Performance of a solid oxide fuel cell CHP system coupled with a hot water storage tank for single household

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Abstract: In this paper a solid oxide fuel cell (SOFC) system for cogeneration of heat and power integrated with a stratified heat storage tank is studied. Thermal stratification in the tank increases the heat recovery performance as it allows existence of a temperature gradient with the benefit of delivering hot water for the household and returning the coldest fluid back to SOFC heat recovery heat-exchanger.

A model of the SOFC system is presented to estimate the energy required to meet the average electric energy demand of the residence. The model evaluates the amount of heat generated and the amount of heat used for thermal loads of the residence. Two fuels are considered, namely syngas and natural gas. The tank model considers the temperature gradients over the tank height.

The results of the numerical simulation are used to size the SOFC system and storage heat tank to provide energy for a small household using two different fuels. In particular it was shown that in the case of syngas, due to larger system heat output, a larger tank volume was required in order to accumulate unused heat over the night.

The detailed description of the tank model will be useful to energy system modelers when sizing hot water tanks.

Keywords: Energy system modeling, Solid oxide fuel cell, Cogeneration, Storage heat Tank

1. Introduction

In residential sector, energy consumption can be considerably reduced by increasing the efficiency of energy supply. Fuel cell technologies are suitable for domestic micro-generation to meet the energy demand requirements for a single-family household. Among the various fuel cell systems, micro combined heat and power (micro-CHP) based on Solid Oxide Fuel Cells (SOFCs) is potentially attractive due to its ability to operate at high efficiency on commonly available hydrocarbon fuels such as natural gas or syngas. micro-CHP can be easily integrated into existing heating system, therefore has been applied in the residential sector extensively [1,2].

In residential application, it is important to match the energy generated by the micro-CHP system with the instantaneous electricity and heat demand. A typical apartment has a relatively low-level of energy consumption for the majority of the day with electrical requirements reaching several kilowatts when high power devices are operated, and even higher heating loads when space heating is required [3]. Thermal Energy Storage systems are useful for maximizing the thermal energy efficiency for meeting the fluctuating cooling demands by shifting energy use from peak to off-peak hours. This is achieved by charging the hot water tank during the off-peak hours and discharging it later during the peak hours.

The size of CHP system is dependent on climate conditions, which determine directly the thermal and electrical demands of residents. It is therefore important to evaluate the performance of the CHP system under difference climate conditions to ensure that it matches well with the local heat-to-power load ratio [4].
2. Energy system modeling

A micro-CHP system based on SOFC fuelled by syngas and natural gas is modelled. The detailed description of this model is provided in [3] and [5]. The heat-to power ratio of the systems when using two these different fuels is compared. Based on the heat recovered by the micro-CHP, a storage heat tank is sized in the two cases. A detailed model of the heat tank is provided in section 2.2. The water tank model Matlab® script is provided in the Appendix.

2.1 micro-CHP heat-to power ratio performance

The mathematical model of the micro-CHP system was detailed presented in Liso et al. [3]. For the purpose of the work in this paper, only the micro-CHP system heat-to-power ratio with two different fuels is analyzed, namely Syngas and natural gas. We define the micro-CHP heat-to-power ratio, $H2P\ Ratio$, as the ratio between the heat output, $\dot{Q}$, and electricity output, $P_{el}$, of the micro-CHP system.

$$H2P\ Ratio = \frac{\dot{Q}}{P_{el}}$$  

(1)

Micro-CHP fuel compositions for Syngas and natural gas are provided in Table 1.

<table>
<thead>
<tr>
<th>Species</th>
<th>Molar fraction</th>
<th>Syngas</th>
<th>Natural gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2$</td>
<td>0.293</td>
<td>0.0</td>
<td>0.287</td>
</tr>
<tr>
<td>$CO$</td>
<td>0.287</td>
<td>0.0</td>
<td>0.0024</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>0.118</td>
<td>0.0042</td>
<td>0.0</td>
</tr>
<tr>
<td>$N_2$</td>
<td>0.030</td>
<td>0.0421</td>
<td>0.9488</td>
</tr>
<tr>
<td>$H_2O$</td>
<td>0.272</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$CH_4$</td>
<td>0.000</td>
<td>0.9488</td>
<td>0.0</td>
</tr>
</tbody>
</table>

In Fig 1, a schematic of the fuel cell system coupled with a water tank for a single household is depicted.

![Figure 1: Fuel cell system for residential application](image-url)
In Fig 2, the micro-CHP system Heat-to-Power ratio against different current density is plotted. The Heat-to-Power Ratio in SOFC-based micro-CHP system fuelled by natural gas ranges between 0.5-1.5. We can see that see that a system fueled by natural gas produces around 30% less heat than a system fueled by syngas when operating at nominal conditions (i.e. 3000-4000 A/m²). In fact, at 3000 A/m², a system fueled by syngas shows a $H2P$ Ratio equal to 1 whereas a system fuelled by natural gas has a $H2P$ Ratio around 0.7. We can conclude that a system fueled by Syngas will have a $H2P$ Ratio range of operation between 0.8 and 1.95.

![Fig 2: Heat to power ratio of a micro-CHP system fueled by Syngas and Natural gas](image)

### 2.2 Stratified storage heat tank model design

The heat accumulation in the storage tank can be defined as the difference over the time between the recovered heat from the SOFC system, $\dot{Q}_{\text{Supply}}$ [kW], and the household heat demand for space heating and hot water, $\dot{Q}_{\text{Demand}}$ [kW]. Both supply and demand varies over the day and are thus function of time. In equation 1 the heat accumulation in the tank, $M_{\text{tank}} C_p \frac{dT}{dt}$ [kW], is defined.

$$M_{\text{tank}} C_p \frac{dT}{dt} = \dot{Q}_{\text{Supply}} - \dot{Q}_{\text{Demand}} (T)$$ (2)

In Fig. 3 the heat accumulation in the tank on a daily bases is shown. We assume that the micro-CHP system is operating in steady state condition; therefore the system recovered heat flow rate is constant. If we take the heat accumulation in one day, we see that the peak heat accumulation is achieved at the end of the night. We can conclude that the tank should be able to accumulate at least the heat produced by the micro-CHP over one night (i.e. 7-8 hours).
In Fig. 3 the heat demand range of variation in European cities at different latitudes is shown as a sum of hot (tap) water and space heating consumption. The range of variation is in grey. It can be seen that, in winter, the demand for space heating change significantly at different latitudes. On the other hand, in summer heat demand is quite similar in different climate and it is shown to be around 7 kWh/Day.

Based on the hot water household demand, tank volume was calculated as shown in figure 5. The calculation is based on simple assumption that considering inlet temperature of 10°C from the water mains, and an average storage temperature of 50°C, a liter of water would hold an amount of energy of around 0.046 kWh. This information is fundamental to roughly size a hot water tank.

Tank volume is calculated in different climatic zone. The volume is based on the peak heat accumulation which occurs at the end of the night. For a detailed derivation of the values, we refer to Liso et al. [2]. It appears that a large tank volume is required to store heat in cold climate zones. The Heat-to-Power Ratio in SOFC-based micro-CHP system fuelled by natural gas ranges between 0.5-1.5; It was concluded that a tank volume of 120 liters is sufficient if the system is operated at low heat-to-power ratio.
The majority of models developed for simulating thermocline storage tanks consider packed-bed systems and are based on Schumann’s one-dimensional model described in [6]. This model uses similar assumptions however it considered different heat transfer equations for fluid and packed-bed particles. A schematic of the tank with connections to the network is shown in Fig. 6.

During the charging process, hot water from the micro-CHP SOFC system enters the tank from the top while cold water, extracted from the bottom, enters the returning network. The corresponding valves are open, while the others are closed. Hot water is extracted from the top and delivered to the users through the supply network. Fig. 6 shows a cross section of the tank as required for the 1D numerical modeling. The energy equation applied to the general volume can be written:

$$\rho C_p A \frac{\partial T}{\partial x} = \dot{m} C_p \frac{\partial T}{\partial x} - h \psi(T - T_{amb}) + \lambda A \frac{\partial^2 T}{\partial^2 t}$$

(3)

where the mass flow rate in the tank top-down direction is \(\dot{m} = \nu A \rho\). In equation 3, \(\rho C_p A \frac{\partial T}{\partial x}\) represents the heat accumulation term, \(\dot{m} C_p \frac{\partial T}{\partial x}\) is heat carried by the flowing water, \(h \psi(T - T_{amb})\) represents the heat loss through the tank insulation and \(\lambda A \frac{\partial^2 T}{\partial^2 t}\) is the heat transferred
throughout the water volume in the tank along the vertical direction. Symbols in equation 3 are described in Table 2.

### Table 2. Symbols used in equation 3

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Constant</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>Distance traversed by the fluid in the tank</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>T</td>
<td>Temperature of the water in the tank</td>
<td></td>
<td>°C</td>
</tr>
<tr>
<td>( T_{amb} )</td>
<td>Ambient temperature</td>
<td>25</td>
<td>°C</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Water density</td>
<td>990</td>
<td>m(^{-3})</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Heat capacity of water</td>
<td>4180</td>
<td>kj/kg K</td>
</tr>
<tr>
<td>A</td>
<td>Tank area</td>
<td></td>
<td>m(^2)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Tank perimeter</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Tank height</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>( m )</td>
<td>Water mass flow rate</td>
<td></td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( v )</td>
<td>Average linear velocity of water</td>
<td></td>
<td>m s(^{-1})</td>
</tr>
<tr>
<td>( h )</td>
<td>Heat transfer coefficient of the walls</td>
<td>0.02</td>
<td>W/m(^2) K</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Effective conductivity of water</td>
<td>0.6</td>
<td>W/m K</td>
</tr>
</tbody>
</table>

This problem can be solved using MATLAB command `pdepe()`, which is generally used for initial boundary value problems in one spatial dimension [7]. The matlab function accept data in the following form: \([c,f,s] = pdefun(x,t,u,dudx)\). We consider the variable \( u \) as the temperature along the tank length. \( c(\cdot), f(\cdot) \) and \( s(\cdot) \) are functions of \( x, t, u, \) and \( \frac{du}{dx} \) where:

- \( x \), a point in the domain;
- \( t \), the current time;
- \( u(\cdot) \), the current solution (temperature along the tank);
- \( \frac{du}{dx}(\cdot) \), the current solution spatial derivative.

The output to be computed by \( pdefun(\cdot) \) is a set of three items:

- \( c(\cdot) \), the coefficients of \( \frac{du}{dt} \);
- \( f(\cdot) \), the term to which \( \frac{d}{dx} \) is to be applied;
- \( s(\cdot) \), the source term.

If we divide equation 3 by the tank area \( A \), Matlab terms, \( c, f \) and \( s \) will be defined as follow:

\[
c = \rho C_p
\]

\[
f = \lambda \cdot \frac{du}{dx}
\]

\[
s = \nu \rho C_p \cdot \frac{du}{dx} - h \psi / A \cdot (T - T_{amb})
\]

In order to solve the heat equation we must give the problem some initial conditions. So we must define the temperature of every point along the tank at time \( t = 0 \), which we do with the function \( u(x, 25) = u_0(x) \) for \( 0 \leq x \leq L \). This function is known as the initial temperature distribution. Since heat can only enter or exit the tank at its bottom and top boundaries we must define some “boundary conditions” for the tank. Therefore we assume at the tank bottom, \( u(0, t) = 25 \), and top, \( u(L, t) = 60 \) for all \( t > 0 \). These are known as Dirichlet conditions.
The fluid effective conductivity \( \lambda \) specific heat \( C_p \) and water density \( \rho \) are considered independent of temperature, being their variation negligible over the working temperature interval, 20-60°C. The effective conductivity acts by increasing the water thermal conductivity to include turbulent mixing in the thermocline which is one of the major contributor to the loss of the thermodynamic availability of the stored energy during the charging and discharging processes. The combined effect of thermal diffusion, heat loss via the tank walls and inlet flow induced mixing was quantified by establishing an interface and allowing the thermocline to degrade over time [7]. Under what conditions and to what extent these assumptions are justified can be determined only by experiment. An inquiry into the experimental applications of the theory was not made in the present paper as the main purpose was to present the mathematical treatment of the problem.

3. Tank sizing

In Fig. 7 a and b, a temperature distribution at each hour for 9 hrs in tank is shown in the 2 cases studies: Micro-CHP fuelled by Syngas and Natural Gas. Water temperature distribution in the stratified tank is characterized by three regions with warm water at the top of the tank, cool water at the bottom and thermocline in the middle region. The water temperature profile over the tank height forms S-Curve consisting of two asymptote curves as shown in Fig. 7. Average cool and warm water temperature is formed by the asymptote values of cold and hot water temperature. The position of the thermocline region defines the boundary line of cool and warm water in the tank. Thermocline degrades if the cool water depth occupied the majority of the tank volume.

In general, a thinner thermocline is desired since a larger thermocline implies a larger degradation of stratification. The thickness of the thermocline indicates the extent of mixing occurred due to inflow hot and cold streams. This factor influences the degradation of stratification as well as heat transfer losses from the tank. Tank design should ensure heat accumulation for roughly 8 hours (i.e. Night period). From Fig. 7 it can be noted that in both cases, after 9 hours thermocline degrades completely.

Tank radius as well as the electric output was fixed in both cases. The heat flow rate, \( \dot{Q} \), was fixed depending on the lower limit Heat-to-Power ratio obtained in the two cases. Water mass flow rate in the tank, \( \dot{m} \), and average linear velocity of water, \( v \), were calculated as \( \dot{m} = \dot{Q}/C_p \Delta T \). \( \Delta T \) represents the difference between water temperature at the main and temperature of hot water (60-20°C). Table 3.a and Table 3.b present the designing values adopted in the two cases.

<table>
<thead>
<tr>
<th>Table 3.a Designing values used for the tank sizing in case of mCHP fueled by Natural Gas</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mCHP electric output</td>
<td>1</td>
<td>kW</td>
</tr>
<tr>
<td>Lower limit Heat to power ratio mCHP fueled by Natural Gas</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>mCHP system heat recovered mCHP fueled by Natural Gas</td>
<td>0.5</td>
<td>kW</td>
</tr>
<tr>
<td>Tank radius</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Tank height</td>
<td>0.96</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.b Designing values used for the tank sizing in case of mCHP fueled by Syngas</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mCHP electric output</td>
<td>1</td>
<td>kW</td>
</tr>
<tr>
<td>Lower limit Heat to power ratio mCHP fueled by Syngas</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>mCHP system heat recovered mCHP fueled by Syngas</td>
<td>0.8</td>
<td>kW</td>
</tr>
<tr>
<td>Tank radius</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Tank height</td>
<td>1.55</td>
<td>m</td>
</tr>
</tbody>
</table>
In this paper, performance of a micro-CHP system based on SOFC where analyzed in terms of Heat-to-power ratio. Two fuels were considered i.e. Natural gas and Syngas. Based on the system heat output a stratified hot water tank was sized. A detailed model of the water tank was provided in this paper. The problem of finding the temperature distributions has been solved assuming the well-known laws governing the transfer of heat from a liquid. The proposed model is useful to roughly estimate the size of the water tank.

We concluded that a system fueled by natural gas produces around 30% less heat than a system fueled by syngas when operating at nominal conditions (i.e. 3000 - 4000 A/m2). This leads to assume that a system fuelled by Syngas will have a $H2P$ Ratio range of operation between 0.8 and 1.95. It was concluded that tank should able to store the heat produced by the micro CHP system when operating at the low hand of the $H2P$ Ratio for around 8 hours (i.e. night period).

Finally a storage heat tank for the two cases was sized. It was shown that in the case of syngas, due to larger heat output, a larger tank volume was required in order to accumulate unused heat over the night. A 60 percent larger volume compared to the case of methane fuelled micro-CHP was estimated.

**Appendix**

Below the MATLAB script for sizing of the heat storage tank is provided.
function water_tank
    close all; clc; clear all;
    L = 1.55;    % Tank Height, [m]
    r_i = 0.2;   % Internal tank radius [m]
    Perimeter = 2*pi*r_i; % Perimeter [m]
    Area_tank = pi*r_i^2; % Cross-sectional area, [m^2]
    Volume_tank = Area_tank*L*1000; % [Liters]
    tfinHR = 9; % Final time [hours]
    x = linspace(0,L,50); % [m]
    stepNR = tfinHR;
    tfin = tfinHR*60*60; % Final Time [s]= [hr*60*60]
    t = linspace(0,tfin,stepNR); % [s]
    m = 0;
    sol = pdepe(m,@pdexpde,@pdexic,@pdexbc,x,t);
    u = sol(:,:,1);

    figure
    plot(u,x,'LineWidth',2)
    xlim([25 60])
    xlabel('T [C]')
    ylabel('Tank Quote [m]')
    legend('1hr','2hr','3hr','4hr','5hr','6hr','7hr','8hr','9hr')
    title(['L = ', num2str(L), ' [m] R = ',num2str(r_i), ' [m] Tank Vol =',num2str(Volume_tank), ' [l]'])
   
    %------------------------------------------------------------------------------
    function [c,f,s] = pdexpde(x,t,u,DuDx)
        rho = 990;            % Average density of water, [kg/m^3]
        cp = 4180;            % Average specific heat capacity of water, [J/kg*K]
        Troom = 25;          % Room temperature, [T]
        K_insulant = 0.02;   % [W/m/K]
        k_water = 0.6;       % Thermal average conductivity of water, [W/m*K] 0.630
        Qdotwater = 800;     % Water heat rate at tank inlet [J/s]= [W]
        u_w = Qdotwater/(Area_tank*rho*cp*(60-20)) % Hot water inlet: Qdot = mdotH2O*cp*DeltaT [J/s]
        mdot_w = u_w*Area_tank*rho % Water mass flow rate , [kg/s]
        c = rho*cp;          % c, the coefficients of du/dt;
        f = k_water*DuDx;    % f, the term to which d/dx is to be applied;
        s = u_w*cp*rho*DuDx - K_insulant*Perimeter/Area_tank*(u-Troom); % s, the source term.
    end
    %------------------------------------------------------------------------------
    function u0 = pdexic(x)
        u0 = 25;
    end
    %------------------------------------------------------------------------------
    function [pl,qi,pr,qr] = pdexbc(xl,ul,xr,ur,t)
        pl = ul-25;
        qi = 0;
        pr = ur-60;
        qr = 0;
    end

References

