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Title: Development of Pore Pressure in Cohesionless Soils with Initial Shear Stresses During Cyclic Loading
Development of pore pressure in cohesionless soils with initial shear stresses during cyclic loading

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ABSTRACT: A number of triaxial tests with the loading harmonically oscillating around an initial and anisotropic stress state have been performed. Hereby the influence of the initial shear stress on the development of pore pressure in a cohesionless sand specimen have been clarified. A simple theory describing the development of the degree of mobilization with the number of cycles have been presented.

INTRODUCTION

In 1966 Seed and Lee wrote the first article on liquefaction in sand during cyclic loading. Since then a large number of articles describing the triaxial testing of sand under different types of loads have been publicized. A great deal of these tests have been performed from an isotropic stress state and shows very little resistance against liquefaction. Should these results be taken seriously, almost all constructions carrying variable loads caused by winds, waves, currents or earthquakes would be likely to suffer from failure due to liquefaction.

One of the reasons why this doesn't happen is the occurrence of shear stresses in the soil under a foundation before the application of a given variable load that should have caused the liquefaction. The shear stresses are caused in part by the anisotropic consolidation of the soil, and in part by the load of the construction. Consequently some articles describe the fact that anisotropic consolidation increases the liquefaction resistance, e.g. Castro and Poulos (1977), and some describe the influence of initial shear stresses in the interval from isotropic stress state to the state of failure e.g. Loung (1980).

This article attempts to link the results of static triaxial testing to the results of cyclic triaxial testing and thereby explaining the influence of initial shear stresses. A theory accounting for this is suggested and put into a simple mathematical form.

CATEGORIZATION OF THE LIQUEFACTION PHENOMENON

The phenomenon initially referred to as "liquefaction" is caused by the change of the pore pressure due to stress variations under undrained conditions. The change in effective stresses caused by positive or negative pore pressure build up is categorized in the following way:
- A positive pore pressure is built up causing the effective stresses to fall until the ultimate bearing capacity is reached. This phenomenon is termed "Liquefaction" and is shown by sequence A in figure 1.

- A positive pore pressure is built up causing the effective stresses to fall until a stable condition is reached. The ultimate bearing capacity of the soil has not been reached. This phenomenon is termed "Cyclic mobility" and is shown by sequence B in figure 1.

- A negative pore pressure is built up causing the effective stresses to increase until a stable condition. The ultimate bearing capacity of the soil is consequently increased. This phenomenon is termed "Stabilization" and is shown by sequence C in figure 1.

The definitions of these phenomena have given cause for argument between the writers of different articles. However, the definition presented in this paper is based on the test results which will be presented in the following.

Figure 1. Categorization of the liquefaction phenomenon.

LABORATORY EQUIPMENT

In triaxial tests the changes in stresses, strains, volume and pore pressure are measured on the outside of the test specimen. Therefore it is essential that the conditions inside the test specimen are homogeneous during the test. This means that the equipment must have smooth pressure heads and the specimen height must be equal to the diameter. This paper is based on tests performed in the static and dynamic triaxial cell of the usual Danish type which meets these requirements. The loading in the cyclic tests are sinus waves with a constant amplitude. This equipment has been
developed at Aalborg University Center by the foundation division.

TESTING MATERIALS

The tests have been performed using two sand sorts Vestbjergsand and Lund No. 0. The grain size curves for these sands are shown in Figure 2.

![Grain size for the sand sorts](image)

Figure 2. Grain size for the sand sorts.

Table 1 contains geotechnical properties of the sands.

<table>
<thead>
<tr>
<th></th>
<th>Vestbjergsand</th>
<th>Lund no 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of uniformity U</td>
<td>3.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Void ratio e</td>
<td>0.62</td>
<td>0.63</td>
</tr>
<tr>
<td>Relative density I_0</td>
<td>0.77</td>
<td>0.7</td>
</tr>
<tr>
<td>Angle of friction ( \phi' )</td>
<td>34.7°</td>
<td>41.8°</td>
</tr>
<tr>
<td>Effective cohesion c'</td>
<td>5.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 1. Geotechnical properties of sands.

The test specimens have been prepared by the pluviation method. The specimens were fully saturated using a total vacuum. Consequently all specimens have been subjected to an isotropic consolidation close to 100 kPa.

STATIC TRIAXIAL TESTS

For a dense sand, the conventional drained triaxial test, the CD test, starts with an isotropic confinement \( \sigma_3 \), which is maintained constant while the deviatoric stress \( \sigma_1 - \sigma_3 \) increases. Initially the volume decreases, then the volume decrease rate slows down and becomes zero. This tendency is then reversed as the volume increases in order to disengage the particle.
interlocking and allow large relative movements. This volume increase is called dilatancy. The point where the volumetric strain rate $\dot{\varepsilon}_v = 0$ is shown for three tests in figure 3.

![Graph showing dilatancy](image)

**Figure 3.** The point where the test specimens dilatate, and $\dot{\varepsilon}_v = 0$

Loung (1980) utilizes the deviatoric stress level at the moment of passage from compressive volume changes to dilatation in order to define a "characteristic state" in the soil.

![Graph showing characteristic line](image)

**Figure 4.** The "characteristic line."
If the deviatoric stress levels at the "characteristic state" are plotted in a \( \sigma_1-\sigma_2,\sigma_3 \) diagram, for different tests with different confinement pressures, see figure 4, they will define the "characteristic line" CL. Under this line the volume will decrease, over the line the volume will increase, provided that the confinement pressure is constant.

**CYCLIC TRIAXIAL TESTS**

A somewhat similar line has been found to exist for undrained, cyclic triaxial tests on dense sands. The test results show the existence of an equilibrium state in which the positive pore pressure generated during loading equals the negative pore pressure generated during unloading. This state is called the "stable state", since cyclic loading with constant stress amplitude does not result in changes of the pore pressure over one cycle. This means that the effective stresses remain constant.

If the mean values of the \( \sigma_1-\sigma_3 \) and \( \sigma_3 \) stresses over one cycle at the "stable state" are plotted into a \( \sigma_1-\sigma_2,\sigma_3 \) diagram for different tests having different anisotropic stress levels, a "stable line" \( M_s \) will form as shown in figure 5.

![Figure 5. Cyclic loading in triaxial tests. Examples of two stable stress sequences.](image)

As can be seen from the figure, positive pore pressure will be built up under the \( M_s \) line which will cause the effective stresses to fall, sequence A (cyclic mobility). Over the \( M_s \) line negative pore pressure will be built up which will cause the effective stress to increase, sequence B (stabilization). Both sequences are characterized by the fact that they
approach a stable condition. Liquefaction can only occur if the amplitude on the cyclic loading is so high that Coulomb's failure criterion is reached before the stable state is reached.

The "stable line" has been found to lie below the "characteristic line" CL introduced by Loung.

**DEGREE OF MOBILIZATION**

In order to interpret tests with different densities or tests with different sand sorts it is necessary to represent the stresses in a non dimensional form. Many researchers use the stress relationship defined by the deviatoric stress $\sigma_1 - \sigma_3$ divided by the confinement pressure $\sigma_3$. However this relationship depends on the strength of the soil which is a function of the density index $I_0$. Therefore the degree of mobilization is used. The degree of mobilization is defined as the deviatoric stress divided by the deviatoric stress at failure found at the same confining pressure $\sigma_3$.

$$ M = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_{\text{fail}}} $$

**THEORY OF STABILIZATION**

Based on the "stable state" phenomenon a simple theory is suggested. The following definitions are introduced:

- $c$ is the drained deviatoric stress level before the cyclic loading begins.
- $M_m$ is the mean value of the degree of mobilization $M$ on each cycle. The initial value is called $M_m^0$, and must correspond to the drained condition that exists before the cyclic loading begins.
- $M_{\text{max}}$ is the maximal value of $M$ at each cycle.
- $N$ is the number of cycles.

During the cyclic loading $M_m$ will attempt to approach the stable state $M$. Is $M_m < M$, corresponding to sequence A in figure 5, a positive pore pressure will be built up. Stable conditions will emerge, when $M_m = M_m$. If however, $M_{\text{max}}$ exceeds 1 before this stable condition is reached, liquefaction will occur.

Since the stress amplitude $a$ is constant $M_{\text{max}}$ can be expressed as

$$ M_{\text{max}} = k \cdot M_m $$

where

$$ k = \frac{a + c}{c}. $$

A formula for $M_m$ must be designed in such a way that $M_m$ remains constant when $M_m = M_m$. Thus the following formula is suggested

$$ M_m = M_m^0 + (M_m - M_m^0) f(N) $$

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where \( f(N) \) is a function which must be zero for \( N = 0 \) and one for \( N = \infty \). The simplest possible expression is:

\[
f(N) = N / (N + N^0)
\]

where \( N^0 \) depends on \( M_0 \).

Figure 6. Development of the degree of mobilization during cyclic loading.

- **a:** Increasing \( M_{\text{max}} \) resulting in the stable state.
- **b:** Increasing \( M_{\text{max}} \) resulting in liquefaction.
- **c:** Decreasing \( M_{\text{max}} \) resulting in the stable state.

Figure 6 shows the three possible developments of \( M_0 \) and \( M_{\text{max}} \) during cyclic loading. These have been calculated from (1) and (2). **a:** shows cyclic mobility, **b:** shows liquefaction and **c:** shows stabilization.

The theory of stabilization as it has been proposed here should not be applied to tests where \( M_0 = 0 \). This however is not a very interesting case in triaxial testing since failure in most cases will be caused by tension, with the specimen becoming hourglass shaped and having an almost horizontal failure line. This type of failure was observed frequently during the earliest investigation, and is known as "necking", Lee (1976).

**INTERPRETATION OF TEST RESULTS**

The development of \( M_{\text{max}} \) in three different tests are shown in figure 7 by circles and triangles. In test 8730 \( M_0 \) was almost equal to \( M_0 \), consequently almost no pore pressure was built up. The difference between \( M_0 \) and \( M_0 \) was bigger in tests 8729 and 8732 causing a relatively great pore pressure build up. Common to these tests are that they approach the "stable state", in the case of test 8729 the "stable state" is reached close to failure.

The variation of \( M_{\text{max}} \) calculated using (1) is compared to test results in figure 7. In the calculation of \( M_{\text{max}} \) has been used a value of \( M_0 = 0.34 \). This value of \( M_0 \) is a mean value of the test results shown in table 2.
Figure 7. Comparison of observed values of $M_{\text{max}}$ with the calculated variation of $M_{\text{max}}$ based on $M_0 = 0.34$. The observed values are marked by circles and triangles.

Table 2 shows some examples of test results, where for each test, the parameters $k$, $N_0$, and $M_0$ have been determined by the method of least squares. As it can be seen $M_0$ varies very little. The tests have almost the same number of loading cycles but have different values of $M_0$ and amplitude. Test number 8731 has $M_0 > M_0$, and in this case decreasing pore pressure was detected. The stable degree of mobilization of this test determines the upper limit of $M_0$.

<table>
<thead>
<tr>
<th>Test nr.</th>
<th>$c_0$</th>
<th>$c$</th>
<th>$a$</th>
<th>$M_0$</th>
<th>$N$</th>
<th>$k$</th>
<th>$N_0$</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8713</td>
<td>35.18</td>
<td>±20</td>
<td>0.1</td>
<td>222</td>
<td>2.6</td>
<td>50</td>
<td>0.32</td>
<td></td>
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<tr>
<td>8719</td>
<td>34.50</td>
<td>±33</td>
<td>0.29</td>
<td>331</td>
<td>1.87</td>
<td>21</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>8720</td>
<td>34.42</td>
<td>±33</td>
<td>0.25</td>
<td>284</td>
<td>2.14</td>
<td>72</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>8726</td>
<td>32.35</td>
<td>±33</td>
<td>0.22</td>
<td>345</td>
<td>2.2</td>
<td>28</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>8728</td>
<td>32.26</td>
<td>±25</td>
<td>0.16</td>
<td>321</td>
<td>2.23</td>
<td>28</td>
<td>0.30</td>
<td></td>
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<tr>
<td>8729</td>
<td>33.22</td>
<td>±25</td>
<td>0.13</td>
<td>315</td>
<td>2.4</td>
<td>28</td>
<td>0.4</td>
<td></td>
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<tr>
<td>8730</td>
<td>33.50</td>
<td>±25</td>
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<td>225</td>
<td>1.5</td>
<td>14</td>
<td>0.34</td>
<td></td>
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<tr>
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<td>31.75</td>
<td>±33</td>
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<td>255</td>
<td>1.32</td>
<td>0.39</td>
<td></td>
<td></td>
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<tr>
<td>8732</td>
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<td>±22</td>
<td>0.11</td>
<td>393</td>
<td>2.95</td>
<td>28</td>
<td>0.30</td>
<td></td>
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<tr>
<td>8740</td>
<td>45.40</td>
<td>±33</td>
<td>0.12</td>
<td>600</td>
<td>2.94</td>
<td>40</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>16*</td>
<td>35.2.5</td>
<td>±15</td>
<td>0.05</td>
<td>27</td>
<td>(4.76)</td>
<td>96</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>18*</td>
<td>30.2.5</td>
<td>±15</td>
<td>0.10</td>
<td>200</td>
<td>2.65</td>
<td>50</td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
</table>

* Vestbjerg sand.

Table 2. Summary of test results.

The difference between the $M_0$ value of the individual test and the mean value is quite small. For the tests 8729 and 8732 shown in figure 7 the
difference between the individual $M_a$ value and the mean value of $M_a$ is relatively great. As can be seen in figure 7 this has very little importance for how well the curve represents the test results.

CONCLUSION

Based on a number of cyclic triaxial tests with the loading harmonically oscillating around an initial and anisotropic stress state, a simple theory, describing the development of the degree of mobilization in relation to the number of cycles, has been presented.

The theory of stabilization clarifies the definition of the three terms: "cyclic mobility", "stabilization" and "liquefaction".

The theory is based on the assumption that at "stable state" exists in which the pore pressure increase during loading is neutralized by the pore pressure decrease during unloading, thus causing a stable condition in which the effective stresses remains constant. The "stable state" is represented by the "stable line" $M_a$.

It has been found that $M_a$ always lies below the "characteristic line" $CL$ defined by Loung (1980).

References


