Abstract: In Denmark it will be a challenge in near future to balance the electrical grid due to a large increase in the renewable energy production mainly from wind turbines. Smart grid solutions which exploit all storage capacities are essential to meet this challenge. In this work single family houses with heat pumps and floor heating are investigated for storage capability. The aim is to shift energy consumption a few hours in time to mitigate the effect of fluctuating production from wind and other renewable energy sources on the grid. Based on measurements in six inhabited houses for approximately a year prediction models are analysed. The main topic of this work is to investigate how behaviour of inhabitants affect the quality of predictions. Unfortunately the output of the models for single inhabited single family houses seems to give large standard deviations of the predictions, and aggregated models for use in a coordinated control scheme could improve the possibility to use thermal capacity in houses as dynamical storages.

1. INTRODUCTION

Problems with balancing electricity supply and demand in the grid have emerged in Denmark in recent years. The problems are mainly due to power production from wind turbines, they can suddenly increase or decrease production depending on weather conditions. These rapid production changes are not always predictable and can therefore have severe consequences for grid stability. These problems will grow in the near future due to an increased number of wind turbines. In Denmark at present wind power meets 30 % of the electricity demand, however this covers variation from a minimum of 2-3 % to peaks more than 100 % of the instantaneous power demand. The Danish government wants the wind energy percentage to be larger than 50 % in 2020, [1]. As conventional power plants are phased out gradually, alternative sources of ancillary services must be found. One of the approaches to obtaining alternative ancillary services is the smart grid concept, where demand-side devices with flexible power consumption take part in the balancing effort.

One flexible power consumption device is a heat pump. In Denmark a great number of single family floor heating systems using small heat pumps (5 kW-15 kW) has been installed in the recent years. These installations have large energy storage capacities in the concrete floors giving a possibility of moving energy consumption to times where a high production of wind energy takes place with only minor discomfort and thereby they are very flexible power consumers.

The Danish Energy Agency estimates that more than 50,000 small heat pumps are installed for heating single-family houses corresponding to 6-7 % of the total electricity consumption. The small heat pumps are used outside the district heating areas which cover about 54 % of the total heat consumption in Denmark.

The major part of the electricity in Denmark is traded on the Nord Pool Elspot market, [2]. The day-ahead market, Elspot, is the main arena for trading power in the Nordic region. Here, contracts are made between seller and buyer for the delivery of power the following day, the price is set and the trade is agreed. Daily trading is driven by the members planning their production or consumption. A buyer needs to assess how much energy he will need to meet the demand the following day, and how much he is willing to pay for this energy, hour by hour. The seller, for example the owner of a power plant, needs to decide how much he can deliver and at what price, hour by hour. These needs are reflected through orders entered by buyers and sellers into the Elspot trading system. It should be noticed that wind power is the greatest uncertainty in this bidding round. Elspot calculates the hour by hour price. Put simply, the price is set where the curves for selling and buying bids meet. The intra-day imbalances must be treated by buying or selling power at the Regulating power market. Trading at the Regulating power market is more expensive than trading on the Elspot market.

In a normal heat pump control system room temperature is controlled automatically to a set point. An ON/OFF control ensures that this room temperature is within a certain temperature band (comfort band). This leaves some freedom to move energy consumption still respecting the comfort band. This has previously been suggested by numerous authors, [4], [5], [6], [7]. A central controller is suggested using an internet connection to each house, furthermore it is chosen to use direct control of the heat pump power. The central controller can be placed and
operated by a balance responsible party (BRP). The BRP is a private retailer company that buys and sells electric power from producers (wind mill owners, decentralized heat plants etc.) and consumers (factories, private households etc.) and trades this on the electricity markets, e.g., the Nord Pool Elspot market. The BRP predicts the wind energy production and this information is used to optimize control of the heat pumps. A 36 hour’s time interval is interesting because this is the time horizon for the Nord Pool day-ahead Elspot marked. It can be shown that Nord Pool Elspot prices are closely connected to wind energy production; this implies that buying most power to the heat pumps when the price is low will move the consumption to times where the wind energy production is high. If the BRP buys power at Nord Pool market for the heat pumps according to this price estimate still respecting user comfort, this can shift consumption according to the wind energy production, resulting in a lower price for heating a house and a lower price for balancing the electricity grid.

The Elspot prices are valid for one hour, so the energy is bought hour by hour. To make an hour by hour energy plan for each heat pump it is a challenge to predict the correlation between the power from the heat pump and the indoor temperature.

2. DEMANDS TO A DYNAMIC MODEL.

To predict the energy demands models of houses are essential. There are two demands for the models, namely to help purchasing energy on the Nord Pool Spot market and to control the individual heat pumps. The latter must ensure the house temperature is within given comfort limits and at the same time ensure the used energy is following the pre-described plan as close as possible. The Nord Pool day-ahead market trades energy in 1 hour slots which calls for a model with a sampling time of 1 hour. The BRP purchase energy hour by hour where each hour purchase must be greater than 10 MWh; this means that purchase for a number of houses must be aggregated and sometimes also aggregated with other energy consuming units.

The relation between power input and house indoor temperature has a time constant significantly larger than the 1 hour interval meaning that a dynamic model is necessary. The regulating power market use 15 minutes slots and thereby demands a sampling time less than a quarter of an hour.

The heat pumps can be turned ON or OFF at any time, but to keep a satisfying performance most manufacturers have implemented a minimum ON-time e.g., 20 minutes. This and a sampling time of 5 minutes for sampling the transducers in the demonstration houses leads to a sampling time of 5 min. for controlling the indoor temperature. The demands for the models are similar therefore it is chosen to use the same continuous model and discretize this using different sampling times.

To purchase energy on the spot market the model must be able to predict the energy consumption necessary to keep the indoor temperature within the comfort limits. The indoor temperature is affected by the weather conditions and the behaviour of the inhabitants. Ordinary control of the temperature can be based on a SISO model, power from heat pump is input and indoor temperature is output, weather conditions and inhabitants may be seen as disturbances. Though in our case we need to move energy consumption and therefore prediction of the impact of moving energy consumption in time is necessary.

The model must fulfil the following demands

- Describe the indoor temperature as function of power from heat pump and disturbances from weather and inhabitants.
- Describe the power needed to keep the temperature within temperature limits.
- Describe the effect of changing power consumption.

The demands will be tested using 300 days data from measurements of six inhabited houses. Here the model will be simulated using measured input data and the output (indoor temperature) will be compared to the measured indoor temperature. The prediction of the needed power can be tested using simulations where the input is the indoor temperature and the output is the needed heat pump power. Similarly the impact of changing power consumption can be tested.

3. DESCRIPTION OF THE TEST SET-UP

In the figure 1 the overall concept in the project is shown. The houses are equipped with add-on sensors measuring room air temperature, water temperatures and flow for the heat pump. A special control box was constructed for the project 'Styr din varmepumpe' [3]. The READY! project has access to a number of boxes in individual houses. The boxes connect the internet to the sensors and to the electronics of the heat pump, making it possible to start and stop the pumps. The BRP has access to various information useful in the control algorithm; this information includes local weather forecast and the actual sensor readings from each house. From the weather forecast and other information (consumption estimates, state of power plants etc.) the BRP is able to make an estimate of hourly prices for electricity. The BRP is in regular contact with the Transmission System Operator (TSO), the latter is the responsible for the grid balance. Finally the BRP can make bids on the Nord Pool Elspot market.

To achieve an appropriate amount of energy to trade on on the Nord Pool spot market a large number of houses must be included in the portfolio. The additional cost of control equipment in each house must be kept low. Therefore the number of sensors must be limited. In the READY! project each house has one indoor temperature sensor, one outdoor temperature sensor, and sensors for measuring the heat power and electrical power to the heat pump. In inhabited houses the indoor temperature is usually varying for different locations. The position of the transducer has been determined by a field engineer and though he has tried to find a spot with representative temperature there is danger that draught and solar radiation makes it inappropriate for the control system. Even with several temperature sensors the temperature profile can be difficult to determine.
4. A SINGLE FAMILY HOUSE PREDICTION MODEL.

The intended use of the model is to predict the power consumption for different scenarios, e.g., necessary power to keep the indoor temperature within the comfort limits for given weather conditions. The model may be based on either first principle or be data driven. A pure first principle model is not appropriate due to the complex structure of the houses, limited number of sensors and the wish for automatic adjustment of the model. Using the pure data driven model the parameters can not be directly related to physics. A grey box model is suitable as it combines a physical model with a parameter identification. Furthermore it is assumed that the individual houses may use the same model structure and the parameters are to be identified.

The model has to be used for estimation of the total energy consumption as well as the distribution in time. The total energy consumption during 24 hours may be calculated using the measured energy consumption during the previous day combined with weather forecast. The distributed energy consumption is necessary to predict effect of hour by hour spot market purchase.

All the houses are normal Danish family houses and all are inhabited. The behaviour of the people will vary from day to day and from family to family. Many houses have other energy sources e.g. wood stoves or solar panels. Domestic appliances and people also submit energy. The previous mentioned may cause large disturbances. Due to the limited number of sensors it is only realistic to use models with a limited number of parameters. Each of the parameters will cover a number of physical properties and therefore it is foreseen that the parameters may vary.

It is assumed that uninhabited houses can be modelled and using parameter identification a fair description of the energy factors [8] can be obtained. The disturbances and parameter variations caused by people may be included in the house parameters. A topic of the work is to investigate if the mentioned model parameters can be recursively identified with small variations and that they have a meaningful physical interpretation. The resulting prediction of the power consumption based on the identified model must be investigated by comparing predicted and measured power consumption.

Based on the available measurements the model can be formulated as:

\[ T_{in}(t) = \text{model}(P_{hp}(t), T_{out}(t), P_{rad}(t)), \]

where \( T_{in} \) is the indoor temperature, \( P_{hp} \) is the power from the heat pump, \( T_{out} \) is the outdoor temperature and \( P_{rad} \) is the solar radiation. The identification of the model parameters is based on measurements in inhabited houses where the heat pump is controlled by the normal control system. Assuming that the normal controller is a proportional type controller identification must be performed in closed loop. We consider the case with an ARX model of the temperature, \( T_{in} \) at the discrete time instance \( t \). \( e(t) \) is an unmeasured input, assumed to be white noise with mean value zero and variance \( \sigma^2 = E[e^2(t)] \). Model and one step predictor are given as:

\[ T_{in}(t) = -aT_{in}(t-1) + b_1 P_{hp}(t-1) + b_2 T_{out}(t-1) + e(t), \]

\[ \hat{T}_{in}(\theta, t) = -\hat{a}T_{in}(t-1) + \hat{b}_1 P_{hp}(t-1) + \hat{b}_2 T_{out}(t-1) = \theta^T \phi(t), \]

which gives a prediction, \( \hat{T}_{in}(t) \) of the temperature at the discrete time \( t \) in dependence of measurements available before time \( t \), \( \phi(t) = [-T_{in}(t-1), P_{hp}(t-1), T_{out}(t-1)] \) and estimated parameters \( \theta = [a, b_1, b_2]^T \). To determine the parameters we formulate a quadratic value function

\[ V(\theta) = \sum_{i=1}^{N} (T_{in}(t) - \hat{T}_{in}(\theta, t))^2. \]

The parameters which minimize this value function can be shown to converge to the correct parameters \( \theta = [a, b_1, b_2]^T \) if the model, e.q. (2) is correct and the number of measurements is large enough, \( N \rightarrow \infty \). The minimizing parameters are found by taking the gradient of the value function to zero,

\[ \frac{\partial V}{\partial \theta} = -2\sum_{i=1}^{N} \phi(i)(T_{in}(t) - \hat{T}^T \phi(t)) = 0. \]

By using \( \hat{T}^T \phi(t) = \phi^T(t) \hat{\theta} \) this leads to:

\[ \sum_{i=1}^{N} \phi(i) \phi^T(t) \theta = \sum_{i=1}^{N} \phi(i) T_{in}(t), \]

or

\[ \hat{\theta} = (\sum_{i=1}^{N} \phi^T(t) \phi(t))^{-1} (\sum_{i=1}^{N} \phi(i) T_{in}(t)) = R_N^{-1} f_N. \]

where

\[ R_N = \sum_{i=1}^{N} \phi(i) \phi^T(t) \]

\[ f_N = \sum_{i=1}^{N} \phi(i) T_{in}(t). \]

The value function can be shown to converge to \( N^2 \sigma^2 \). However if the system is controlled using a proportional controller with constant reference, \( T_r \), the input to the model will be related to the output. For simplicity let us take \( T_r \) to zero (or let temperatures be measured relative to \( T_r \)):

\[ P_{hp}(t-1) = k(T_r - T_{in}(t-1)) = -kT_{in}(t-1). \]

Fig. 1. Test set-up with BRP computer and individual houses.
In this case we will have
\[
R_N = \sum_{t=1}^{N} \begin{bmatrix}
T_2^2 & kT_2^2 & -T_{in}T_{out} \\
-kT_{in}T_{out} & kT_{in}T_{out} & -kT_{in}^2T_{out}
\end{bmatrix}.
\] (10)

Obviously in this case \( R_N \) is rank deficient and not invertible since the second row (or second column) is proportional to the first row (column). Therefore \( \theta \) can not be determined by eq. (7) and the feedback results in identifiability problems.

In practice the heat pumps are controlled ON/OFF, but control will obviously still reduce identifiability and result in \( R_N \) being close to rank deficient with large uncertainty of the parameters as a result. It was chosen to use the data for parameter identification, but for practical use this will imply that the heat pumps need to be excited occasionally for instance in night time where comfort is less critical to obtain better quality of the model.

The measurements are performed in houses dispersed in Denmark and probably missing data and outliers may occur. Some data reconstruction tools are used to make the data acceptable for identification and control. Another item concerning model identification is slow variations in the operating points caused by e.g., the inhabitants. In the work we will test first and second order models.

The following table summarize the test scenario.

| Data validation | Missing data, outliers |
| Trends in data | DC values, slow variations |
| Model type | linear, non-linear, 1st principle, black-, grey-box |
| Model order | 1’ and 2’ order |
| Identification horizon | 3, 7 days or fixed model |

5. IDENTIFICATION OF STATIC MODELS

Using the test set-up a large number of measured data has been collected. In this section we will focus on data for one house from April to January in total 300 days. The measurements are representative for many houses. The heat transfer coefficient from the inside to the ambient will be investigated using static models. The data are sampled with 5 minutes. To avoid the daily variations mean values of all data are used.

In figure 2 the one day mean of heat pump power consumption for one house is illustrated (blue). For the same days the difference between mean values of measured indoor temperature and outdoor temperature is shown (green). Assuming that the only energy supplied to the house is the heat pump power and that the energy transferred from the house is proportional to the said temperature difference the equation
\[
P_{hp} = hA(T_{in} - T_{out}) \Rightarrow hA = \frac{(T_{in} - T_{out})}{P_{hp}}
\] (11)
is given, where \( hA \) is the heat transfer coefficient.

The variations in the heat pump power is surprisingly large from day to day and not always correlated with

![Fig. 2. Measured heat pump power and measured temperature difference for one house.](image)

6. IDENTIFICATION OF DYNAMIC MODELS

To move energy consumption within 24 hours and fulfill comfort demands in a house a dynamic model is needed.

![Fig. 3. Heat transfer based on equation 11](image)
The purpose of the model is to predict the day ahead temperature variations and necessary power. In figure 2 a close connection between outdoor temperature and heat pump power. A new model is identified every day based on data from the three previous days. Measured data of heat pump power, outdoor temperature and solar radiation are used as input to the model and the output with the best fit identified model using data from the same day (red) is plotted on figure 5 and figure 6. In the same figures the measured indoor temperatures are plotted. In figure 5 a month with fair identification in one house is illustrated. Figure 6 shows poor results from the same house another month.

The heat capacity may be assumed constant for an uninhabited house, but in inhabited houses e.g., an open door can change the size of the room and thereby the heat capacity. The power from the heat pump $P_{hp}$ is measured as the flow of the floor water multiplied with the temperature difference between the inlet and outlet and is assumed to be equally distributed in the house. In the equation it is assumed that all power goes from the room, where $T_{in}$ is measured, to the outdoor; here some parts may go to un-heated neighbour rooms. Adjustments of the individual room temperatures may also influence on the $C$ and $hA$ values. A second order model could be

$$C_{fl} \frac{dT_{fl}}{dt} = P_{hp} - hA(T_{fl} - T_{in})$$

$$C_{ro} \frac{dT_{ro}}{dt} = hA(T_{fl} - T_{in}) - hA(T_{in} - T_{out}) + r_a P_{rad} + P_{added},$$

where subscripts $fl$ and $ro$ refers to floor and room. The second order system may be reformulated to a transfer function with four combined parameters.

The system identification of the first and second order models has been performed by the matlab function n4sid. This algorithm is preferred rather than PEM methods because it does not need initial parameter values and gives a suggestion to system order. Identification to determine a proper system order has been performed using 9 month data from 6 different houses. The predominant model-order of the identification is one, and similar result are obtained comparing first and second order models.

The literature [8] thermal models of houses are presented. These models describe uninhabited houses and assume several sensors. In this case all houses have different sizes, ages and partitions; they are inhabited with ordinary families causing disturbances from electric appliances, wood stoves, open/close doors and windows. A first principles model will not be able to describe the behaviour of the people. In a black box model the knowledge of the structure of the model of an empty house is not incorporated. A grey box approach where the parameters can be interpreted is chosen. It is assumed that the energy may be stored in one or two control volumes, where one common volume is the concrete construction and the second could be a volume with lower heat capacity e.g., the air plus a part of the inner walls. A first order model could be

$$C \frac{dT_{in}}{dt} = P_{hp} - hA(T_{in} - T_{out}) + r_a P_{rad} + P_{added}.$$  

where

$$C_{fl} \frac{dT_{fl}}{dt} = P_{hp} - hA(T_{fl} - T_{in})$$

$$C_{ro} \frac{dT_{ro}}{dt} = hA(T_{fl} - T_{in}) - hA(T_{in} - T_{out}) + r_a P_{rad} + P_{added},$$

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The models identified from the most well-suited data are found. To give a representation of these models 1 kW step responses are calculated and the indoor temperatures are plotted in figure 7.

As seen large variation in the time constants appears, the gain differs very much and the even have different signs. These 'best models' are unsuited for prediction of temperatures and power consumption.

Fig. 7. Heat transfer based on equation 12

7. AGGREGATED MODELS

In steady state equation 13 gives a possibility to calculate the added heat if $hA$ and $r_a$ are constant and known. Using mean values for 24 hours the added heat is shown in figure 8.

![Stepresponse (1000w) for the validated models, House 1, from heat pump](image)

Fig. 8. Variations and mean value of added heat.

We assume that the daily energy consumption in one house consists of the measurable heat pump power and solar radiation power and the stochastic added heat. The daily energy consumption $P_{day}$ may be modelled as a normally distributed stochastic process with mean heat pump power, $\mu_{hp}$ plus solar radiation power $\mu_{rad}$ plus mean value of the added heat $\mu_{ah}$ and a variance $\sigma_{day}^2$ as

$$P_{day} = \mathcal{N}(\mu_{hp} + \mu_{rad} + \mu_{ah}, \sigma_{day}^2). \quad (15)$$

If $N$ houses are added the distribution will be

$$P_{day,N} = \mathcal{N}\left(\sum_{i=1}^{N} (\mu_{hp,i} + \mu_{rad,i} + \mu_{ah,i}), \sum_{i=1}^{N} (\sigma_{i}^2)\right), \quad (16)$$

and the standard deviation of $P_{day,N}$ will be

$$\sigma_{day,N} = \sqrt{\sum_{i=1}^{N} (\sigma_{i}^2)}. \quad (17)$$

This mean that if e.g., for equal $\sigma$ $\sigma_{day,N} = \sqrt{N}\sigma = \sqrt{N}\sigma$. Adding a large number (N) of houses the standard deviation on the sum will grow in proportion to $\sqrt{N}$ and the total energy consumption in proportion to $N$. This indicates that an aggregated model for $N$ houses may be suitable.

8. CONCLUSION

In this work it has been investigated if it is possible to construct a model that gives reliable predictions for single family inhabited houses. Individual models appear to be difficult to find due to large disturbances and parameter variations. Large parameter variations occur even for a steady state model only having one heat transfer coefficient as parameter. Identified dynamic models show large variations in gain and time constants. Unfortunately the inhabitants cause large parameter variations and give large disturbances. The large variation of the daily energy consumption indicate that aggregate models would be more feasible for obtaining a total power consumption for several houses with acceptable standard deviation.

REFERENCES