Resonant-Inductor-Voltage-Feedback Active Damping Based Control for Grid-Connected Inverters With \textit{LLCL}-filters

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Abstract—\textit{LLCL}-filter is recently emerging into grid-connected inverters due to its high attenuation of high-frequency harmonics with a smaller size. Active damping methods have been proposed to reduce the resonance peak caused by the \textit{LLCL}-filter to stabilize the whole system without extra losses. The active damping method with an extra feedback provides a high rejection of the resonance so that the dynamic is improved. In this paper, taking a Proportional-Resonant (PR) together with a harmonic compensator (HC), resonant-inductor-voltage-feedback active damping is applied on an \textit{LLCL}-filter based three-phase grid-connected voltage source inverter (VSI). The design method is described through the analysis in the s-domain and the z-domain. Then the robustness and harmonic rejection of the grid voltage with the active damping method is analyzed considering the processing delay. Finally, the performance of the proposed method is investigated in simulation and by experimental results.

I. INTRODUCTION

Over the last decade, the growing energy demand and greenhouse effect have prompted a renewed interest in the use of renewable energy, such as solar energy, wind energy and fuel cell, etc. Voltage Source Inverter (VSI) has increasingly been used to enable the integration of renewable power generations into grids. Due to the switching frequency harmonics produced by the Pulse-Width Modulation (PWM) of VSI, a low-pass power filter is generally inserted between the VSI and the grid to attenuate PWM harmonics to an acceptable limit. An \textit{L}-filter or an \textit{LCL}-filter is usually used for the low-pass filter. Compare with \textit{L} -filter, \textit{LCL}-filter has higher attenuation of the switching frequency harmonics and allows smaller inductance. However, to further reduce the total inductance and volume of filter an \textit{LLCL}-filter has been reported with the 10% -25% reduction compared to an \textit{LCL}-filter [1], [2]. Hence, the \textit{LLCL}-filter may become attractive for the grid-connected VSIs [3], [4].

Similarly to the \textit{LCL}-filter, the \textit{LLCL}-filter resonance is challenging the stability of grid-connected VSI. A direct way to dampen the filter resonance is to add a dissipative element in parallel or series with the filter inductors or filter capacitor which is called as passive damping. However this method will bring extra cost and reduces the overall system efficiency [5]-[8]. Another way is to actively dampen the resonance by introducing current or voltage feedbacks or control algorithm. Active damping methods are more flexible and lossless, but they need extra sensors and bring control complexity [9]-[12]. Moreover, the performance and the stability of control system are closely related to the ratio of the resonance frequency to the control frequency, due to the computation and PWM delays in digital control system [13] - [17]. The capacitor–current-feedback active-damping for \textit{LCL}-filter considering the effect of the delay is studied [14]. It showed how the delay has the effect to the active damping and proposed a way to reduce the delay. At the same time, the design of active damping is also influenced by the variation of the grid impedance. The robustness of the system with \textit{LCL}-filter based on the grid impedance variation is analyzed in [18], [19].

The stability of the \textit{LLCL}-filter is studied in [20]. It showed when the ratio of the resonant frequency to the control frequency is high the system can be stable without damping but the system robustness is not good if the resonant frequency is around the \(1/6\) of the control frequency. Compare with the \textit{LCL}-filter, the \textit{LLCL}-filter has an extra resonant inductor which is possible also be sensed to attenuate the resonance. In this paper, resonant-inductor-voltage-feedback active damping method is analyzed for the \textit{LLCL}-filter using integral feedback coefficient considering the effect of the delay and the grid impedance variation.

In additional to the active damping method, the Proportional-Resonant with the Harmonic-Compensation (PR+HC) controllers is also used in this paper. PR can provide larger gain at the fundamental frequency to eliminate the steady sate error compared with PI regulator and HC performs well to reject the grid harmonic distortion [21], [22].
In Section II, the model of the grid-connected inverter with the LLCL-filter in the s-domain is introduced. Section III shows the analysis in the z-domain. The design procedures of current control and resonant-inductor-voltage-feedback are introduced in section IV. Based on the design example in section IV, section V analyzes the robustness and grid harmonic rejection with the proposed active damping. Last, simulated and experimental results are shown to verify the proposed design method.

II. MODELING THE GRID-CONNECTED INVERTER WITH THE LLCL-FILTER IN THE S-DOMAIN

A. The Mathematical Model

Fig. 1 shows the topology of three-phase grid-connected inverter with LLCL-filter, where the parasitic resistances are ignored. The LLCL-filter parameters are designed according to [2]. \(L_1\) is the inverter-side inductor, \(C_f\) is the filter capacitor, \(L_f\) is the filter inductor, \(L_2\) is the grid-side inductor, \(L_g\) is the grid impedance, \(i_1\) is the inverter-side current, \(i_c\) is the capacitor current, \(i_g\) is the grid-side current, \(u_1\) is the output voltage of inverter, \(u_{cG}\) is the output voltage of the capacitor, \(u_L\) is the output voltage of the resonant inductor, \(u_g\) is the voltage of \(L_f C_f\) circuit and \(u_g\) is the grid voltage.

\[\begin{align*}
L_1 \frac{di_1}{dt} &= u_1 - u_{cG} - u_g \\
\left(L_2 + L_f\right) \frac{di_g}{dt} &= u_{cG} + u_{cG} - u_g \\
C_f \frac{di_c}{dt} &= i_c - i_g \\
u_c &= L_f \left(\frac{di_1}{dt} - \frac{di_g}{dt}\right)
\end{align*}\]  

\(\omega_r = \frac{1}{\sqrt{\frac{L_1 \left(L_2 + L_g\right)}{L_1 + L_2 + L_g + L_f} \cdot C_f}}\)  

The resonance frequency of the LLCL-filter \(\omega_r\) is derived in (2). The open loop transfer functions from \(i_g\) to \(u_1\) and \(i_c\) to \(u_1\) are expressed in (3) and (4), respectively.

\[G_{i_g \rightarrow u_1}(s) = \frac{\left(L_2 + L_f\right) s^2}{\left[L_1 \left(L_2 + L_g\right) + \left(L_1 + L_2 + L_g + L_f\right) \cdot L_f\right] \left(s^2 + \omega_r^2\right)}\]  

B. Model of Resonant-Inductor-Voltage-Feedback Active Damping

According to [12] different variables feedbacks are feasible for damping the resonant problem. The structure of the active damping method based on the capacitor current feedback for grid-connected inverter with the LLCL-filter is shown in Fig. 2 which is equal to a resistance paralleled with the capacitor for grid-connected inverter with the LCL-filter. \(K_i\) is the capacitor current feedback coefficient, \(G_c(s)\) is the delay part in series with the forward path. \(G_i(s)\) is the current controller. \(K_{PWM}\) is the transfer function of the inverter bridge and \(i_g^*\) is the reference current.

\[G_{i_g \rightarrow u_1}(s) = \frac{\left(L_2 + L_f\right) s^2}{\left[L_1 \left(L_2 + L_g\right) + \left(L_1 + L_2 + L_g + L_f\right) \cdot L_f\right] \left(s^2 + \omega_r^2\right)}\]

C. Equivalent Transformation

Fig. 4 can be transformed into the model depicted in Fig. 5 [23], [24]. \(H(s)\) is the injected grid current sensor. In order
to simplify the analysis, it is assumed to be one. \( G_1(s) \) and \( G_2(s) \) can be expressed in (5) and (6).

\[
G_1(s) = \frac{K_{PWM}(1+C_L L_s s^2)G_r(s)G_u(s)}{(L_f + L_f)C_f s^2 + K_{PWM}K_{ult} G_r(s)L_f C_f s + 1}
\]

(5)

\[
G(s) = \frac{L_f + L_f C_f s^2 + K_{PWM}K_{ult} G_r(s)L_f L_f C_f s + 1}{[L_f + L_f + (U_f + L_f) L_f]C_f s^2 + K_{PWM}K_{ult} G_r(s)L_f [L_f + L_f]C_f s + (U_f + L_f + L_f]s)}
\]

(6)

The closed loop expression of the grid current \( i_g \) can be derived as:

\[
i_g(s) = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)} \cdot i_g(s) - \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)} u_g(s)
\]

(7)

The loop gain of the system is:

\[
T(s) = G_1(s)G_2(s)H(s)
\]

\[
T(s) = K_{PWM}(1+C_L L_s s^2)G_r(s)G_u(s)
\]

(8)

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS OF SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC link voltage ( U_d )</td>
<td>650 V</td>
</tr>
<tr>
<td>Grid phase voltage ( U_x )</td>
<td>220 V</td>
</tr>
<tr>
<td>Resonant frequency ( f_r )</td>
<td>2.6 kHz</td>
</tr>
<tr>
<td>Inductor ( L_f )</td>
<td>1.8 mH</td>
</tr>
<tr>
<td>Grid-side inductor ( L_f )</td>
<td>2 mH</td>
</tr>
<tr>
<td>Capacitor ( C_f )</td>
<td>4 ( \mu )F</td>
</tr>
<tr>
<td>Resonant inductor ( L_f )</td>
<td>64 ( \mu )H</td>
</tr>
<tr>
<td>Switching frequency ( f_s )</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Sampling period ( T_s )</td>
<td>100 us</td>
</tr>
</tbody>
</table>

III. ANALYSIS IN THE DISCRETE DOMAIN

A. The Influence of delay for Resonant-Inductor-Voltage-Feedback Active Damping

When the grid-side current is used for feedback, the system is unstable without considering delay regardless of the controller. Because of the delay caused by the computation and PWM pattern generator, the system is no longer a minimum phase system. The stability of the system should be analyzed considering the delay. Ref. [15] - [16] have discussed the effect of the delay and methods to utilize the delay. In this paper, to calculate these lag phase shifts the computational delay is modeled as a pure delay of \( T_s \), and the PWM is equivalent to a Zero Order Hold (ZOH), as a pure delay of 0.5\( T_s \).

Fig. 6 shows the bode plot of the forward path transfer function \( T(s) \) with the resonant-inductor-voltage-feedback active damping based on the example in Table I. \( f_r \) is the resonant frequency and \( f_0 \) is the critical frequency when the phase crosses -180° [15]. As seen in Fig. 6, the feedback of the resonant-inductor-voltage can effectively dampen the resonance peak and a larger resonant-inductor-voltage feedback coefficient leads to a better damping of the resonance peak. Delay has significant impact on the phase of the loop gain and the phase decreases at the frequencies lower than the resonance frequency. And if \( f_r \) is larger than \( f_0 \) the phase of \( T(s) \) will cross -180° twice, so the resonant-inductor-voltage-feedback coefficient should be small to satisfy the magnitude margin both at the resonant frequency and the critical frequency.

![Bode plot of the forward path transfer function for the single loop](image)

**B. Analysis in the Discrete-Time Domain**

For digital algorithm implementation, the proposed controller needs to be transformed from \( s \)-domain to \( z \)-domain [25]. The active damping is implemented in the digital controller, see Fig. 7.

When the grid current and the resonant inductor voltage are in practice sampled at the same time instants sampling period, (3) and (4) can be transformed to \( z \)-domain by applying ZOH transform [17] to be:

\[
i(z) = \left( z^{-1} - 1 \right) u_0 + \left( z^{-1} - 1 \right) u_0
\]

\[
i(z) = \left( z^{-1} - 1 \right) G(z) u_0
\]

![Block diagram for active damping in the digital controller](image)
It can be seen from (16) when considering the delay of 1.5Ts, the critical frequency is 1/6 of the sampling frequency. T(z) has right-half plane poles if $f_r > f_s/6$. Hence, to ensure the closed-loop system stably, the resonance peak of the LLCL-filter should be damped below 0dB when $f_r < f_s/6$, or the resonance peak of the LLCL-filter should remain above 0dB when $f_r > f_s/6$ [14]. Next section will introduce the design of specific example.

IV. DESIGN OF THE CURRENT CONTROL AND RESONANT-INDUCTOR-VOLTAGE-FEEDBACK WITH LLCL-FILTER

A. PR+HC Controller Gain Design

The crossover frequency $f_s$ is usually set lower 1/10 of the switching frequency considering the effect of high-frequency attenuating noise [17]. Thus the influence of the filter capacitor can be ignored when calculating the magnitude of the loop gain at $f_r$ and the frequencies lower than $f_r$. The LLCL-filter is approximate to an L filter and $G_c(s)$ can be regarded as $K_p$ at crossover frequency. $K_p$ is determined by considering the bandwidth of the whole system and the compensated frequencies are shown in (17) and (18). The cross-over frequency $\omega_p$ can be determined in (19) considering the phase lag caused by the delay.

$$G_c(\omega_p) = \frac{K_p}{\omega_p(L_r+L_s)}$$ \hspace{1cm} (17)

$$\omega_p = \frac{\pi}{2T_s} - \phi_m$$ \hspace{1cm} (18)

$$K_p = \frac{\omega_p(L_r+L_s)}{2T_s}$$ \hspace{1cm} (19)

where $\phi_m$ is the desired phase margin. Usually, if the value of $K_{ih}$ is larger, the steady-state error is smaller and robustness of grid frequency is better [20], but large $K_{ih}$ would lead to poor phase margin and affects overall system performance. Hence $K_{ih}$ should be designed to satisfy the phase margin.

B. Resonant-Inductor-Voltage-Feedback Coefficient Design

Due to the influence of the delay, the design of resonant-inductor-voltage-feedback active damping coefficient should be complicated by the value of resonant frequency. According to Fig. 6 and (16), it can be deduced that the gain margin at $f_r$ and $f_s/6$ is very important to the system stability when $1.5T_s$ is considered. $GM_1$ and $GM_2$ are the magnitude of $T(z)$ at $f_r$ and $f_s/6$ that can be calculated based on (8) [17]. $GM_1 > 0$ means the magnitude is lower than 0 dB at the corresponding frequency. Then the relation between $K_{ih}$ and $GM_1, GM_2$ can be obtained as (20) and (21).

$$K_{ih} = 10^{GM_1/20} \frac{2\pi f_r L_r}{L_c K_{PWM}}$$ \hspace{1cm} (20)

$$K_{ih} = 10^{GM_2/20} \frac{6f_s f_r L_r}{L_c K_{PWM}} + \frac{2\pi f_r L_c}{L_c K_{PWM}} \left(\frac{f_s}{6} - \frac{f_r}{6}\right)^2$$ \hspace{1cm} (21)

When $f_r < f_s/6$, the resonant frequency will increase with $K_{ih}$ increasing then $K_{ih}$ should satisfy (16) to make sure the
phase only across -180° once at $f_r$, $GM_1 > 0$. If $K_{ud/f}$ does not satisfy (16), there are unstable poles, $GM_1<0$ and $GM_2<0$; when $f_r > f_r/6$, $GM_1 < 0$ and $GM_2 > 0$ to make sure the magnitude of the loop gain at $f_r/6$ must be lower than 0 dB and at $f_r$ must be larger than 0 dB.

C. Design Example

Taking the system given in Table I as an example, the basic design procedures can be addressed as:

1. Determine the parameters of the loop gain. According to Table I, the resonant frequency is larger than $f_r/6$, $GM_1<0$ and $GM_2>0$. The desired phase margin $\Phi_m$ should be larger than 30° and $GM_1 = -4$ dB, $GM_2 = 2.5$ dB to get a good dynamic response and stability margin.

2. Obtain the value of $K_p$ and $f_c$ to satisfy all the requirements according to (17), (18) and (19). $f_c = 1.0$ kHz is chosen to obtain fast dynamic response. Then $K_p$ is calculated as 0.06 according to (18).

3. Taking $K_p=0.06$ into the open loop $T(z)$, root locus of the system with $K_{ud/f}$ increasing can be drawn in Fig. 8 to find the satisfactory feedback gain in this example. There is a region for $K_{ud/f}$ to keep the system stable. Take $f_c = 1.0$ kHz in (20), (21) to make sure the gain margin. $K_{ul/f}=110.5$ is chosen.

4. Then decide $K_1 = K_5 = K_r = 25$ by bode plot to get a small steady error with appropriate grid harmonic rejection.

Fig. 8. Root locus of the system with $K_{ud/f}$ increasing.

V. STABILITY AND ROBUSTNESS ANALYSES

A. Robustness Analysis With Impedance Variation

The Robustness of the system is in respect to the variation of damping coefficient, system parameters and grid transformer impedance. The system is capable of rejecting low frequency harmonic distortion, which is existing in the grid voltage. Based on the control parameters designed before, Fig. 9 shows bode plots of system open-loop transfer function $T(s)$ under different grid inductances: 0.5 mH, 1.8 mH, and 4 mH. The gain will be decreased and the effect of the active damping will be decreased with $L_g$ increasing, but it is still stable.

![Bode Diagram](image)

Fig. 9. System open-loop bode plots with $L_g$ increasing.

While designing a controller, a typical specification is the gain margin in root locus. The performance of the resonant-inductor-voltage-feedback active-damping control with the grid impedance variation is studied by closed-loop pole maps, as shown in Fig.10. The pair closed-loop poles introduced by the PR regulator are not drawn. With the increasing of $L_g$, the damping ratio of the system is reduced. The resonant poles are inside the unit circle but getting close to the border. The system has enough robustness towards the grid impedance variation in this case.

![Root Locus](image)

Fig. 10. Root locus of the system with $L_g$ variation.

B. Robustness Analysis With Variation of Inductor $L_f$

For the LLCL-filter, $L_f$ is designed according to the switching frequency. It can be seen from (20) and (21), the $K_{ul/f}$ is related to $L_f$.

When $K_{ul/f}$ is designed if the parameter drift of $L_f$ is in a range of ± 20%, then the actual feedback value of $K_{ul/f}$
would also have a variation of ±20%. This will influence the
damping effect of the resonant-inductor-voltage-feedback
and could cause instability. Hence when $K_{udf}$ is designed
according to (20), (21) and the root locus of the system, the
enough margins should be reserved considering the possible
variation of the inductor $L_f$.

C. Grid Harmonics Rejection

The grid voltage is seen as the disturbance term in the
design of the current loop controller. PR with HC can be
utilized. The transfer function from $u_g$ to $i_g$ is expressed as:

$$i_g(s) = \frac{G_z(s)}{u_g(s) + G_z(s)G_i(s)H(s)}$$ (22)

As shown in Fig. 11, with the increase of the grid
inductors, the characteristics do not change too much with
the active damping. It shows the system has sufficient grid
harmonic rejection capability at the compensated frequencies, 250 Hz, and 350 Hz. At the resonant frequency,
the system has sufficient damping.

VI. SIMULATION RESULTS

In order to test the current control and the active
damping method of LLCL-filter based grid-connected
inverter for the stability and robustness analysis, a three-
phase inverter with 6 kW rated power is simulated using
MATLAB. For the integral feedback control, the noise and
switching harmonics presented in the inductor current may
be amplified, and therefore, a low-pass filter is normally
used in practical applications, as shown in Fig.12.

![Fig. 11. Characteristic plots of grid harmonic rejection.](image)

**Fig. 11.** Characteristic plots of grid harmonic rejection.

![Fig. 12. Block diagram of grid-side current control with resonant inductor voltage feedback.](image)

**Fig. 12.** Block diagram of grid-side current control with resonant inductor voltage feedback.

This paper uses PR+HC controller with resonant-
inductor-voltage-feedback under the system shown in Table

I. According to the designed parameters before, some
simulations can be done. The resonant frequency is 2.6 kHz
which is higher than $f_s/6$. Theoretically, the system can be
stable without active damping. Fig. 13 shows the dynamic
performance of the grid current without active damping
when change the reference peak current from 7 A to 12.9 A.

![Fig. 13. Grid-side waveform without damping when change the reference current from 7A to 12.9A.](image)

**Fig. 13.** Grid-side waveform without damping when change the reference current from 7A to 12.9A.

![Fig. 14. Grid-side currents waveforms with active damping when change the reference current from 7A to 12.9A.](image)

**Fig. 14.** Grid-side currents waveforms with active damping when change the reference current from 7A to 12.9A.

![Fig. 15. Grid current waveform when active damping is enabled at time 0.1s under grid impedance is 4mH.](image)

**Fig. 15.** Grid current waveform when active damping is enabled at time 0.1s under grid impedance is 4mH.
VII. EXPERIMENTAL RESULTS

As shown in Fig. 16, the experimental setup consists of a Danfoss FC302 converter connected to the grid through an isolating transformer and the DC-link supplied by Delta Elektronika power sources. The control algorithm has been implemented in a dSPACE DS1103 real time system. The parameters of the system are shown in the Table I. The leakage inductance of the transformer is 2mH.

Fig. 16. Photograph of set-up for experiments with the three-phase grid-connected converter.

Fig. 17 shows the experimental results of $u_c$ and $i_g$ under the steady state situation with the resonant-inductor-voltage-feedback active damping.

Fig. 17. Experimental results with active damping under the steady state.

Fig. 18 shows the dynamic transition of grid-side currents when the power is increased with resonant-inductor-voltage-feedback active damping.

Fig. 18. Experimental results when the reference of the grid-current is increased with active damping.

Fig. 19 shows the experimental results when the active damping feedback coefficient $K_{udf}$ is increased and makes the system unstable. Fig. 20 shows the grid-side currents and $L_f/C_f$ circuit voltage when active damping is enabled with the grid impedance is 4 mH. The system is unstable without the active damping under the influence of the grid-impedance and then become stable when the resonant-inductor-voltage-feedback is enabled.

Fig. 19. Experimental results when active damping feedback coefficient $K_{udf}$ is increased.

Fig. 20. Experimental results when active damping is enabled with $L_f= 4$ mH.

VIII. CONCLUSION

This paper has presented a resonant-inductor-voltage-feedback active damping method for LLCL-filter based grid-connected inverter considering the delay effect. The ratio of the resonance frequency to the control frequency has important influence on the system stability.

In order to enhance the output harmonic current rejection and reduce steady state error, PR+HC control algorithm was used in the current control. The work also shows the impact of resonant frequency on the choosing of active damping coefficient to make the system stable. The design method is given based on the analysis in the s-domain and z-domain. The analysis and the design example demonstrate a system with the resonant-inductor-voltage-feedback active damping that has a good robustness against grid impedance variation.

REFERENCES


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