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Bayesian Compressed Sensing with Unknown Measurement Noise Level

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Introduction

- In Bayesian compressed sensing (BCS) we apply approximate Bayesian inference to estimate a sparse vector $\mathbf{w} \in \mathbb{C}^M$ from noisy measurements $\mathbf{y} \in \mathbb{C}^N$ taken as

$$\mathbf{y} = \mathbf{\Phi}\mathbf{w} + \mathbf{n},$$

with dictionary $\mathbf{\Phi} \in \mathbb{C}^{N\times M}$ and white Gaussian noise $\mathbf{n} \in \mathbb{C}^N$.

- In most BCS literature it is not tractable to estimate the noise precision λ (inverse variance) as an integral part of the Bayesian inference. Heuristic methods are instead employed resulting in increased computational complexity.
- In this work we propose to modify the three-layer hierarchical prior model in [1] such that the estimation of the noise precision can be included in the inference scheme with no penalty in terms of complexity.

Graphical Models

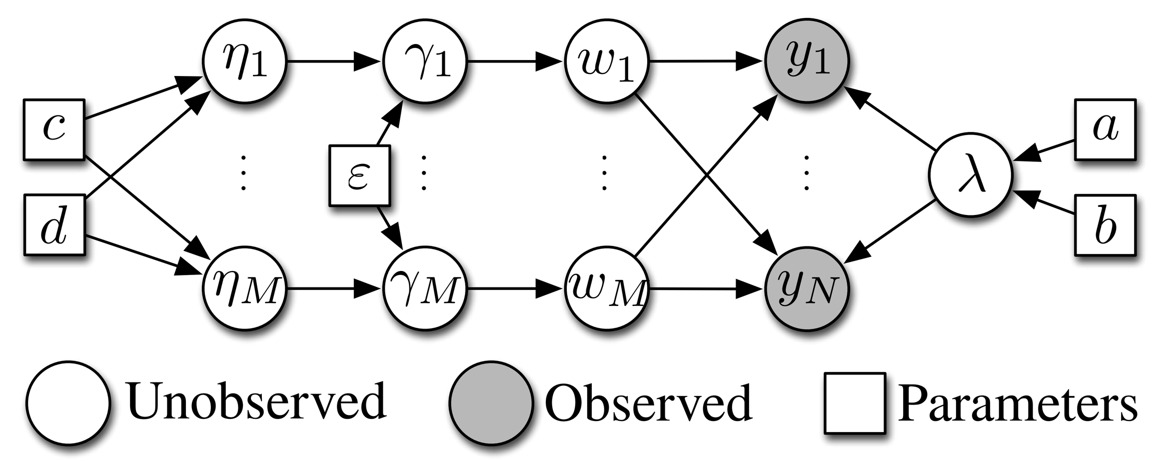


Fig 1: Hierarchical prior model as proposed by Lovmand et. al. [1].

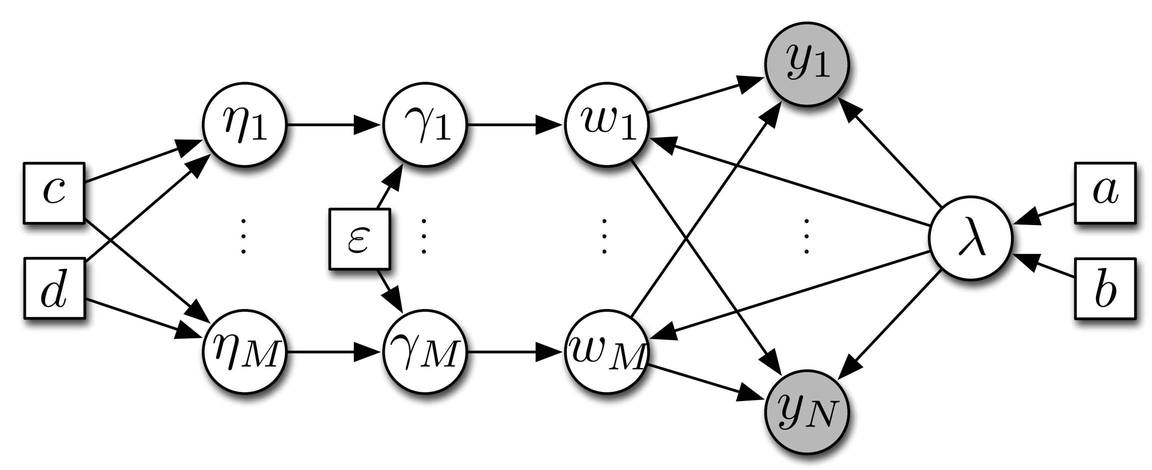


Fig 2: Proposed model with the modification inspired by [2].

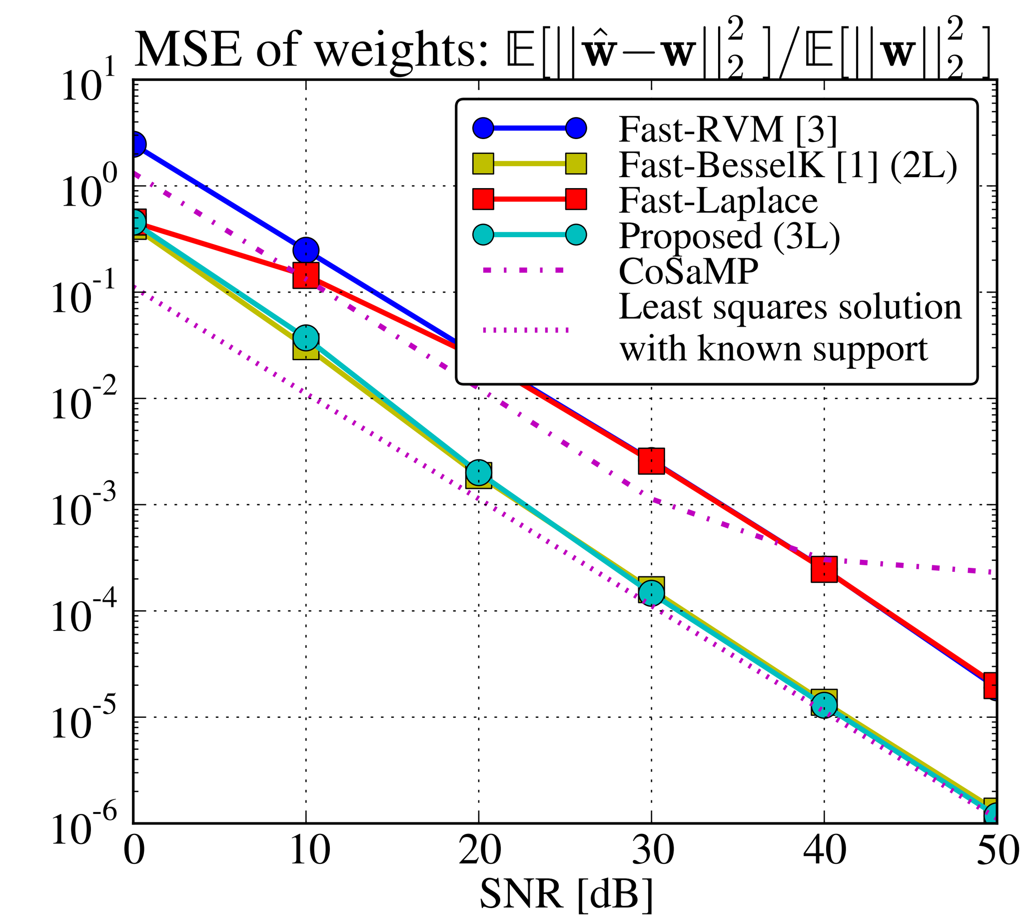
The relevance vector machine (RVM), its fast variant [3] and other algorithms proposed in the literature can be derived from special cases of the model in Fig. 1.

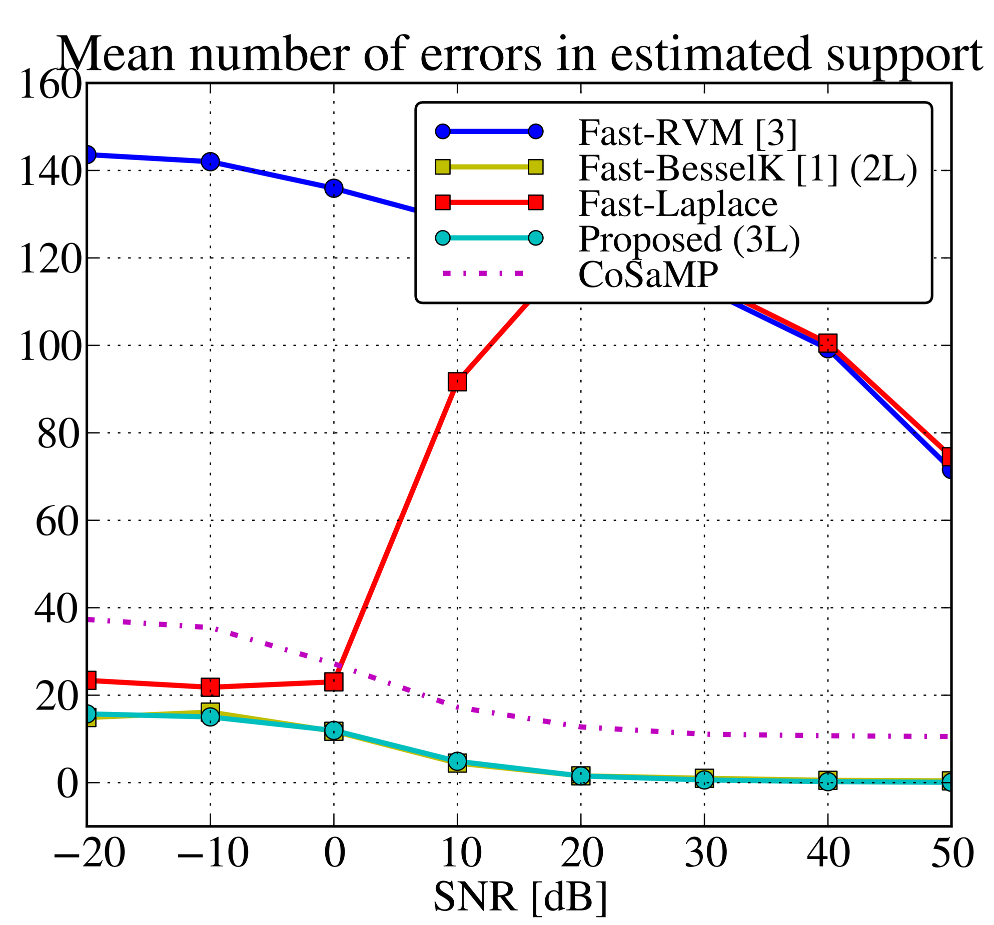
Hierarchical Prior Modelling

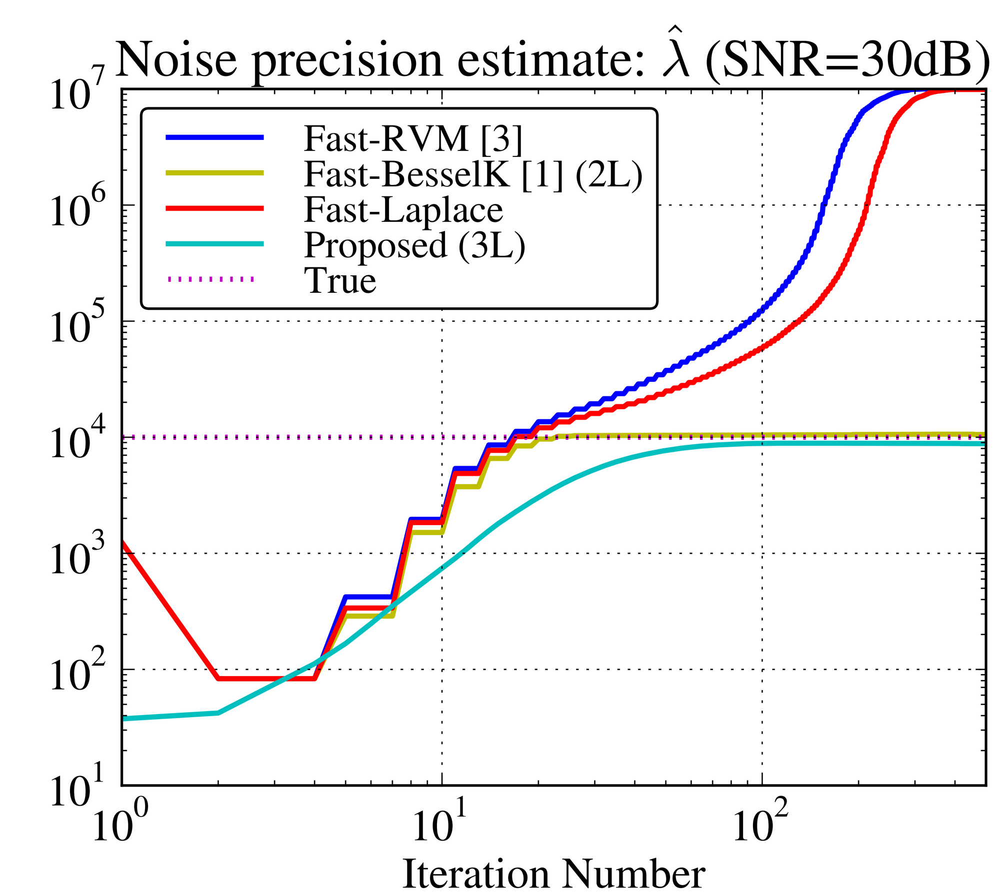
With $\mathbf{\Gamma} = \mathrm{diag}(\boldsymbol{\gamma})$, the probability distributions in the prior models are:

Density	Model in [1]	Proposed changes
Observation model $p(\mathbf{y}\|\mathbf{w},\lambda)$	$\mathrm{N}(\mathbf{y}\|\mathbf{\Phi}\mathbf{w},\lambda^{-1}\mathbf{I})$	-
Prior on λ, $p(\lambda)$	$\mathrm{Ga}(\lambda\|a,b)$	-
Layer 1 on weights, $p(\mathbf{w}\|\boldsymbol{\gamma})$	$\mathrm{N}(\mathbf{w}\|\mathbf{0},\mathbf{\Gamma})$	$\mathrm{N}(\mathbf{w}\|\mathbf{0},\mathbf{\Gamma}\lambda^{-1})$
Layer 2 on weights, $p(\boldsymbol{\gamma}\|\boldsymbol{\eta})$	$\prod_{i=1}^{M}\mathrm{Ga}(\gamma_i\|\varepsilon,\eta_i)$	-
Layer 3 on weights, $p(\boldsymbol{\eta})$	$\prod_{i=1}^{M}\mathrm{Ga}(\eta_i\|c,d)$	-

Numerical Results







Computational Complexity

- The fast implementation of the RVM [3] can only use its computationally efficient matrix-vector updates when the noise precision estimate is held fixed.
- Using our proposed model, these updates become independent of the noise precision estimate.
- We assume $S \leq N \leq M$, with N compressed measurements, M basis vectors in the dictionary and S nonzero entries in $\mathbf{w}$.
- Computational complexity per iteration:

	Model in [1] & [3]	Proposed
Fixed $\hat{\lambda}$	$\mathcal{O}(MN)$	$\mathcal{O}(MN)$
Updating $\hat{\lambda}$	$\mathcal{O}(MNS)$	$\mathcal{O}(MN)$

References

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[2] S. Ji, D. Dunson, and L. Carin, "Multitask compressive sensing," *IEEE Transactions on Signal Processing*, vol. 57, no. 1, pp. 92–106, Jan. 2009.

[3] M. E. Tipping and A. Faul, "Fast marginal likelihood maximisation for sparse Bayesian models," in *Proceedings of the International Workshop on Artificial Intelligence and Statistics*, 2003, pp. 3–6.