle in a hexagon. When a signal is injected into the CMV, each phase voltage curve should draw one closed curve, which should be inside the circle from Fig. 4 (a), otherwise the fundamental *l-l* voltage will be distorted.

III. Injection of a Sinusoidal Signal into the Common Mode Voltage

As presented in the introduction, adding the same signal to all the three phase voltages, the fundamental *l-l* voltage will not be affected. The equations of the three phase reference voltages with the injected sinusoidal signal are:

$$\begin{cases} u_{ag} = U_{dc}(M\cos(\omega_0 t) + M_z \sin(z\omega_0 t)) \\ u_{bg} = U_{dc}(M\cos(\omega_0 t - \frac{2\pi}{3}) + M_z \sin(z\omega_0 t)) \\ u_{cg} = U_{dc}(M\cos(\omega_0 t + \frac{2\pi}{3}) + M_z \sin(z\omega_0 t)) \end{cases} \quad (3)$$

where u_{ag} is the phase to ground voltage of phase *A*, *M* is the modulation index, M_z is the amplitude of the injected harmonic, while *z* is the number of the fundamentals upper harmonic.

The *l-l* output voltages can be calculated from eq. (3), and they are given by the differences between the phase leg voltages:

$$\begin{cases} u_{ab} = u_{ag} - u_{bg} = M\sqrt{3}U_{dc}\cos(\omega_0 t + \frac{\pi}{6}) \\ u_{bc} = u_{bg} - u_{cg} = M\sqrt{3}U_{dc}\cos(\omega_0 t - \frac{\pi}{2}) \\ u_{ca} = u_{zg} - u_{ag} = M\sqrt{3}U_{dc}\cos(\omega_0 t + \frac{5\pi}{6}) \end{cases} \quad (4)$$

It can be noticed that the injected signal in phase to ground voltages disappears from (4). Injecting a sine wave with the amplitude of $M_z=0.15M$ and frequency of $f=7\omega_o t$ into the CMV (*M* and ω_o same as it was for SVM from Fig. 4), the plot of the three phase voltages in polar and Cartesian coordinates becomes as presented in Fig. 5.

Representation of the modulation with the 7th harmonic injection in *α-β* plane, presented for SVM in Fig. 4 (b), will not differ from the representation of the same modulation without the 7th harmonic injected.

The injection of the 7th harmonic spreads the position of the active vector in a carrier period, which results in a higher motor current ripple and reduces the linear range of the modulation index. The advantage of injecting such a signal in CMV is that new frequency components appears in the *l-l* voltage spectrum, which may improve the EMC and the acoustical performance of the motor [6].

IV. Determination of the *L-L* Voltage Spectrum Using Double Fourier Series

A mathematical approach for determination of the Fourier spectrum of the phase or *l-l* voltages generated by PWM is described in [8]. From the Fourier transform theory of decomposition, a time varying signal *f(t)* can be expressed as an infinite series of sine and cosine harmonic components:

$$f(t) = \frac{a_0}{2}\sum_{m=-\infty}^{\infty}\left(a_m\cos(m\omega t) + b_m\sin(m\omega t)\right) \quad (5)$$

where a_m and b_m are the amplitudes of each individual frequency component and they can be determined using a Fourier integral:

$$\begin{cases} a_m = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot\cos(m\omega t)dt \\ b_m = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot\sin(m\omega t)dt \end{cases} \quad (6)$$

A PWM switching signal is generated by the combination of a reference signal (usually sinusoidal) and a carrier waveform

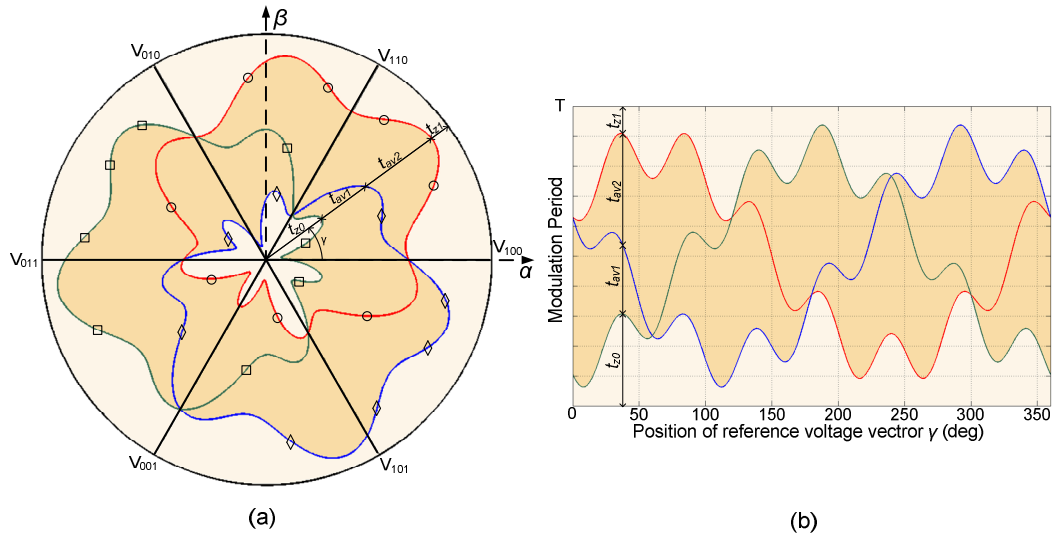

Fig. 5 Phase to ground voltage with the 7th harmonic injected during a fundamental period: (a) polar coordinates (b) Cartesian coordinates

(usually triangular). Mathematically the description of a PWM signal, which is generated by the comparison of two time variable functions can be described as:

$$\begin{cases} f(t) = f(x(t), y(t)) \\ x(t) = \omega_c t + \theta_c, y(t) = \omega_o t + \theta_o \end{cases} \tag{7}$$

where ω_c is the carrier angular frequency, ω_0 is the fundamental angular frequency, while θ_c and θ_o are the phase offsets.

The expansion in Fourier harmonics of the PWM switching function can be done by using the extension of (5) for a double variable function called double Fourier series method. The general equation of a decomposed PWM signal in harmonics as [3] is:

$$\begin{aligned} f(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \begin{pmatrix} A_{0n} \cos\left(n(\omega_0 t + \theta_o)\right) + \\ B_{0n} \sin\left(n(\omega_0 t + \theta_o)\right) \end{pmatrix} + \\ \sum_{m=1}^{\infty} \begin{pmatrix} A_{m0} \cos\left(m(\omega_c t + \theta_c)\right) + \\ B_{m0} \sin\left(m(\omega_c t + \theta_c)\right) \end{pmatrix} + \\ \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \begin{pmatrix} A_{mn} \cos\left(m(\omega_c t + \theta_c) + n(\omega_0 t + \theta_o)\right) + \\ B_{mn} \sin\left(m(\omega_c t + \theta_c) + n(\omega_0 t + \theta_o)\right) \end{pmatrix} \end{aligned} \tag{8}$$

where the amplitude of the harmonic components can be defined as:

$$A_{mn} + jB_{mn} = \frac{U_{dc}}{\pi^2} \int_{-\pi}^{\pi} \int_{-\frac{\pi}{2}(1+M\cos(y)+M_z\cos(z\cdot y))}^{\frac{\pi}{2}(1+M\cos(y)+M_z\cos(z\cdot y))} e^{j(ny+qx)} dxdy \tag{9}$$

for a sinusoidal signal injected into the CMV. By setting $q = m + n(\omega_0/\omega_c)$ the equation becomes valid for a regular sampled asymmetrical modulation, while $q = m$ represents the case of a natural sampled modulation.

In (8) and (9), m is the carrier index variable, n is the baseband index variable.

Solving the double integral from (9), the closed-form solution can be expressed like:

$$A_{mn} + jB_{mn} = \frac{4U_{dc}}{q\pi} \left(\begin{aligned} & J_0\left(q\frac{\pi}{2}M\right) J_0\left(q\frac{\pi}{2}M_z\right) \sin\left(q\frac{\pi}{2}\right)\Bigg|_{n=0} + \\ & J_0\left(q\frac{\pi}{2}M_z\right) \sum_{k=1}^{\infty} \begin{pmatrix} J_k\left(q\frac{\pi}{2}M\right) \times \\ \sin\left((q+k)\frac{\pi}{2}\right) \end{pmatrix}\Bigg|_{k=|n|} + \\ & J_0\left(q\frac{\pi}{2}M\right) \sum_{h=1}^{\infty} \begin{pmatrix} J_h\left(q\frac{\pi}{2}M_z\right) \times \\ \sin\left((q+h)\frac{\pi}{2}\right) \end{pmatrix}\Bigg|_{M_z h=|n|} \\ & \sum_{k=1}^{\infty} \sum_{h=1}^{\infty} \begin{pmatrix} J_k\left(q\frac{\pi}{2}M\right) \times \\ J_h\left(q\frac{\pi}{2}M_z\right) \times \\ \sin\left((q+k+h)\frac{\pi}{2}\right) \end{pmatrix}\Bigg|_{\substack{k+M_z h=|n| \\ k-M_z h=|n| \\ M_z h-k=|n|}} \end{aligned} \right) \tag{10}$$

where k and h are the index values of Jacobi-Anger expansion.

From (10), only those amplitude values have to be considered, where n, k and h fulfills the conditional limitations. For calculation of one amplitude value, an infinite summation of the Bessel function terms is required. However, considering the rapid roll off of the Bessel functions, it is enough to calculate only for the first 10 terms of h and k sums [3]. Fig. 6 (c) presents a graphical representation of the *l-l* voltage spectrum expressed by eq. (10).

V. Measurements and Validation

To show the coincidence between the measured and analytically calculated spectra, Fig. 6 presents the spectrum of the *l-l* voltage around 10kHz for a 25Hz fundamental, with 30th harmonic (750Hz) injected sinusoidal signal and a 5kHz carrier wave frequency. The setup used for measurements consists a 2.2kW asynchronous motor driven by a 2.2kW Danfoss FC302 VLT. The *l-l* voltage and vibration on the motor shell were measured with a Bruel and Kjaer Pulse Multi-analyzer type 3560.

When comparing the theoretical calculations with the experimental results, should be taken into consideration that for the measured results discrete Fourier transform (DFT) was used. The drawback of using DFT is that the windowing will introduce sidelobes in the frequency domain; aliasing will introduce non-existing frequency components. The magnitude of the spectral components will be influenced by the limits of the analogue to digital (A/D) conversion and by the external noise.

Injection of the harmonic into the CMV slightly reduces the amplitude of the carrier and its sideband harmonics. The maximum frequency of the sine wave which can be injected in the CMV is given by:

$$z_{\max} = \frac{f_{sw}}{2 f_{fund}} \qquad (11)$$

where f_{sw} is the switching frequency, f_{fund} is the frequency of the fundamental signal.

Adding the same sinusoidal signal to the three phase voltages, will introduce a group of harmonic components as shown in Fig. 6 (c) with the amplitude and frequency dependent only on the injected sinusoidal signal.

By varying the frequency of the injected signal in the CMV in a random manner, the active vectors are going to have random position during the modulation period, which gives rise to similar voltage spectrum as FCF-RPWM methods. As it was shown in Fig. 6, by injection of a sinusoidal signal in the CMV with a defined amplitude and frequency, several well defined amplitude and frequency components appears in the *l-l* voltage spectrum. Having control on the amplitude and frequency of the injected harmonics in the CMV, the *l-l* voltage spectrum can be predicted using (10) and shaped as it is desired. The spectrum of the *l-l* voltage from Fig. 7 was obtained by varying the frequency of the injected sinusoidal signal randomly in the range of 500-600Hz and 1-1.5kHz. The amplitude of the injected sinusoidal signal in the CMV corresponding to the lower frequency range was set four times lower than that of the corresponding higher frequency range.

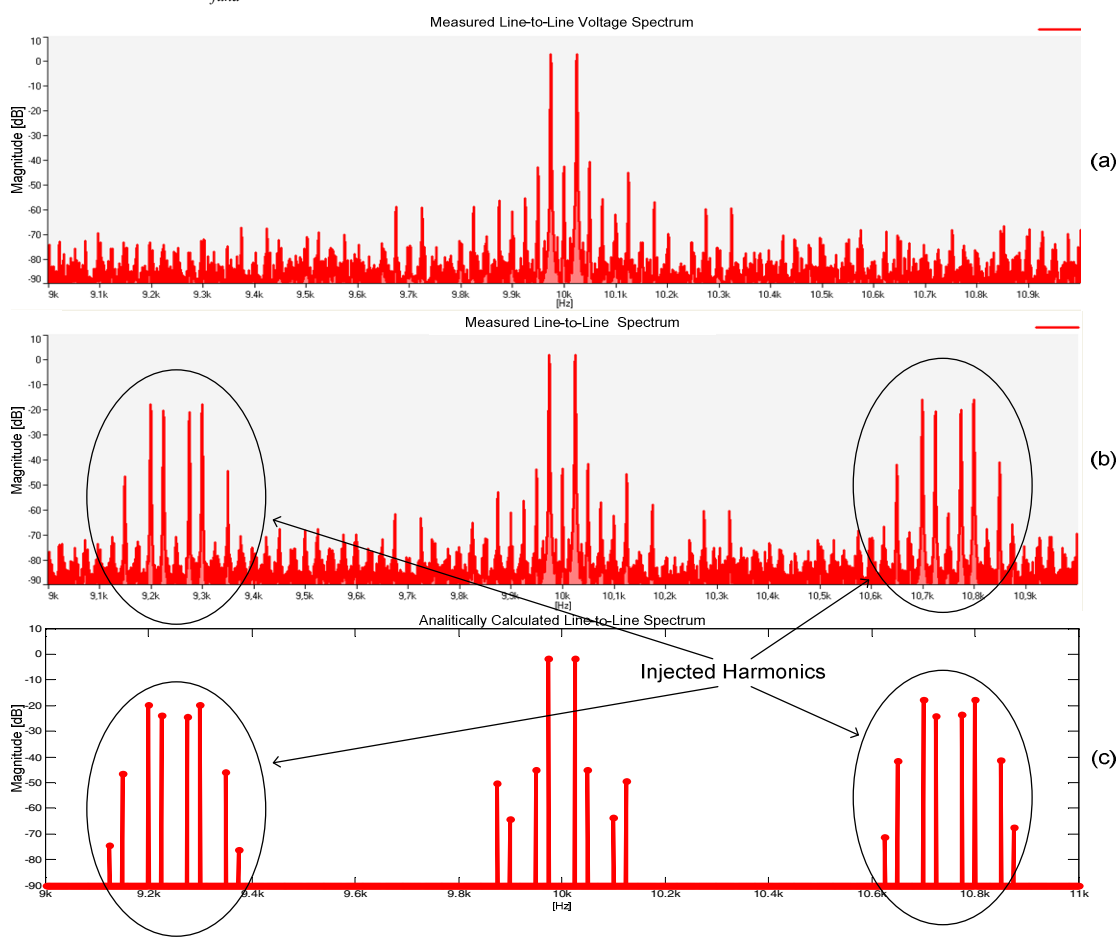

Fig. 6 Sine-triangle PWM *l-l* voltage spectra: (a) measured results with no harmonic injected, (b) measured results with 30th harmonic injected, (c) analytically calculated spectrum; All dB values referenced to magnitude of sine wave fundamental

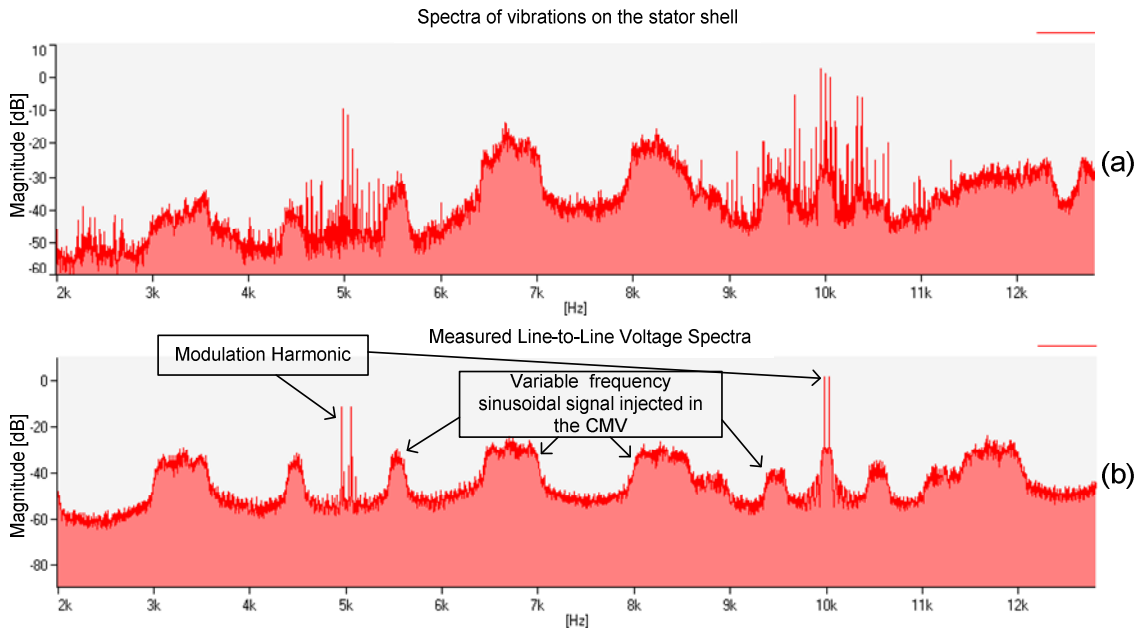

Fig. 7 Measured spectra of (a) vibrations on the stator shell (dB/1.0m/s^2), (b) *l-l* voltage using sine wave injection in CMV with randomly selected frequency (dB values referenced to magnitude of sine wave fundamental); the frequencies were selected from two defined ranges

A limitation of the method is that the maximum amplitude of the injected sinusoidal signal is inversely proportional to the modulation index. At high modulation index, the time length of the zero vectors is small; in fact the zero-state vectors can even disappear, which results in limited range for replacement of the active vectors.

VI. Conclusions

A new fixed carrier frequency quasi random PWM method has been presented and analyzed in this paper. The measured results show that the unwanted frequency components, which may excite electrical or mechanical resonances, can be avoided. This method offers full control over the spectra of the line-to-line voltage, and therefore the acoustical characteristics of the entire drive.

An analytical method for calculation of the spectra of the line-to-line voltage has also been presented. The harmonic spectra calculated analytically shows an excellent agreement with the experimentally measured spectra.

The graphical interpretation of the phase voltages presented in this paper, visualize the distribution of the zero vectors during a fundamental period. The authors hope that this new way of representation of the phase voltages, will be useful for understanding the link between the zero-state vector placement and common mode voltage.

VII. Acknowledgment

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