



Robust Pitch Estimation Using an Optimal Filter on Frequency Estimates

Karimian-Azari, Sam; Jensen, Jesper Rindom; Christensen, Mads Græsbøll

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Robust Pitch Estimation Using an Optimal Filter on Frequency Estimates

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Sam Karimian-Azari, Jesper Rindom Jensen,
and Mads Græsbøll Christensen
{ska, jrj, mgc}@create.aau.dk

Audio Analysis Lab, AD:MT,
Aalborg University, Denmark

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AALBORG UNIVERSITY
DENMARK

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In speech and audio signal processing applications such as:

- ▶ hearing-aids
- ▶ teleconference systems

we can design filters based on parametric models.

Pitch estimate is often necessary in some applications such as:

- ▶ Coding
- ▶ Enhancement
- ▶ Separation
- ▶ Compression

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Harmonic Signal Model:

$$s(n, \theta) = \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \varphi_l)}, \quad (1)$$

$$\theta = [\alpha_1, \varphi_1, \omega_1, \dots, \alpha_L, \varphi_L, \omega_L]^T$$

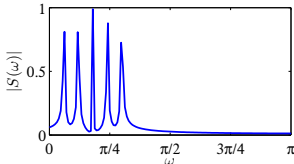
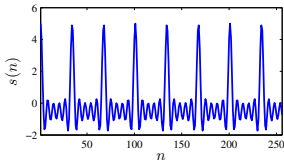
where $\omega_l = l\omega_0$ (for $l = 1, \dots, L$),

L : number of sinusoids

α_l : real magnitudes

ω_0 : fundamental frequency

φ_l : phases of harmonics



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Existing approaches*:

- **Filtering Methods:** Based on filtering the observed signal that pass power undistorted frequency of harmonics, e.g., optimal adaptive designs.
- **Subspace Methods:** Based on the principles of subspace orthogonality, e.g., MUSIC method
- **Statistical Methods:** Based on maximizing the likelihood function of pitch, e.g., the non-linear least-squares (NLS), and weighted least squares (WLS) [H. Li, P. Stoica, and J. Li 2000].

* [M.G.Christensen and A.Jakobsson 2009]

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The Weighted Least Squares (WLS) Parameter Estimator

[H. Li, P. Stoica, and J. Li 2000]

The observed signal is contaminated by Gaussian noise $v(n)$ with variance of σ^2

$$x(n) = \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n). \quad (2)$$

With the assumption of white Gaussian noise, the WLS pitch estimator yields

$$\hat{\omega}_0 = \frac{1}{\sum_{l=1}^L (l \hat{\alpha}_l)^2} \sum_{l=1}^L l \hat{\alpha}_l^2 \hat{\omega}_l, \quad (3)$$

where $\hat{\omega}_l$ and $\hat{\alpha}_l$ can be estimated using different methods, e.g., the DFT, MUSIC, ESPRIT, NLS, Capon, and APES.



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Drawbacks

The WLS pitch estimator is **computationally efficient** with good **statistical performance**,

BUT! It is not optimal for cases:

- ▶ nonidentical noise variances among harmonics, e.g., colored noise.
- ▶ spurious frequency estimates.
- ▶ missing harmonics.



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At a high narrowband SNR, i.e.,

$$\text{SNR}(\omega_l) = \frac{\alpha_l^2}{\sigma^2} \gg 1, \quad (4)$$

the harmonic frequency ω_l is perturbed with a real angular noise [S. Tretter 1985]

$$\Delta\omega_l(n) = \frac{v(n)}{\alpha_l} \sin(l\omega_0 n + \varphi_l), \quad (5)$$

which has a normal distribution with zero mean and variance

$$\text{E}\{(\Delta\omega_l)^2\} = \frac{1}{2 \text{SNR}(\omega_l)}, \quad (6)$$

where the white Gaussian noise has a homogeneous power spectrum $\Phi(\omega) = \sigma^2$ across frequencies $\omega \in [0, \pi]$.

In general,

$$\text{E}\{(\Delta\omega_l)^2\} = \frac{\Phi(\omega_l)}{2 \alpha_l^2}. \quad (7)$$

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We can approximate $x(n) = \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n)$ like,

$$x(n) \approx \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \Delta\omega_l(n) + \varphi_l)}. \quad (8)$$

Harmonic frequencies:

$$\mathbf{\Omega} = [\omega_1, \omega_2, \dots, \omega_L]^T = \mathbf{d}_L \omega_0, \quad (9)$$

where $\mathbf{d}_L = [1, 2, \dots, L]^T$.

Unconstrained frequency estimates (UFEs):

$$\begin{aligned} \hat{\mathbf{\Omega}} &= [\hat{\omega}_1, \hat{\omega}_2, \dots, \hat{\omega}_L]^T \\ &= \mathbf{\Omega} + \Delta\mathbf{\Omega} = \mathbf{d}_L \omega_0 + \Delta\mathbf{\Omega}, \end{aligned} \quad (10)$$

where $\Delta\mathbf{\Omega} = [\Delta\omega_1, \Delta\omega_2, \dots, \Delta\omega_L]^T$.

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Apply a filter $\mathbf{h} \in \mathbb{R}^L$

$$\hat{\omega}_0 = \mathbf{h}^T \hat{\Omega} = \mathbf{h}^T \mathbf{d}_L \omega_0 + \mathbf{h}^T \Delta \Omega. \quad (11)$$

With the distortionless constraint that $\mathbf{h}^T \mathbf{d}_L = 1$, the mean squared error (MSE) of the unbiased estimator is given by

$$\begin{aligned} \text{MSE}[\hat{\omega}_0] &= \text{E}\{(\hat{\omega}_0 - \omega_0)^2\} \\ &= \text{E}\{(\mathbf{h}^T \Delta \Omega)(\Delta \Omega^T \mathbf{h})\} = \mathbf{h}^T \Phi_{\Delta \Omega} \mathbf{h}. \end{aligned} \quad (12)$$

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Independent UFEs would be implicitly uncorrelated, i.e., $E\{\Delta\omega_i\Delta\omega_k\} = 0$ for $i \neq k$, when harmonics are well-separated and the narrowband SNRs are high enough;

$$\begin{aligned}\Phi_{\Delta\Omega} &= E\{\Delta\Omega \Delta\Omega^T\} \\ &= \text{diag}\left\{\left[\frac{\Phi(\omega_1)}{2\alpha_1^2}, \frac{\Phi(\omega_2)}{2\alpha_2^2}, \dots, \frac{\Phi(\omega_L)}{2\alpha_L^2}\right]\right\}.\end{aligned}\quad (13)$$

In practice:

$$\Phi_{\Delta\Omega} = E\{(\hat{\Omega} - E\{\hat{\Omega}\})(\hat{\Omega} - E\{\hat{\Omega}\})^T\}, \quad (14)$$

where $E\{\cdot\}$ is the mathematical expectation, and

$$E\{\hat{\Omega}(n)\} = \frac{1}{M} \sum_{m=0}^{M-1} \hat{\Omega}(n-m). \quad (15)$$

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Optimal filter is given as the solution to

$$\begin{aligned} \min_{\mathbf{h}} \quad & \mathbf{h}^T \boldsymbol{\Phi}_{\Delta\Omega} \mathbf{h} \\ \text{subject to} \quad & \mathbf{h}^T \mathbf{d}_L = 1 \end{aligned}$$

Minimum variance distortionless response (MVDR):

$$\mathbf{h}_{\text{MVDR}} = \boldsymbol{\Phi}_{\Delta\Omega}^{-1} \mathbf{d}_L (\mathbf{d}_L^T \boldsymbol{\Phi}_{\Delta\Omega}^{-1} \mathbf{d}_L)^{-1}. \quad (16)$$

\Downarrow

Maximum likelihood (ML):

Assuming white Gaussian noise with the variance σ^2

$$\boldsymbol{\Phi}_{\Delta\Omega}^{-1} = \frac{2}{\sigma^2} \text{diag}\{\alpha_1^2, \alpha_2^2, \dots, \alpha_L^2\}, \quad (17)$$

$$\mathbf{h}_{\text{ML}} = \frac{1}{\sum_{l=1}^L (l\alpha_l)^2} [\alpha_1^2, 2\alpha_2^2, \dots, L\alpha_L^2]^T. \quad (18)$$

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Asymptotic phase estimates of harmonics at $n \in [0, N - 1]$:

$$\hat{\Psi}(n) = \mathbf{d}_L \omega_0 n + \Xi_L + \Delta\Omega(n) \quad (19)$$

Likelihood Function:

$$P(\hat{\Psi}, \xi) \sim \mathcal{N}(\mathbf{d}_L \omega_0 + \Xi_L, \Phi_{\Delta\Omega}) \quad (20)$$

where $\xi = [\omega_0, \Xi_L]^T$ and $\Xi_L = [\varphi_1, \varphi_2, \dots, \varphi_L]^T$.

CRLB of pitch estimate:

$$[\mathbf{I}(\xi)^{-1}]_{1,1} = \frac{12}{N(N^2 - 1)} (\mathbf{d}_L^T \Phi_{\Delta\Omega}^{-1} \mathbf{d}_L)^{-1}, \quad (21)$$

where $\mathbf{I}(\xi) = -E\left\{\frac{\partial^2 \ln p(\Psi, \xi)}{\partial \xi \partial \xi^T}\right\}$ is the Fisher information matrix.

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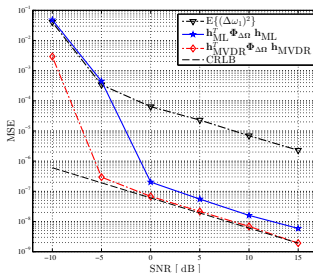
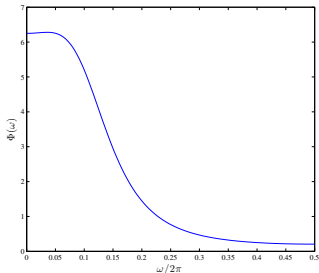
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Experimental Results

Experiment 1 (synthetic signal)

A synthetic harmonic signal $\omega_0 = 0.15\pi$, $L = 5$ sinusoids with identical amplitudes. We estimated UFEs from $N = 60$ samples using the subspace orthogonality method based on the MUSIC method.

- Additive colored Gaussian noise, different SNR:



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Experiment 2 (synthetic signal)



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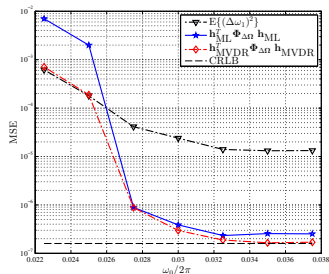
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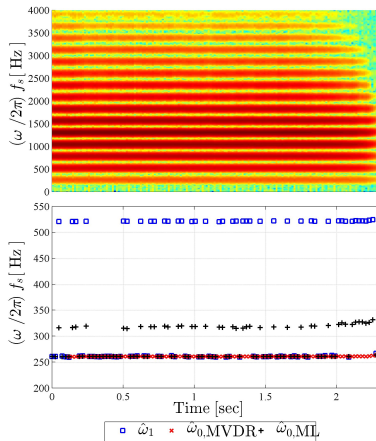
- White Gaussian noise in SNR= 5 dB, different ω_0 :



Experimental Results

Experiment 3 (real signal)

- ▶ A trumpet signal contaminated by colored Gaussian noise, $f_s = 8.0$ kHz, $N = 128$
- ▶ We estimated L using the subspace orthogonality method [M.G.Christensen et al. 2007]



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Summery and Conclusion



- ▶ We estimated pitch from unconstrained frequency estimates (UFEs).
- ▶ We designed an optimal filter based on noise statistics which is estimated from UFEs.
- ▶ The optimal filter design is a general method that we can derive the WLS pitch estimator.
- ▶ Pitch estimation using the optimal filter is robust against different noise situations, e.g., colored noise.
- ▶ Pitch estimation using the optimal filter is robust against missing harmonics in some time-frames and outperforms the WLS pitch estimator.

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Thank you!



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