Robust DOA Estimation of Harmonic Signals Using Constrained Filters on Phase Estimates

Sam Karimian-Azari\(^1\), Jesper Rindom Jensen\(^2\), and Mads Grønbæk Christensen

email: {skkar, jrij, mgcc}@create.aau.dk

Introduction

Existing approaches to direction of arrival (DOA) estimation:

- Time-difference of arrival (TDOA) based estimators that scale the TDOA of successive microphones.
- Beamforming based methods that steer the array in a range of possible directions, and maximize output power versus the DOA.
- High-resolution estimators based upon spatiotemporal correlation matrix estimates.

The TDOA estimators possess an advantage over the two other methods in terms of computational complexity. Conventional TDOA estimators are designed assuming a single-source. However, the harmonic characteristics of audio signals facilitates a remarkable ability to estimate TDOAs of multiple sources which do not have spectral overlap.

We design optimal filters based on estimated noise statistics to apply on multi-channel phase estimates.

Formulation

Observed signal in an array (M microphones):

\[
y(n) = \sum_{l=1}^{L} \alpha_l e^{j(\omega_0 \tau_l + \phi)} \mathbf{d}_l(\omega_0) + \mathbf{v}(n) \tag{1}
\]

\[
\mathbf{d}_l(\omega) : \text{ a steering vector for DOA of } \theta \text{ at } \omega \in [0, \pi]
\]

\[
\beta = \text{diag}[\{\beta_1, \beta_2, \ldots, \beta_M\}] : \text{ magnitude attenuations}
\]

\[
\mathbf{v}(n) = [\mathbf{v}_1(n), \mathbf{v}_2(n), \ldots, \mathbf{v}_M(n)]^T \in \mathbb{C}^M : \text{ Gaussian noise}
\]

\[
\mathbf{v}_0(n) \text{ has the real and imaginary uncorrelated parts with the variance of } \sigma_n^2/2
\]

SNR\(_l\) = \(\frac{\sigma_d^2}{\sigma_n^2}\) : narrowband SNR

If SNR\(_l\) \(\gg 1\), the additive Gaussian noise can be converted to a normally distributed phase noise \(\Delta \varphi(l, \omega_0)\) with the variance of \(E[\{\Delta \varphi(l, \omega_0)^2\}] = \frac{\sigma_n^2}{2}\) [1]:

\[
\Delta \Phi = [\Delta \varphi(1, \omega_0), \Delta \varphi(2, \omega_0), \ldots, \Delta \varphi(L, \omega_0)]^T : \text{ phase noise vector}
\]

\[
\mathbf{R}_{\Delta \Phi} = E[\Delta \Phi \Delta \Phi^T] = \text{diag}\left(\frac{1}{2 \text{SNR}_1}, \frac{1}{2 \text{SNR}_2}, \ldots, \frac{1}{2 \text{SNR}_M}\right) \tag{3}
\]

Approximate noisy signal model:

\[
y(n) \approx \sum_{l=1}^{L} \alpha_l e^{j(\omega_0 \tau_l + \phi)} \mathbf{d}_l(\omega_0) \beta \mathbf{d}_l(\omega_0), \tag{4}
\]

with \(\mathbf{d}_l(\omega_0) = \text{diag}(\exp(j \Phi_l))\).

Phase Shift Estimate \(\hat{\psi}_l\) (step 1)

Multi-channel phase estimates:

\[
\Phi_l = \mathbf{p} = \left[\begin{array}{c} \hat{\psi}_0 \\
\hat{\psi}_1 \end{array}\right] + \Delta \Phi \tag{5}
\]

\[
\Phi_l = [\Phi_{l1}, \Phi_{l2}, \ldots, \Phi_{LM}]^T : \text{ collection of phase estimates}
\]

\[
\Phi_{LM} \in \mathbb{R}^{M \times 2} \text{ is a known matrix based on the number of microphones and the linear relationship between phases.}
\]

Apply a filter \(W \in \mathbb{R}^{M \times 2}\):

\[
\hat{\psi}_l = W^T \hat{\Phi}_l = W^T \Phi_l + W^T \Delta \Phi. \tag{6}
\]

With the constraint that \(W^T \Phi_l = I_{2 \times 2}\), MSE:

\[
\text{MSE}\left[\begin{array}{c} \hat{\psi}_1 \\
\hat{\psi}_2 \end{array}\right] = \text{tr}\{W^T R_{\Delta \Phi} W\}. \tag{7}
\]

Design:

\[
\min W \text{ tr}[W^T R_{\Delta \Phi} W] \text{ subject to } W^T \Phi_l = I_{2 \times 2}.
\]

\[
W_{\text{MVDR}} = R_{\Delta \Phi}^{-1} \Phi_l (\Phi_l^T R_{\Delta \Phi}^{-1} \Phi_l)^{-1}. \tag{7}
\]

DOA Estimate \(\hat{\theta}\) (step 2)

While \(\hat{\psi}_l = \sin^{-1}(\hat{\psi}_l/\omega_0 f_0\tau_l)\) for \(l = 1, \ldots, L\), the DOA of the harmonic source can be estimated from \(L\) phase shift estimates:

\[
\hat{\psi} = \omega_0 f_0 \tau_l \sin(\theta) \tau_l + \Delta \psi_l, \tag{8}
\]

\[
\Delta \psi_l = [\Delta \psi_1, \Delta \psi_2, \ldots, \Delta \psi_L]^T : \text{ phase shift noise}
\]

\[
\tau_l = \{1, 2, \ldots, L\} \tag{8}
\]

Apply a filter \(h \in \mathbb{R}^L\):

\[
(\hat{\psi}) = h^T \hat{\psi} = \omega_0 f_0 \tau_l \sin(\theta) \tau_l + h^T \Delta \psi_l. \tag{9}
\]

With the constraint that \(h^T \tau_l = 1/\omega_0 f_0 \tau_l\), MSE\{(\hat{\psi})\} = h^T R_{\Delta \Phi} h.

Design:

\[
\min h \text{ h}^T R_{\Delta \Phi} h \text{ subject to } h^T \tau_l = 1/\omega_0 f_0 \tau_l. \tag{10}
\]

DOA estimate:

\[
\hat{\theta} = \sin^{-1}(h_{\text{MVDR}}^T \hat{\psi}). \tag{11}
\]

Simulation Results

DOA estimates of a synthetic signal, i.e., \(\omega_0 = 0.15\pi\), \(L = 5\), and \(M = 5\), in different SNRs of colored noise and using different number of microphones (SNR\(_{10}\) = 20 dB):

Compared to:

- Weighted least-squares (WLS) DOA estimator [2]
- MVDR beamforming with harmonic emphasis (8H-MVDR) [3]

Covariance matrix:

\[
R_{\Delta \Phi} = E\{(\hat{\Phi}_l - E(\hat{\Phi})) (\hat{\Phi}_j - E(\hat{\Phi}))^T\} \tag{12}
\]

\[
E(\hat{\Phi}) \approx \frac{1}{B} \sum_{b=0}^{B-1} (\hat{\Phi}_l(b) - bl_01_M) \tag{13}
\]

where \(\hat{\Phi}(b)\) are estimated from \(Y(b) = [y(b), y(b+1), \ldots, y(b+N-1)]\).

Conclusion

We have estimated the DOA of a harmonic signal source from multi-channel phase estimates.

- We designed optimal filters based on spatial and spectral noise statistics.
- The designed filters are robust against different noise scenarios, e.g., colored noise.
- Results of the proposed method approach to the CRLB.

References