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Characterization of Coded Random Access with Compressive Sensing based Multi-User Detection

Yalei Ji(1), Čedomir Stefanović(2), Carsten Bockelmann(1), Armin Dekorsy(1), Petar Popovski(2)
(1)Department of Communications Engineering University of Bremen, Germany
{ji, bockelmann, dekorsy}@ant.uni-bremen.de
(2)Department of Electronic Systems Aalborg University, Denmark
{cs, petarp}@es.aau.dk

***Abstract*—The emergence of Machine-to-Machine (M2M) communication requires new Medium Access Control (MAC) schemes and physical (PHY) layer concepts to support a massive number of access requests. The concept of coded random access, introduced recently, greatly outperforms other random access methods and is inherently capable to take advantage of the capture effect from the PHY layer. Furthermore, at the PHY layer, compressive sensing based multi-user detection (CS-MUD) is a novel technique that exploits sparsity in multi-user detection to achieve a joint activity and data detection. In this paper, we combine coded random access with CS-MUD on the PHY layer and show very promising results for the resulting protocol.**

I. Introduction

In recent years there has been a revival of the research interest in random access protocols, instigated by the growth of machine-to-machine (M2M) communications. This is particularly valid for cellular networks, where the random access mechanisms are used to establish the initial connection between the terminals and the base station (BS), and facilitate access to data services. Cellular random access is commonly based on the traditional slotted ALOHA (SA), a simple distributed access method that provides satisfactory performance for human-oriented traffic. However, the M2M traffic has fundamentally different requirements, primarily seen in the expected number of accessing terminals, and using traditional SA may create bottlenecks already in the access network.

A promising approach for enhancing the performance of SA by using interference cancellation (IC) was demonstrated recently in [1]–[3]. In brief, application of IC potentially unlocks the collision slots, radically boosting the throughput of SA. Of a particular importance is Liva's paper [2], where it was shown that the use of successive IC in SA for recovering user transmissions is analogous to the iterative belief-propagation (BP) erasure decoding, promoting the use of the erasure coding theory to design "coded slotted ALOHA" schemes.

Despite the similarities, there are important differences between erasure coding and SA with IC due to the effects of the wireless medium in the latter. When the power of one of the colliding signals is stronger than the rest, a capture effect may occur, i.e., the corresponding transmission may be successfully received. Therefore, the capture effect may significantly affect both the access scheme design and performance, as the collision slots may potentially be exploited both through captures and IC. In that case, the analysis and design of coded SA schemes requires incorporation of the capture effect in the model that is inspired by erasure coding. A brief treatment of the capture effect in coded SA was presented in [2], introducing the general modification of the and-or tree evaluation [4], a tool used to assess the asymptotic performance of the erasure codes when decoded by the iterative BP algorithm. This analysis was extended further in [5], showing how to actually evaluate the asymptotic performance of coded SA for narrowband systems and Rayleigh fading.

Besides the work on MAC random access protocols there has also been a renewed interest in multi-user detection (MUD) in the context of random access. In classical MUD, it is assumed that the set of active users is known a priori and the focus on reliable data detection. However, in random access schemes the main ingredient is the uncertainty about the set of active users, such that both the user activity as well as data have to be estimated. Considering that the setup leads to so-called *sporadic communication* the active users only constitute a small subset of all users, such that the problem is inherently sparse and motivates the use of compressive sensing (CS) to facilitate a low-overhead PHY scheme for low data rate M2M communication. This novel compressive sensing based multi-user detection (CS-MUD) achieves a joint detection of activity and data of the subset of active users in a slot and exhibits performance close to the genie-upper bound when the user activities are known a priori [6]–[8].

In this paper we focus on the coded SA with capture effect in broadband, MUD systems, which, to the best of our knowledge, has not been analyzed in the literature available so far. First, we show how to generally model and incorporate the capture effect in MUD systems into the and-tree evaluation, applicable to any coded slotted ALOHA scheme. In the next step, we deviate from the simple PHY-layer, commonly used in ALOHA schemes, and introduce the details of the receiver based on CS-MUD. Finally, we apply the obtained analytical and numerical results to the frameless ALOHA, a simple but effective variant of coded SA [3], [9], and demonstrate how the capture effect and features of the CS-MUD impact the design and performance of the scheme.

The paper is organized as follows. Section II introduces the most important concepts of coded SA and capture effect.

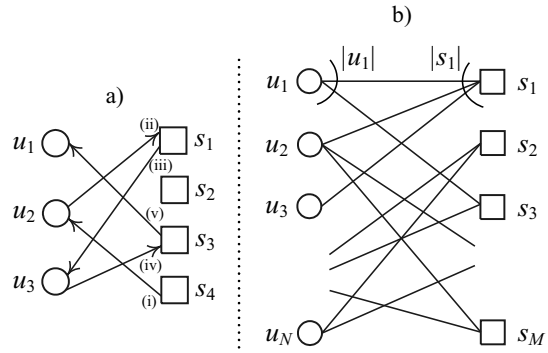

Fig. 1: Graph representation of coded slotted ALOHA.

Section III elaborates the system model. The analysis of the proposed access method is performed in Section IV. Section V presents the asymptotic performance of frameless ALOHA with CS-MUD. Finally, the paper is concluded in Section VI.

II. Background

A. Coded Slotted ALOHA

The basic principles of coded SA are illustrated in the example presented in Fig. 1a). The contention process is depicted by a bipartite graph, where nodes on the left represent users (i.e., contending terminals), the nodes on the right represent slots, and the edges connect users with the slots in which they transmitted (for instance, user u_2 transmitted in slots s_1 and s_4). Each time the user makes a transmission, it is the replica of the same data packet that, in addition, contains the information in which slots the other replicas occurred[1]. In traditional SA with no capture effect, only the singleton slot s_4 is usable and the packet of u_2 becomes recovered, denoted by (i) in Fig. 1a); the collision slots s_1 and s_3 and the idle slot s_2 are wasted. On the other hand, the use of interference cancellation allows for removal of packet replica of u_2 from s_1 (ii), which reduces s_1 to a singleton slot and enables recovery of the packet of u_3 (iii). In the same way, the packet replica of u_3 is removed from s_3 using IC (iv), enabling recovery of the packet of u_1 (v). The above representation and the successive application of IC are analogous to the representation of codes-on-graphs and iterative BP erasure-decoding.

We now briefly introduce the notation used in the rest of the paper. Denote users by u_i, $1 \leq i \leq N$, and slots by s_j, $1 \leq j \leq M$, see Fig. 1b). By $|u_i|$ ($|s_j|$) denote the degree of u_i (s_j), where the degree is the number of edges incident to the node.[2] Further, denote by $\Lambda_{i,k}$ ($\Omega_{j,l}$) the probability that a degree of user (slot) node is k (l). An important feature of coded SA is that one can design only the user degrees $|u_i|$, $1 \leq i \leq N$, while the slot degrees $|s_j|$, $1 \leq j \leq M$, are random, being outcomes of the contending process. A typical assumption is that users select slots in which they transmit with a uniform probability [2], [3], [9], [10]. This implies that $|s_j|$, $1 \leq j \leq M$, are independent and identically distributed (i.i.d.), i.e., $\Omega_{j,l} = \Omega_l$. As shown in [2], [9], the probabilities Ω_l, $l \geq 0$, can be approximated by a Poisson distribution:

$$\Omega_l \approx \frac{\beta^l}{l!} e^{-\beta}, \tag{1}$$

where $\beta = \mathrm{E}[|s_j|]$, $1 \leq j \leq M$, i.e., β is the average slot degree. The same assumption is typically used in sporadic communication setups with CS-MUD [6], enabling the combination of the two approaches.

B. Capture Effect

As already stated in the introductory section, a capture effect occurs when one or more user transmissions may become recovered despite the interference originating from other users. In narrowband systems, the capture effect leads to recovery of a single user transmission, while in broadband MUD systems it may lead to recovery of more than just one transmission.

The capture effect has been extensively studied in the traditional SA framework, both in narrowband and in broadband systems; c.f. [11]–[16]. A typical premise is that the capture occurs for all transmissions whose signal-to-interference-plus-noise ratio (SINR) is above capture threshold b, where $b \geq 1$ for narrowband, and $b < 1$ for broadband systems [14]–[16]. However, the features of the CS-MUD receiver studied in this paper and presented in detail in Section III-B, can not be described by such a simplistic model. Specifically, the considered receiver exploits compressive sensing, and its performance depends on the sparsity of the input observation, as well as the correlation among users' spreading sequences and noise. We note that these dependencies pose significant analytical difficulties and we therefore numerically evaluate the capture probabilities for the CS-MUD receiver and the scenario of interest, as presented in Section IV-C.

III. System Description

In this paper we adopt the frameless ALOHA strategy [3], [9]. We assume that there are N users in the system, synchronized on a slot basis. The users contend for the access to the base station (BS) by transmitting replicas of the same packet in randomly selected slots of the contention period. The start and end of the contention period are denoted by downlink beacons sent by the BS, as shown in Fig. 2. The duration of the contention period in slots, denoted by M, is a priori unknown and chosen such that the expected throughput is maximized. Every user transmits with a predefined activation probability $p_A = \beta/N$ in every slot of the contention period, where β is a suitably chosen constant. Therefore, the probabilities $\Lambda_{i,k}$ are the same for all users u_i, and it can be shown that:

$$\Lambda_{i,k} = \Lambda_k \approx \frac{((1+\epsilon)\beta)^k}{k!} e^{-\beta},\ 1 \leq i \leq N, \tag{2}$$

where $\epsilon = \frac{M}{N} - 1$. The average user degree is:

$$\mathrm{E}[|u_i|] = (1+\epsilon)\beta,\ 1 \leq i \leq N. \tag{3}$$

To ease the following analytical as well as numerical investigations the following assumptions are made: (i) the

[1] A practical way how to achieve this is elaborated in [3].

[2] Henceforth, we use notions of user/user node, slot/slot node and transmission/edge interchangeably.

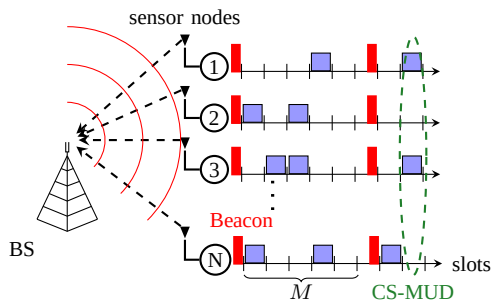

Fig. 2: Sporadic uplink transmission of multiple devices controlled by BS beacons. Each time slot corresponds to one data frame. The system is assumed to be synchronous.

user channels are assumed to be i.i.d. and constant for the duration of a contention period, (ii) channel state information is perfectly known at the BS and (iii) the received power of all users is the same on average, e.g., through power control.

A. Receiving Operation at BS

In every slot s_j, $1 \leq j \leq M$, the BS receives and stores the composite signal $\mathbf{y}_j$, which combines the colliding transmissions and the noise, and executes the following steps:

1) All previously recovered transmissions whose replicas occur in s_j (if there are such), are removed from s_j, i.e., the BS performs the *inter-slot* IC.
2) The BS applies the CS-MUD algorithm.
3) The BS removes all the newly recovered transmissions in step 2) from s_j, i.e., performs *intra-slot* IC.
4) The BS removes all the replicas of the newly recovered transmissions in step 2) from all the previous slots where they occur, i.e., performs *inter-slot* IC in s_k, $1 \leq k \leq j-1$.
5) The BS repeats steps 2)-4) until there are no newly recovered transmissions in s_j.

Furthermore, steps 2)-5) are also executed on all the slots s_k, $1 \leq k \leq j-1$, that experienced inter-slot IC in previous runs, potentially resulting with new candidate slots on which the same operation cycle, i.e., CS-MUD, intra- and inter-slot IC, is executed again. The processing at the BS ultimately stops when there are no new slots affected by the inter-slot IC.

An assumption made in this paper is that both inter- and intra-slot IC are perfect, i.e., the recovered transmissions and their replicas are removed leaving no residual interference[3].

B. Physical Layer CS-MUD

For a general introduction to CS-MUD in sporadic communication please refer to [6]. Here, we focus on a summary of the most important parts. We model the physical layer transmission through a typical simplified synchronous baseband description, in which a linear input-output relation is used to express the multi-user wireless transmission in slot s_j [17]:

$$\mathbf{y}_j = \mathbf{A}\mathbf{x} + \mathbf{n}_j, \tag{4}$$

[3]In [2] it was demonstrated that this assumption holds under moderate-to-high SNRs.

where $\mathbf{A}$ summarizes the influences of channel and medium access. The stacked multi-user vector $\mathbf{x}$ contains all symbols from all N user nodes in one slot, independent of their activity in that slot. Due to the probabilistic activity of users, i.e., the number of active users in one slot is given by the random slot degree, the activity in the system is unknown at the BS and needs to be estimated. Accordingly, inactive users who do not transmit any data are modeled as "transmitting" a frame of L zeros. Active users transmit frames of L data symbols from a discrete modulation alphabet $\mathcal{A}$. Thus, the elements of the multi-user vector are $\mathbf{x} \in \mathcal{A}_0^{LN}$ with $\mathcal{A}_0$ being the so-called *augmented* alphabet given by $\mathcal{A}_0 = \mathcal{A} \cup 0$. For simplicity, we assume that BPSK is applied in our system, which leads to $\mathcal{A}_0 = \{\pm 1, 0\}$. However, this is not a general restriction and higher order modulations are easily incorporated. Finally, $\mathbf{n}_j$ denotes additive white Gaussian noise with zero mean and variance σ_{n}^2 and the vector $\mathbf{y}_j$ represents the measurements of the signal $\mathbf{x}$ available at the receiver in slot s_j.

In sporadic communication the basic assumption is intermittent (sporadic) user activity, which usually leads to a small number of active users compared to the total number of users. Here, the SA scheme determines how many users are accessing a slot at any given time, which may lead to less sparse systems compared to the current CS-MUD literature. Nonetheless, many elements in $\mathbf{x}$ may be zero or, more specifically, block-zero in blocks of L consecutive symbols. In CS literature this property is called "block-sparse" and enables improved detection algorithms which exploit this structure. Thus, the sparse multi-user detection problem (4) can be solved by CS, which facilitates a joint activity and data detection. A very interesting feature of this CS-MUD is that (4) may even be highly under-determined. The reconstruction of $\mathbf{x}$ from the noisy received signal $\mathbf{y}_j$ is still possible due to the sparsity of $\mathbf{x}$.

For the remainder of the paper, CDMA will be applied as an exemplary medium access scheme, which is quite attractive for M2M communications due to its adaptive and flexible support of different number of devices, as well as variable Quality of Service. Specifically, $\mathbf{A}$ describes the spreading of $\mathbf{x}$ by user-specific real-valued PN sequences, which are assumed to be known at the BS and could serve as a user ID. All transmitted frames are assumed to have the same length of F chips after spreading each of the L symbols by a PN sequence with spreading factor N_{s}. Here, the spreading factor N_{s} determines the resource efficiency of the PHY layer. As the slot degree l is typically much smaller than N, we choose $N_{\mathrm{s}} < N$, which leads to an overloaded CDMA system. In this case, (4) is an under-determined linear equation system. Further, the matrix $\mathbf{A}$ also describes the convolution with the user-specific channel impulse responses $\mathbf{h}_i \in \mathbb{C}^{L_{\mathrm{h}}}$ of length of L_{h}. After convolution $F' = F + L_{\mathrm{h}} - 1$ chips will be received. Thus, $\mathbf{A} \in \mathbb{C}^{F' \times NL}$ summarizes both the spreading and channel convolution.

The details of CS-MUD algorithms are not our main focus and readers who are interested may refer to [6]. In order

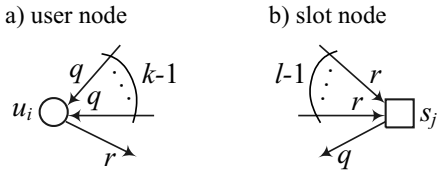

Fig. 3: And-or tree evaluation.

to have reasonable complexity and exploit the discussed group sparsity, we choose the well studied Group Orthogonal Matching Pursuit (GOMP) as our detection algorithm. Usually, physical layer algorithms are analyzed by bit or frame error rate plots over the signal-to-noise ratio. However, here we are more interested in the capture probability of this specific PHY layer setup for the overall evaluation of coded random access with CS-MUD. In Section IV-C, we will detail how these probabilities are numerically obtained.

IV. Analysis

A. And-or Tree Evaluation

For the general introduction to the and-or tree evaluation in the erasure coding framework, we refer the reader to [4], [18]. Hereafter, we focus on the most relevant aspects that are subject to the properties of the proposed access method and the CS-MUD receiver.

And-or tree evaluation provides the asymptotic probability (i.e., when number of users $N \to \infty$) of user transmission recovery. The evaluation is based on the graph that represents the contention process, Fig. 1b), and it is performed via iterative updates of the probabilities of transmission recoveries over the corresponding graph edges. As elaborated in Section III, the access strategy is uniform both over users and slots, users' channels are statistically equivalent, and the expected received powers for all users are the same. Therefore, we can model the and-or tree evaluation through a message exchanges among a referent (exemplary) user and slot, depicted in Fig. 3. In Fig. 3a), q denotes the probability that a user transmission corresponding to the incoming edge *is not* recovered in the incident slot in the previous iteration. Therefore, the probability r that a replica corresponding to the outgoing edge *is not* recovered is $r = q^{k-1}$, i.e., the outgoing edge is not recovered if all the incoming edges are not recovered. Averaging over user degrees yields:

$$r = \sum_{k=1}^{\infty} \lambda_k q^{k-1}, \tag{5}$$

where λ_k denotes the probability that an edge is connected to a user node of degree k, which can be calculated as [18]:

$$\lambda_k = \frac{k\Lambda_k}{\sum_{v=1}^{\infty} v\Lambda_v}. \tag{6}$$

For frameless ALOHA, using (2) and (6) transforms (5) into:

$$r = \sum_{k=1}^{\infty} \frac{((1+\epsilon)\beta)^{k-1}}{(k-1)!} e^{-(1+\epsilon)\beta} q^{k-1} = e^{-(1+\epsilon)\beta(1-q)}. \tag{7}$$

The analysis of the probability updates performed in slots is more involved, due to the way that receiver operates. The probability that the outgoing edge *is* recovered is:

$$1 - q = \sum_{t=0}^{l-1} C(t) \binom{l-1}{t} (1-r)^{l-1-t} r^t, \tag{8}$$

where t is the number of unrecovered incoming edges, $l-1-t$ is the number of incoming edges that have been recovered in other slots and removed via inter-slot IC, $C(t)$ is the probability that the outgoing edge is recovered when t unrecovered edges remain, and $\binom{l-1}{t}$ is a combinatorial argument representing the symmetry of the problem setting. We deal with computation of $C(t)$ in Section IV-B.

Averaging (8) over slot degrees yields:

$$q = 1 - \sum_{l=1}^{\infty} \omega_l \sum_{t=0}^{l-1} C(t) \binom{l-1}{t} (1-r)^{l-1-t} r^t, \tag{9}$$

where ω_l denotes the probability that an edge is connected to a slot of degree l [18]:

$$\omega_l = \frac{l\Omega_l}{\sum_{v=1}^{\infty} v\Omega_v}. \tag{10}$$

Combining (1), (10) and (9) yields:

$$q = 1 - \sum_{l=1}^{\infty} \frac{\beta^{l-1} e^{-\beta}}{(l-1)!} \sum_{t=0}^{l-1} C(t) \binom{l-1}{t} (1-r)^{l-1-t} r^t. \tag{11}$$

Finally, the and-or tree evaluation is performed in an iterative manner:

$$q_0 = 1; \tag{12}$$

$$r_m = f(q_{m-1}) \text{ and } q_m = g(r_m),\ m \geq 1, \tag{13}$$

where the subscript denotes the iteration number, and $f(\cdot)$ and $g(\cdot)$ are given by (5) and (9), respectively. The probability of user transmission recovery is:

$$P_R = 1 - \lim_{m \to \infty} r_m. \tag{14}$$

A central measure of system efficiency is the throughput T, defined as the number of recovered users within M slots of the contention period. T can be computed as:

$$T = \frac{NP_R}{M} = \frac{P_R}{1+\epsilon}. \tag{15}$$

B. Derivation of Capture Probabilities

Assume that t_{A} users are active in the slot, i.e., there are $t = t_{\mathrm{A}} - 1$ interfering transmissions from the perspective of the referent outgoing edge. Denote by $p(s|t_{\mathrm{A}})$ the probability that s users out of t_{A} have been recovered from the slot using CS-MUD. Further, by $c(s|t_{\mathrm{A}})$ denote the probability of the event that among these s is the user that corresponds to the outgoing edge. Likewise, by $u(s|t_{\mathrm{A}})$ denote the event among s recovered users is not the one corresponding to the outgoing edge. Obviously:

$$p(s|t_{\mathrm{A}}) = c(s|t_{\mathrm{A}}) + u(s|t_{\mathrm{A}}). \tag{16}$$

The recovery of the outgoing edge can happen in any chain of fortunate successive applications of the CS-MUD and the intra-slot IC, as elaborated in Section III-A, which can be stated as:

$$C(t) = C(t_\mathrm{A} - 1) = \sum_{s_1, s_2, \ldots, s_q} u(s_1|t_\mathrm{A}) u(s_2|t_\mathrm{A} - s_1) \cdots \\ \cdots u\left(s_{q-1}|t_\mathrm{A} - \sum_{i=1}^{q-2} s_i\right) c\left(s_q|t_\mathrm{A} - \sum_{i=1}^{q-1} s_i\right), \tag{17}$$

where q is the number of intra-slot IC steps and:

$$t_\mathrm{A} \geq 1,\ 1 \leq q \leq t_\mathrm{A},\ \sum_{i=1}^{q} s_i \leq t_\mathrm{A},\ 1 \leq s_1 \leq t_\mathrm{A}, \\ 1 \leq s_j \leq t_\mathrm{A} - \sum_{i=1}^{j-1} s_i,\ \text{for } 2 \leq j \leq q.$$

In other words, the pivotal idea in (17) is that the chain of the events, whose probabilities constitute summands, ends up with the outgoing edge being recovered.[4]

As all the transmissions colliding in the slots are statistically equivalent, probabilities $c(s|t_\mathrm{A})$ can be obtained from $p(s|t_\mathrm{A})$ in the following way:

$$c(s|t_\mathrm{A}) = \frac{\binom{t_\mathrm{A}-1}{s-1}}{\binom{t_\mathrm{A}}{s}} p(s|t_\mathrm{A}), \tag{18}$$

for $t_\mathrm{A} \geq 1$ and $1 \leq s \leq t_\mathrm{A}$; $u(s|t_\mathrm{A})$ can be obtained using (16). The analytical derivation of probabilities $p(s|t_\mathrm{A})$ for the considered CS-MUD receiver represent a formidable task that is out of the paper scope. Instead, we obtain them numerically, as described in the next subsection.

C. Numerical Evaluation of Capture Probabilities

We focus on the physical layer processing with CS-MUD in one slot, as described in Section III-B. In general, the performance of physical layer algorithms is analyzed by bit or frame error rate results, which are obtained by Monte Carlo simulations. Here, however, we define the event of s users being successfully decoded as $\Xi_s^{t_\mathrm{A}}$ in a slot of degree t_A. Then, the capture probability can be numerically evaluated by extending the usual average frame error rate evaluation such that all user frames *without frame error* in the sense of $\Xi_s^{t_\mathrm{A}}$ are counted. Evaluating T_sim Monte Carlo simulations, the capture probability can be approximated as:

$$p(s|t_\mathrm{A}) \approx \frac{\#\{\Xi_s^{t_\mathrm{A}}\}}{T_\mathrm{sim}}. \tag{19}$$

where $\#\{\Xi_s^{t_\mathrm{A}}\}$ counts how many times the event has happened; this approximation enhances with increasing T_sim. According to (17), $p(s|t_\mathrm{A})$ is required for all combinations of slot degrees $t_\mathrm{A} = 1, 2, \ldots, t_\mathrm{max}$ and captures $s = 1, \ldots, t_\mathrm{A}$, where t_max should be sufficiently high to evaluate all non-zero probabilities $p(s|t_\mathrm{A})$ up to the achievable accuracy.

[4]It can be also shown that the summands in (17) are mutually exclusive.

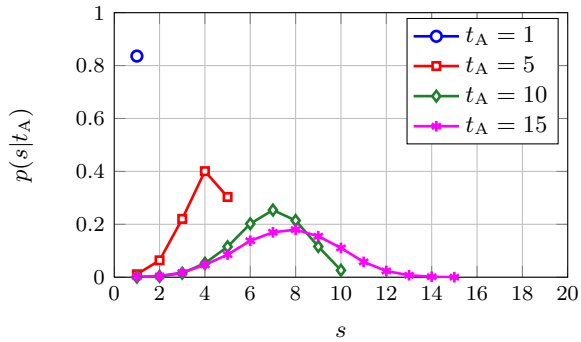

Fig. 4: Probability of s users recovered from a slot of degree t_A, for $1/\sigma_\mathrm{n}^2 = 10$ dB, and $T_\mathrm{sim} = 10^4$ simulations.

In order to numerically evaluate the capture probability for the and-or tree evaluation, we choose a specific physical layer setup. As mentioned in Section III, the capture probabilities are decided by the performance of the CS-MUD scheme. Thus, the choices of the length of PN sequence N_s and average slot degree β highly impact the numerical results. In CS theory, the properties of $\mathbf{A}$ in (4) strongly determine the recovery performance. The correlation of columns in $\mathbf{A}$ is determined by PN sequences and user-specific channels and should be minimal to achieve the best performance. However, for the sake of resource efficiency the spreading factor N_s should be chosen as small as possible, which requires a compromise. Finally, due to the interaction of CS-MUD and coded SA optimization in terms of average slot degrees the overall optimum choice of N_s is unknown and beyond the scope of this paper.

Therefore, based on the general description given in Section III-B, we focus on a basic setup to gain insight into the complex interaction of SA and CS-MUD. We assume an overloaded CDMA system with $N = 128$ users and PN spreading sequences of length of $N_\mathrm{s} = 32$ and $L = 8$ symbols per frame. Furthermore, the transmit data will be encoded by a convolutional code with code rate $R_\mathrm{c} = 0.5$ and modulated to BPSK symbols. All user specific channels to the BS are modeled as $L_\mathrm{h} = 6$ independent and identically Rayleigh distributed taps with an exponential decaying power delay profile. At the receiver, GOMP is applied as the CS-MUD algorithm and $T_\mathrm{sim} = 10^4$ simulations are performed to approximate the capture probability. All the parameters are chosen according the general system model in [6].

Fig. 4 presents an example of the obtained probability $p(s|t_\mathrm{A})$ for $1/\sigma_\mathrm{n}^2 = 10$ dB with different slot degrees t_A. If there is only one active user, i.e., $t_\mathrm{A} = 1$, the probability of recovering this is $p(1|1) \approx 0.85$. Furthermore, increasing t_A leads to lower capture probability. Particularly, the higher t_A is, the lesser the sparsity of the vector $\mathbf{x}$ in (4), which decreases the overall success probability[5].

Based on the capture probabilities provided by CS-MUD,

[5]The and-or tree evaluation method implicitly assumes $N \to \infty$, which cannot be evaluated with specific PHY layer processing. However, Fig. 4 and Fig. 5 clearly show a strong decrease in the probability of capture such that the results are a reasonable indication of the limit performance.

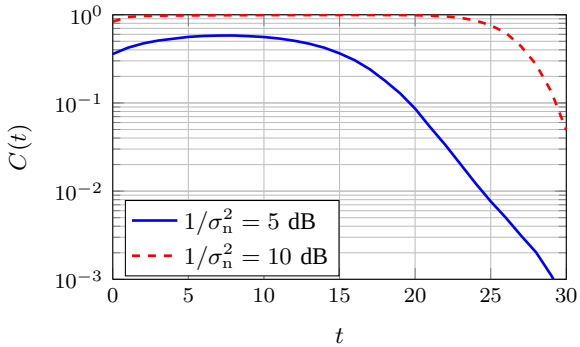

Fig. 5: Capture probability $C(t)$ for and-or tree evaluation at $1/\sigma_n^2 = 5$ and 10 dB.

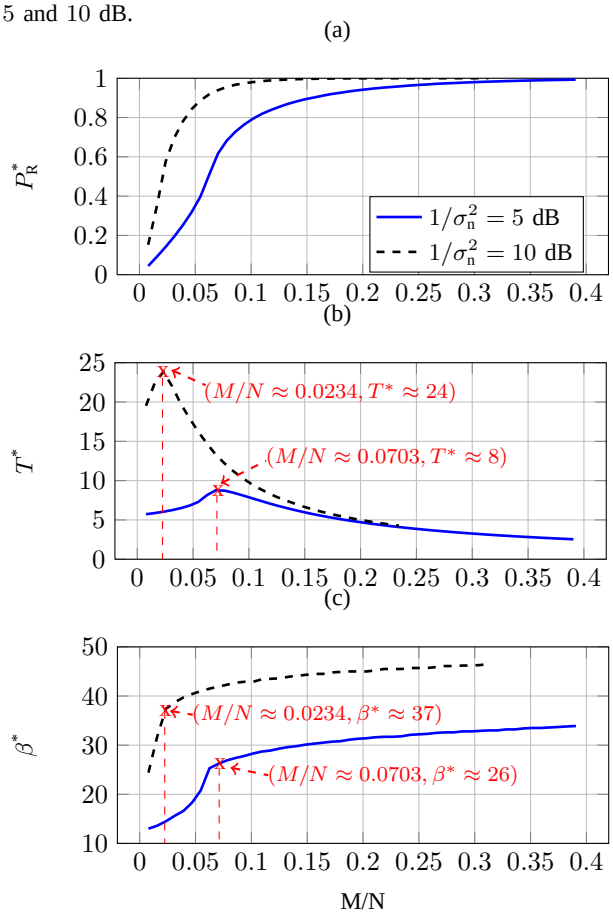

Fig. 6: a) Maximum user resolution probability P_R^*, b) maximum expected throughput T^*, c) optimum average slot degree β^*, as functions of M/N, at $1/\sigma_n^2 = 5$ and 10 dB.

the probability $C(t)$ for and-or tree evaluation can be obtained via (17). Fig. 5 presents $C(t)$ for $1/\sigma_n^2 = 5$ and 10 dB. Obviously, the probabilities of a transmission recovery are higher for higher SNR. Furthermore, the range of the number of interfering users t for which the transmission recovery $C(t)$ is highly probable is significantly wider for higher SNR.

V. Asymptotic Performance

In this section we present asymptotic performance results obtained using and-or tree evaluation with numerically evaluated capture probabilities. Specifically, we present the maximum user resolution probability P_R^*, the corresponding maximum expected throughput T^* and the corresponding optimum average slot degree β^* for which P_R^* and T^* are obtained, as functions of the ratio of the numbers of users and slots M/N.

Fig. 6 shows the asymptotic performance for $1/\sigma_n^2 = 5$ and 10 dB; obviously, the trends are the same for both values. However, the increase of $1/\sigma_n^2$ highly affects the performance due to the higher capture probabilities. As M/N increases, P_R^* steeply increases at first and then saturates; this behavior is analogous to the typical behavior of iterative BP erasure decoding [19]. Correspondingly, T^* at first increases and then, after some critical M/N, starts to drop. This critical value of M/N actually defines the asymptotically optimal length of the contention period, for which the overall maximum expected throughput can be achieved; Fig. 6c) shows which β^* should be used for the given M/N, in order to achieve this overall maximum expected throughput. For example, if one chooses $M \approx 0.023N$ and $\beta^* \approx 37$ when $1/\sigma_n^2 = 10$ dB, then the expected throughput is $T^* \approx 24$. The critical value of M/N is higher for lower $1/\sigma_n^2$, due to slower increase in P_R^*.

A closer inspection of the results reveals the following. For $1/\sigma_n^2 = 10$ dB, using (3) it can be shown that for the critical M/N the expected user degree, i.e., the average number of transmitted replicas per user, is only about 0.87. This actually means that there are not enough replicas to exploit inter-slot IC and that most of the throughput gain is achieved through intra-slot IC. On the other hand, for $1/\sigma_n^2 = 5$ dB, the expected user degree is about 1.83 for the critical M/N, implying that both inter- and intra-slot IC contribute to throughput performance.

Finally, the optimum average slot degrees β^* are significantly higher in comparison to scenarios with no capture or narrowband capture effect [5], [9]. This could be expected, as MUD receivers generally favor higher degrees of collision.

VI. Conclusion

In this paper we presented a study of the coded slotted ALOHA (SA) with capture effect combined with a a broadband Compressive Sensing based Multi-user Detection (CS-MUD) scheme. This novel access method for coded SA with iterative interference cancellation (IC) exhibits a complex interaction of MAC and PHY layer processing. We analyze the scheme using and-or tree evaluation and the numerically obtained capture probabilities of CS-MUD. The results show that the proposed access method significantly improves the throughput performance by increasing the number of decoded users per slot for coded slotted ALOHA.

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