Robust DOA Estimation of Harmonic Signals Using Constrained Filters on Phase Estimates
Karimian-Azari, Sam; Jensen, Jesper Rindom; Christensen, Mads Græsbøll

Published in:
2014 Proceedings of the 22nd European Signal Processing Conference (EUSIPCO 2014)

Publication date:
2014

Document Version
Preprint (usually an early version)

Link to publication from Aalborg University

Citation for published version (APA):
Robust DOA Estimation of Harmonic Signals Using Constrained Filters on Phase Estimates

Sam Karimian-Azari¹, Jesper Rindom Jensen², and Mads Grønbæk Christensen

email: {ska, jjr, mgo}@create.aau.dk

Introduction

Existing approaches to direction of arrival (DOA) estimation:
- Time-difference of arrival (TDOA) based estimators that scale the TDOA of successive microphones.
- Beamforming based methods that steer the array in a range of possible directions, and maximize output power versus the DOA.
- High-resolution estimators based upon spatiotemporal correlation matrix estimates.

The TDOA estimators possess an advantage over the two other methods in terms of computational complexity. Conventional TDOA estimators are designed assuming a single-source. However, the harmonic characteristics of audio signals facilitates a remarkable ability to estimate TDOAs of multiple sources which do not have spectral overlap.

We design optimal filters based on estimated noise statistics to apply on multi-channel phase estimates.

Formulation

Observed signal in an array (M microphones):

\[ y(n) = \sum_{l=1}^{L} \alpha_l e^{(j\omega_0 n - \omega l)} \beta d_l(\omega_0) + v(n) \]  (1)

\[ d_l(\omega) : \text{a steering vector for DOA of } \theta \text{ at } \omega \in [0, \pi] \]
\[ \beta = \text{diag}([\beta_1, \beta_2, \ldots, \beta_M]) : \text{magnitude attenuations} \]
\[ v(n) = [v_1(n), v_2(n), \ldots, v_M(n)]^T \in \mathbb{C}^M : \text{Gaussian noise} \]
\[ v_\delta(n) \text{ has the real and imaginary uncorrelated parts} \]
\[ \text{with the variance of } \sigma^2_\delta/2 \]
\[ \text{SNR}_l = \frac{(\beta_0 \delta)^2}{\sigma^2_\delta} : \text{nearband SNR} \]

If SNR_l >> 1, the additive Gaussian noise can be converted to a normally distributed phase noise \( \Delta \phi_m(\omega_0) \) with the variance of \( E[(\Delta \phi_m(\omega_0))^2] = \frac{1}{2 \text{SNR}_l} \) [1].

\[ \Delta \Phi_l = [\Delta \phi_1(\omega_0), \Delta \phi_2(\omega_0), \ldots, \Delta \phi_M(\omega_0)]^T : \text{phase noise vector} \]  (2)
\[ R_{\Delta \phi} = E(\Delta \Phi_l \Delta \Phi_l^T) = \text{diag}\left(\frac{1}{2 \text{SNR}_1}, \frac{1}{2 \text{SNR}_2}, \ldots, \frac{1}{2 \text{SNR}_M}\right) \]  (3)

Approximate noisy signal model:

\[ y(n) \approx \sum_{l=1}^{L} \alpha_l e^{(j\omega_0 n - \omega l)} D_l(\omega_0) \beta d_l(\omega_0), \]  (4)

with \( D_l(\omega_0) = \text{diag}(\exp(j\Delta \phi_l)) \).

Phase Shift Estimate \( \hat{\phi}_l \) (step 1)

Multi-channel phase estimates:

\[ \Phi_l = \Pi_M \begin{bmatrix} \hat{\phi}_1 \\ \vdots \\ \hat{\phi}_M \end{bmatrix} + \Delta \Phi_l \]  (5)

\[ \Phi_l = [\hat{\phi}_{l,1}, \hat{\phi}_{l,2}, \ldots, \hat{\phi}_{l,M}]^T : \text{collection of phase estimates} \]
\[ \Phi_{l,m} \in \mathbb{R}^{M \times 2} \text{ is a known matrix based on the number of microphones and the linear relationship between phases}. \]

Apply a filter \( W \in \mathbb{R}^{M \times 2} \):

\[ \hat{\phi}_l = W^T \Phi_l = W^T \Pi_M \begin{bmatrix} \hat{\phi}_1 \\ \vdots \\ \hat{\phi}_M \end{bmatrix} + W^T \Delta \Phi_l \]  (6)

With the constraint that \( W^T \Pi_M = I_{2 \times 2} \), MSE:

\[ \text{tr}(W^T R_{\Delta \phi} W) = \text{tr}(W^T \Pi_M W) \]

Design:

\[ \min_W \text{tr}(W^T R_{\Delta \phi} W) \text{ subject to } W^T \Pi_M = I_{2 \times 2} \]

\[ W_{MVDR} = R_{\Delta \phi}^{-1} \Pi_M (\Pi_M^T R_{\Delta \phi}^{-1} \Pi_M)^{-1} \]  (7)

DOA Estimate \( \hat{\theta} \) (step 2)

While \( \hat{\theta}_i = \sin^{-1}(\hat{\phi}_l/\omega_0 f_{l0}) \) for \( l = 1, \ldots, L \), the DOA of the harmonic source can be estimated from \( L \) phase shift estimates:

\[ \hat{\psi} = \omega_0 f_{l0} \sin(\hat{\theta}) \Gamma_L + \Delta \psi, \]  (8)

\[ \Delta \psi = [\Delta \psi_1, \Delta \psi_2, \ldots, \Delta \psi_L]^T : \text{phase shift noise} \]
\[ \Gamma_L = [1, 2, \ldots, L]^T \]

Apply a filter \( h \in \mathbb{R}^L \):

\[ \sin(\hat{\theta}) = h^T \hat{\psi} = \omega_0 f_{l0} \sin(\hat{\theta}) h^T \Gamma_L + h^T \Delta \psi. \]  (9)

With the constraint that \( h^T \Gamma_L = 1/\omega_0 f_{l0} \), MSE(\sin(\hat{\theta})) = \( h^T R_{\Delta \phi} h \).

Design:

\[ \min_h h^T R_{\Delta \phi} h \text{ subject to } h^T \Gamma_L = 1/\omega_0 f_{l0}. \]  (10)

DOA estimate:

\[ \hat{\theta} = \sin^{-1}(W_{MVDR}^T \hat{\psi}), \]  (11)

Simulation Results

DOA estimates of a synthetic signal, i.e., \( \omega_0 = 0.15\pi, L = 5, \) and \( M = 5 \), in different SNRs of colored noise and using different number of microphones (SNR= 20 dB):

Compared to:
- Weighted least-squares (WLS) DOA estimator [2]
- MVDR beamforming with harmonic emphasis (BH-MVDR) [3]

Covariance matrix:

\[ R_{\Delta \phi} = E(\hat{\Phi}_l - E(\hat{\Phi}_l))(\hat{\Phi}_l - E(\hat{\Phi}_l))^T \]  (12)

\[ E(\hat{\Phi}_l) \approx \frac{1}{B} \sum_{b=0}^{B-1} \hat{\Phi}_l(b) - bl_{\omega_0}1_M \]  (13)

where \( \hat{\Phi}_l(b) \) are estimated from \( Y(b) = [y(b), y(b + 1), \ldots, y(b + N - 1)] \).

Conclusion

We have estimated the DOA of a harmonic signal source from multi-channel phase estimates.
- We designed optimal filters based on spatial and spectral noise statistics.
- The designed filters are robust against different noise scenarios, e.g., colored noise.
- Results of the proposed method approach to the CRLB.

References