Stability analysis and active damping for LLCL-filter based grid-connected inverters

Min, Huang; Blaabjerg, Frede; Loh, Poh Chiang; Wu, Weimin

Published in:

DOI (link to publication from Publisher):
10.1109/IPEC.2014.6869958

Publication date:
2014

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
Stability Analysis and Active Damping for LLCL-filter Based Grid-Connected Inverters

Min Huang, Frede Blaabjerg, Poh Chiang Loh
Department of Energy Technology
Aalborg University
Aalborg, Denmark
hmi@et.aau.dk, fbl@et.aau.dk, pcl@et.aau.dk

Weimin Wu
Electrical Engineering
Shanghai Maritime University
Shanghai, China
wmwu@cle.shmtu.edu.cn

Abstract—A higher order passive power filter (LLCL-filter) for the grid-tied inverter is becoming attractive for the industrial applications due to the possibility to reduce the cost of the copper and the magnetic material. To avoid the well-known stability problems of the LLCL-filter it is requested to use either passive or active damping methods. This paper analyzes the stability when damping is required and when damping is not necessary considering sampling and transport delay. Basic LLCL resonance damping properties of different feedback states are also studied. Then an active damping method which is using the capacitor current feedback for LLCL-filter is introduced. Based on this method, a design procedure for the control method is given. Last, both simulation and experimental results are provided to validate the theoretical analysis of this paper.

Keywords—LLCL-filter, grid converter, active damping, current control, resonant frequency, stability.

I. INTRODUCTION

Recently, due to the energy crisis, distributed generation (DG) systems using clean renewable energy such as solar energy, wind energy, etc., have become an important issue in the technical research. Typically, a simple series inductor L is inserted between a voltage source inverter (VSI) and the grid to attenuate the high-frequency PWM harmonics to a desirable limit. But the high value of L-filter needs to be adopted to reduce the current harmonics around the switching frequency, which would lead to a poor dynamic response of the system and a high power loss. Hence, a low-pass passive power filter, LCL-filter, can achieve a high harmonic attenuation performance with less total inductance \((L_1 + L_2)\) [1], [2].

In order to further reduce the total inductance, the LLCL-filter was proposed [3]. Compared with the LCL-filter, the total inductance and volume of the LLCL-filter can be reduced a factor of 25% –40%. So, the LLCL-filter for the grid-tied inverter is becoming attractive for industrial applications [4]. The application of LLCL-filter for a three-phase three-wire Shunt Active Power Filter (SAPF) [5] and a Large-Scale Wave Power Plant [6] were analyzed.

As a high order filter, the LLCL-filter has also a resonant problem. To suppress the possible resonances of an LCL-filter or LLCL-filter, active damping [7-11] or passive damping [12-15] measures may be adopted. Passive damping is realized by adding additional components in the system but it causes a decrease of the overall system efficiency. For a stiff grid application, a passive damping strategy is more attractive. Due to high efficiency and flexibility, the active damping method might be preferred, although at the risk of higher cost of sensors and more control complexity. Normally, the digital sampling and transport delays caused by the controller and modulation, as well as discretization effects are taken into account. The delay will influence the stability of the system and when resonant frequency varies, active damping is usually required in an LCL-filter [8].

Proportional Resonant (PR) compensator and Proportional Integral (PI) compensator are widely used to control the injected grid current in single-phase grid-connected inverters [16]-[18]. The converter control commonly consists of an outer dc-link voltage PI control loop and an inner current control loop. In this paper, all possible feedback states of currents and voltages of LLCL-filter capacitors and inductors with different feedback transfer functions are considered and compared in the continuous Laplace domain. The results show how the various feedback signals need to be fed back in order to achieve resonance damping. Based on the available choices of feedback variables, active damping of the capacitor current control strategy is used for LLCL-filter.

First, the system is described and its stability is analyzed in Section II. In Section III, a more general analysis of different active resonance damping solutions is carried out. The basic LLCL resonance damping properties of different feedback states are studied. In Section IV, the design of current controller and capacitor current feedback damping are described. Last, a 6-kW three-phase grid-connected inverter is built to verify the proposed design method.

II. MODELING AND STABILITY OF LLCL-FILTER-BASED GRID-CONNECTED INVERTER

A. Modeling of LLCL-filter-based Grid-connected Inverter

A three-phase voltage source converter connected to the grid via an LLCL-filter is studied as shown in Fig. 1. The inverter output voltage and current are represented as \(u_g\) (phase voltage) and \(i_{g}\), and the grid voltage and current are represented as \(u_{g}\) and \(i_{g}\). \(L_g\) is the grid impedance.

Grid side current control will be discussed in this paper [1].
Neglecting the influence of the grid impedance and Equivalent Series Resistances (ESRs) of the inductors and capacitors, the transfer function \( \frac{i_g(s)}{u_i(s)} \) of the \( \text{LLCL}-\text{filter} \) can be derived in (1).

\[
G_{h\rightarrow i}(s) = \frac{L_j C_f s^2 + 1}{(L_j L_2 C_f + (L_j + L_2)(L_j + L_2 - L_j + L_2)) s^3 + (L_j + L_2) s} \quad (1)
\]

As shown in (2), \( \omega_r \) is the resonant frequency (in radians per second), \( f_r \) is the resonant frequency. If the inductance of \( L_j \) is set to zero, then the transfer functions of the \( \text{LCL}-\text{filter} \) can also be calculated. Fig. 2 shows bode plots of transfer functions \( i_g(s)/u_i(s) \) of \( \text{LCL}-\text{filter} \) and \( \text{LLCL}-\text{filter} \) when they meet the same harmonic requirement of grid-injected current according to IEEE 519-1992[19].

Fig. 4 shows the bode plot of the forward path transfer function for the single loop.

Table I shows the parameters of the selected test system. Table II shows the parameters of the \( \text{LLCL}-\text{filter} \) in different resonant frequencies cases according to the design method given in [19].

---

**B. Stability of LLCL-Filter-Based Grid-Connected Inverter with Different Resonant Frequencies**

The inverter can be modeled as a linear gain \( k_{PWM} \). The control block diagram of a single loop controller without any damping methods of the three-phase grid-connected inverter with \( \text{LLCL}-\text{filter} \) is shown in Fig. 3.

\[
G_i(s) = k_p + \frac{k_i}{s} \quad (3)
\]

\[
G(s) = G_{\text{delay}}(s) k_{PWM} G_i(s) G_{h\rightarrow i}(s) \quad (4)
\]
It can be seen from Fig. 4, that the LLCL-filter resonance has no influence on system stability when the resonant frequency is high, since the phase is already well below -180° due to the sampling and transport delay.

This analysis identifies that there is also a resonant frequency critical for LLCL-filter, and above it, active damping can be avoided by adjusting the controller gain. When a Zero-Order-Hold (ZOH) is in series of the open loop, discretization of the system introduces delay too, as shown in (5).

\[
G_0(s) = H_1(s)G_{n=1}(s) = \frac{(1 - e^{-T_d})G_{n=1}(s)}{s} \tag{5}
\]

When \( \angle G(j\omega) = -\pi \), the function is shown in (6). \( \omega_c = \pi/3T_d \) is the root. If a phase angle is already below -180° at this resonant frequency, the system can be stable. It can also deduce that \( f_c = 1.68 \text{kHz} \) is the critical frequency.

Hence, a single loop is sufficient to be stable when the resonant frequency is above the critical frequency and active damping is necessary when the resonant frequency is below the critical frequency. For the example in Table I, the critical frequency \( f_c \) is 1.68 kHz.

\[
\angle G(j\omega) = \angle \left\{ \frac{e^{-\omega_c T_d}}{j\omega T_d} \right\} = -\pi
\]

\[
\frac{1 - e^{-2\pi f_c}}{\sqrt{(L_1 + L_2)\omega_c^2 - (L_1L_2 + L_1L_3)\omega_c^4}} = -\pi
\]

Fig. 5 shows the closed loop root loci of the three cases in Table II for the single loop grid current feedback.

Fig. 5 (a) depicts the LLCL resonant frequency is above the critical frequency. The poles initially track inside the unit circle with the proper proportional gain \( K_p \). Fig. 5(b) shows the LLCL resonant frequency at the critical frequency. The system is on the edge to be unstable. When the resonant frequency is less than the critical frequency, the system will always be unstable regardless of what the proportional gain is without damping, as shown in Fig. 5(c). In this case, a damping method is necessary to be used.

III. STUDY OF ACTIVE DAMPING WITH DIFFERENT FEEDBACK STATES

A. Notch Filter Concept

Active damping methods can be classified into two main classes: multi-loop and filter-based active damping [8]. For passive damping, the resonance frequency of an LCL-filter can be damped by connecting a resistor to the filter. The virtual resistance without reducing the efficiency was introduced, which is the equivalent as the passive damping resistors of the filter that resonance damping techniques.

The control structure of inner current loop with a notch concept [22] is shown in Fig. 6.

![Fig. 6. Active damping based on a notch filter concept.](image)

\[
B(s) = \frac{k_{\text{FIRM}}}{1 + k_{\text{FIRM}}K(s)N(s)} = \frac{s^2 + 2\omega_c s + \omega_c^2}{s^2 + 2\zeta\omega_c s + \omega_c^2} \tag{7}
\]

In order to eliminate the resonant peak at the frequency \( \omega_c \), the notch \( B(s) \) should have a negative peak in \( \omega_c \), so it can be expressed as (7).

There are different variables can be chosen as control object for LLCL- of detection variables, it has different function \( K(s) \) with different feedback variables. Table III shows the expressions filter. The structure of \( N(s) \) depends on the selection using the filter capacitor voltage \( u_c \), filter capacitor current \( i_c \), filter resonant inductor...
voltage in grid side $u_{ig}$, inductor voltage in resonant circuit $u_{j}$. It can be seen from the Table III, the feedback function of grid side inductor voltage is a little complex. The choice of the sensor location depends strongly on the application situations. For different ratios between the resonance frequency and control frequency, the selected approaches behave differently [10].

### B. Filter Capacitor Current Feedback

For the LCL-filter, capacitor current feedback and capacitor voltage feedback are often used [10]. When the capacitor current feedback is sensed, $K(s)$ can be easily configured as a proportion link, which is isolated with the system parameters. Fig. 7 shows the capacitor current feedback control scheme according to Fig. 6.

\[ N(s) = G_{u \rightarrow i}(s) = \frac{\omega_0^2 L C_i s}{(s^2 + \omega_0^2)(L_1 + L_2)} \]  

Then the open loop transfer function of the capacitor current feedback control from $i_g$ to $i_i$ is shown in (9)

\[ G(s)_{\text{open}} = \frac{k_{PWM} \omega_0^2 (L_j C_j s^2 + 1) G_{i}(s) G_{d_{l_{pf}}}(s)}{(s^2 + \omega_0^2)(L_1 + L_2) + K_i G_{d_{l_{pf}}}(s) k_{PWM} L_2 \omega_0^2 s} \]  

As shown in Fig. 4, the capacitor current feedback can dampen the resonant peak and make the system stable.

### C. Filter Capacitor Voltage Feedback

When the capacitor voltage is sensed, a derivative filter capacitor voltage feedback is required for the resonance damping. Some papers have illustrated this method for an LCL-filter. As shown in Table III, a differential feedback is necessary but it may cause noise problems in the control because it will amplify high-frequency signals [23].

### D. Filter Resonant Inductor Voltage Feedback

When the resonant inductor voltage is sensed, the integral feedback can be expected to keep the system stable. Normally, the filter capacitor current is sensed for the LCL-filter but the voltage sensor is cheaper than current sensor. This is a new active damping method for LLCL-filter which can reduce the cost of sensor. The resonant inductor voltage feedback can also be transferred in Fig. 7. The feedback coefficient is integral and the structure of $N(s)$ is the transfer function from $u_i$ to $u_ig$.

The next section will analyze the design of capacitor current feedback only.

### TABLE III

**TRANSFER FUNCTIONS OF DIFFERENT VARIABLES FEEDBACK**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$u_{C_i}$</th>
<th>$i_i(s)$</th>
<th>$u_{L_j}$</th>
<th>$u_{L_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(s)$</td>
<td>$\frac{\omega_0^2 L_i C_i s}{(s^2 + \omega_0^2)(L_1 + L_2)}$</td>
<td>$\frac{\omega_0^2 L_i C_i s^2}{(s^2 + \omega_0^2)(L_1 + L_2)}$</td>
<td>$\frac{\omega_0^2 L_i L_j C_i s^2}{(s^2 + \omega_0^2)(L_1 + L_2)}$</td>
<td>$\frac{\omega_0^2 L_i (L_i C_i s^2 + 1)}{(s^2 + \omega_0^2)(L_1 + L_2)}$</td>
</tr>
<tr>
<td>$K(s)$</td>
<td>$sK_{u_i}$</td>
<td>$K_i$</td>
<td>$K_{u_i}$</td>
<td>$K_{u_i}$</td>
</tr>
</tbody>
</table>

The system open-loop gain achieves unity at $\omega_c$. Then the maximum gain can be calculated as

\[ \left| G(j\omega) \right| = \frac{k_{PWM} \sqrt{(\omega_c \tau)^2 + 1}}{\omega_c \tau} \left| \frac{e^{-\omega_c \tau}}{j L_1 + L_2} \right| \frac{k_{PWM}}{k_{PWM} / \tau} \]  

\[ k_p \approx \frac{\omega_c (L_1 + L_2)}{k_{PWM}} \]  

**B. Capacitor current Feedback Coefficient Gain**

The denominator of closed loop transfer function of LCL-filter is shown in (12) based on (9). Base on (12) the minimum value of $K_i$ can be found using the limiting ratio of proportional gain $k_p$ to dampen using Routh’s Stability Criterion, as shown in (13).

\[ D(s) = \left[ L_1 L_j C_j + (L_1 + L_2) L_j C_j \right] s^4 + K_i k_{PWM} L_j L_2 C_j s^3 + (L_1 + L_2) s^2 + k_p k_{PWM} s + k_p k_{PWM} / \tau \]  

2613
K_i \geq \frac{k_p}{(L_1 + L_2)L_f} \left[ L_iL_f + (L_1 + L_2)L_f \right] \quad (13)

GM_1 and GM_2 are defined by the magnitude of (9) at \( f_r \) and \( f_r/6 \) [27]. Then the limitation of \( K_i \) can be obtained from (14) and (15).

\[
K_i = 10^{\frac{\text{GM}_1}{20}} \frac{2\pi f_i L_i}{k_{PWM}} \quad (14)
\]

\[
K_i = 10^{\frac{\text{GM}_2}{20}} \left( \frac{6f_c}{f_d} \right) \frac{2\pi f_i L_i}{k_{PWM}} + \frac{2\pi f_i L_i (f_d/6)^2 - (f_i)^2}{k_{PWM} f_d} \quad (15)
\]

\( K_i \) should satisfy this region. Hence, the basic design procedures can be addressed as:

1. Determine the specifications of the loop gain.
   Desired phase margin \( \phi_m \) is 40°, \( \text{GM}_2 \) is 5.1 dB, and \( \text{GM}_1 \) is -5.8 dB.
2. Obtain the value of \( k_p \) and \( f_c \) to satisfy all the requirements according to (10), (11) and (13), \( f_c = 1.1 \) kHz is chosen to obtain fast dynamic response. Then \( k_p \) is calculated according to (11).
3. Taking \( f_c = 1.1 \) kHz in (13), (14) and (15), calculate the satisfactory feedback gain in this example.

Fig. 8 shows the loci branches of the system with the capacitor current coefficient increasing. There is a stable range of \( K_i \) as shown in Fig. 8. The calculated gain should not pass the limitation.

V. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

In order to illustrate the stability and verify the active damping method of \( LLCL \)-filter based grid-connected inverter, a three-phase inverter with 6 kW rated power is simulated using MATLAB. This paper uses a PI controller and SVPWM modulation. The detailed system parameters are listed in Table I and Table II.

First, in order to investigate stability of the system without damping in different resonant frequencies, the \( LLCL \)-filter is analyzed into three cases with different parameters, one with a high resonance frequency, one with a critical resonance frequency and the other with a low resonance frequency. According to (11), the controller gain is \( k_p = 0.065 \) and \( \tau = 1.59 \) ms.

(1) Case I: high frequency, \( f_r = 3.69 \) kHz.

In the case I, the resonant frequency is high (3.69 kHz) and crossover frequency is set to 1.1 kHz in order to get a fast response and to meet phase margin limitation. It can be seen from Fig. 9 the system is stable.

(2) Case II: critical frequency \( f_\text{res} = 1.68 \) kHz.

As it mentioned before, there is a critical frequency for \( LLCL \)-filter. It is calculated as \( f_d/6 \) based on the function (6). It can be seen from Fig. 10, the system is almost unstable at the critical frequency. When the grid impedance is increased, the resonant frequency will be deduced more.

As shown in Fig. 11, when the grid-side inductance is set to 2.4 mH, the resonant frequency is reduced to 1.6 kHz which is under the critical frequency and the system is unstable.

As shown in Fig. 12, when the grid-side inductance is set to 1.2 mH, the resonant frequency is increased to 1.95 kHz and the system is changed from critical state to stable.
Fig.12. Grid-side current waveform of Case II when increase $L_2$ to 2.4mH ($f_{oc}=1.95$ kHz).

(3) Case III: Low frequency $f_{oc}=1.523$ kHz
In case III, the resonant frequency is low (1.523 kHz). It can be seen from Fig. 13 that system is completely unstable without damping.

Fig.13. Grid-side current waveform of Case III ($f_{oc}=1.523$ kHz).

So, when designing the parameters it is better to make the resonant frequency higher in order to get a better stability and robustness. When the resonant frequency is lower or nearby the critical frequency, the active damping method is necessary to be used. In this paper, take Case III as an example, the active damping with a capacitor current feedback is used.

Applying the parameters described in Table I and Table II, the crossover frequency is set to 1.1 kHz to get a high response and meet the phase margin limitation.

In case III, the resonant frequency is low (1.53 kHz). In order to get enough phase margin crossover frequency should be set below the resonant frequency. According to (12), the PI controller gains $K_p = 0.043$ and $r = 3.18$ ms.

Fig. 14 and Fig. 15 show the dynamic performance of Case I and Case III respectively when the reference current changes from 7 A to 12.9 A at time 0.1s to test the dynamic performance. It can be seen from Fig. 14, there is no oscillation during the transient in the high resonant frequency without damping methods. Fig. 8 shows the same dynamic performance in the low resonant frequency when the capacitor current feedback active damping method is used. It can also be seen that the current ripple in Fig. 15 is smaller than the current ripple in Fig. 14, because the inductance and capacitance in the high resonant frequency case is larger than that in the low resonant frequency case.

Fig. 16 and Fig. 15 show the dynamic performance of Case I and Case III respectively when the reference current changes from 7 A to 12.9 A at time 0.1s to test the dynamic performance. It can be seen from Fig. 14, there is no oscillation during the transient in the high resonant frequency without damping methods. Fig. 8 shows the same dynamic performance in the low resonant frequency when the capacitor current feedback active damping method is used. It can also be seen that the current ripple in Fig. 15 is smaller than the current ripple in Fig. 14, because the inductance and capacitance in the high resonant frequency case is larger than that in the low resonant frequency case.

Fig. 16. Grid-side currents of Case III when active damping is enabled at time 0.1s.

Fig. 17. Grid-side currents of Case III when $K_{oc}$ is increased at time 0.1s.

B. Experimental Results
The control algorithm is implemented on a dsPACE DS 1006 board. Due to the limitation of the setup, the power in experiment is lower than the simulation. Fig. 18
shows the dynamic transition of grid-side currents in high resonant frequency case when the power is increased without active damping. Fig. 19 shows the grid-side currents when active damping is enabled in the low resonant frequency case.

![Grid-side Current(A)](image)

Fig. 18. Experimental grid-side currents in high resonant frequency case without active damping.

![Grid-side Current(A)](image)

Fig. 19. Experimental grid-side currents in low resonant frequency case when active damping is enabled.

VI. CONCLUSION

Compared with the LCL-filter, the resonant frequency of LLCL-filter is higher and it is easier to be stable. The work presented in this paper shows when damping method is not required in an LLCL-filter as the resonant frequency varies considering different sampling and transport delays. The critical frequency is determined by the delay time and sample frequency.

In the low resonant frequency cases, or critical cases, the resonant frequency is easy to be changed due to the parameter variation and grid impedance variation. Then, damping methods are necessary to be used. Based on the analysis of possible feedback variables, capacitor current feedback active damping control strategy is chosen for the LLCL-filter. Simulation and experiment results prove the influence of the resonant frequency and design of capacitor current feedback. The control of LCL-filter and LLCL-filter are similar and the additional inductor of LLCL-filter brings no extra control difficulties.

REFERENCES


