Proportional Derivative Based Stabilizing Control of Paralleled Grid Converters with Cables in Renewable Power Plants

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Abstract—Resonant interactions of grid-connected converters with each other and with cable capacitance are challenging the stability and power quality of renewable energy sources based power plants. This paper addresses the instability of current control of converters with the multiple resonance frequencies consisting in LCL filters and cables. Both grid and converter current controls are analyzed. The frequency region, within which the system may be destabilized, is identified by means of the impedance-based stability analysis and frequency-domain passivity theory. A proportional derivative control strategy is then proposed to stabilize the system. Simulation case studies on four paralleled grid converters and experimental tests for two paralleled grid converters are carried out to validate the performance of the proposed control.

I. INTRODUCTION

The share of renewable energy sources, particularly wind turbines and photovoltaic panels, has been keeping a rapid growth in electrical grids. The large renewable energy based power generation plants are emerging into power grids [1]. A widespread use of power electronics converters in renewable energy sources furnishes full controllability and flexibility for renewable power plants. However, harmonics generated from high-frequency switching operation of converters may make influence on other devices and even trigger resonances [2]. Using LCL filters for switching harmonics attenuation may lead to the instability of grid converters and subsequent resonance propagation in power plants [3]. This challenge is getting worse with the multiple resonance frequencies caused by power transmission cables and other paralleled converters [4]. Hence, the control of grid converters with the multiple resonance frequencies instead of single LCL filter resonance becomes important to keep the stability and power quality of renewable power plants.

Generally, the stability of grid converters is dependent on multiple control loops, including the current control loop, active/reactive power control loops, and grid synchronization loop [5]. Since the bandwidth of the power control and grid synchronization loops are usually well below the resonance frequencies of LCL filters and cables, the current control loop becomes more sensitive to resonance peaks [6].

The interaction of current control loop with single LCL resonance frequency has been thoroughly discussed in many research works [6]-[10]. A number of passive dampers [7] or active damping control methods [8]-[10] have been reported to address the LCL resonance. However, only a few of them can be found for the stability of current control with multiple resonance frequencies [4], [9]. In [4], the active damping control with the cascade notch filters in series with current controller is introduced to stabilize the system. However, it is worth noting that the multiple resonance frequencies do not imply the appearance of multiple oscillations in the current control loop. To clarify the instability effect of multiple resonance frequencies, the numerical analysis with pole zero map is reported in [9], the analytical approach is however missing. Moreover, the use of notch filters for resonance damping is sensitive to the parameter uncertainties, which has to be adaptive to the varied resonance frequencies [10].

This paper addresses the resonant interactions among the current control of grid converters and the multiple resonance frequencies caused by cable capacitances and the paralleled converters. Both grid current and converter current control are analyzed considering the effect of the digital computation and Pulse Width Modulation (PWM) delays. The frequency region for unstable current control is identified by means of impedance-based analysis and frequency-domain passivity theory [6], [15]-[16]. To stabilize the current control with the multiple resonance frequencies, an active damping control strategy based on a backward Euler derivative controller is proposed. For converter current control, the Euler derivative controller is used with an additional zero compensation. For grid current control, the derivative controller is implemented with a positive feedback. Simulations of four grid converters interconnected with power cables and laboratory tests of two paralleled converters validate the proposed approach.

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II. SYSTEM DESCRIPTION

Fig. 1 illustrates a simplified one-line diagram of a cable-based distribution feeder in a renewable power plant. Four renewable energy sources interfaced with grid converters are interconnected by the 1km \( \Pi \)-equivalent cable models. LCL filters are used to attenuate switching harmonics produced by converters. Table I gives the main circuit parameters of the system. For simplicity, the converters are assumed to have the same parameters, and their DC-link voltages are constant.

Fig. 2 shows a general control diagram of the converter. The Phase-Locked Loop (PLL) is used to synchronize with the Point of Connection (PoC) voltage. To avoid unwanted low-frequency instabilities induced by grid synchronization, the PLL is designed to have a lower bandwidth than the grid fundamental frequency [11]-[13]. Both the converter current \( i_1 \) and grid current \( i_2 \) can be controlled, whose diagrams are detailed in Fig. 3. \( G_c(s) \) is the current controller implemented with a Proportional Resonant (PR) controller. \( G_d(s) \) denotes the computation and Pulse Width Modulation (PWM) delays in the digital control system [14].

\[
G_c(s) = \frac{k_p}{s} + \frac{k_s s}{s^2 + \omega_1^2 s + \omega_1^2 T_s}, \quad (1)
\]
\[
G_d(s) = e^{-\frac{s T_s}{2}}, \quad (2)
\]

where \( \omega_1 = 2\pi f_1 \), \( f_1 \) is the grid fundamental frequency, and \( T_s \) is the sampling period of control system.

![Fig. 1. Simplified one-line diagram of a distribution feeder with four grid converters.](image1)

![Fig. 2. General diagram of grid converter with single-loop current control.](image2)

![Fig. 3. Block diagrams of current control of grid converters. (a) Converter current control loop. (b) Grid current control loop.](image3)

TABLE I. MAIN CIRCUIT PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Electrical Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_g )</td>
<td>Grid voltage</td>
<td>400 V</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>Grid fundamental frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>( L_g )</td>
<td>Grid inductance</td>
<td>2 mH</td>
</tr>
<tr>
<td>( f_{sw} )</td>
<td>Converter switching frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Converter sampling frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>Converter DC-link voltage</td>
<td>750 V</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>LCL filter - converter-side inductor</td>
<td>2.7 mH</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>LCL filter - grid-side inductor</td>
<td>0.9 mH</td>
</tr>
<tr>
<td>( C_f )</td>
<td>LCL filter - capacitor</td>
<td>9.4 ( \mu )F</td>
</tr>
<tr>
<td>( L_c )</td>
<td>Cable inductance</td>
<td>0.48 mH/km</td>
</tr>
<tr>
<td>( C_c )</td>
<td>Cable capacitance</td>
<td>0.46 ( \mu )F/km</td>
</tr>
<tr>
<td>( r_c )</td>
<td>Cable resistance</td>
<td>0.025 ( \Omega )/km</td>
</tr>
</tbody>
</table>
III. IMPEDANCE-BASED STABILITY ANALYSIS

A. Modeling of Grid Converters

Fig. 4 shows the Norton equivalent models of converter and grid current control loops at the PoC of converters. To preserve the electric property at the PoC of the converter, the equivalent model of converter current control is derived at the filter capacitor, as shown in Fig. 4 (a), where the “plant” of control loop is the converter-side inductor \( L_1 \). Thus, the terminal behavior of converter current control at the PoC can be derived in the following

\[
T_i = G_1 G_2 Y_{ip}, \quad Y_{ip} = Y_o = \frac{1}{Z_{11}} \tag{3}
\]

\[
G_{cl} = \frac{T_i}{1+T_i}, \quad Y_o = \frac{Y_o}{1+T_i} - \frac{1}{G_1 G_2}, \quad Y_{id} = \frac{1}{G_1 G_2} \tag{4}
\]

\[
k_i = \frac{Z_{12} + Z_{22}}{Y_o Z_{12} + Z_{12} + Z_{22} - G_{cl} Y_o - Y_o Z_{12} + Z_{12} + Z_{22} + V_{\text{ref}}} \tag{5}
\]

where \( T_i \) and \( G_{cl} \) are the open-loop and closed-loop gains of control loop, respectively. \( Y_{ip} \) and \( Y_o \) are the open-loop and closed-loop output admittances of control loop, respectively. \( Y_{ip} \) is the transfer function derived from the converter voltage to converter current. From (4), it is seen that the closed-loop output admittance \( Y_{id} \) is equivalent to the open-loop output admittance \( Y_o \) in series with an admittance introduced by the control loop, i.e., \( Y_{id} = 1/(G_1 G_2) \).

In contrast, the terminal behavior of grid current control loop is directly equivalent to a Norton circuit, as shown in Fig. 4 (b), and the “plant” of control loop is the LCL filter.

\[
Y_{ip} = \frac{i_p}{V_{\text{ref}}} \bigg|_{\omega \to 0} = \frac{Z_{12}}{Z_{ij} Z_{12} + Z_{12} + Z_{ij} Z_{12}} \tag{6}
\]

\[
Y_{ds} = \frac{i_s}{V_{\text{ref}}} \bigg|_{\omega \to 0} = \frac{Z_{22}}{Z_{ij} Z_{12} + Z_{12} + Z_{ij} Z_{12}} \tag{7}
\]

where \( Y_{ip} \) is the transfer function from the converter voltage to grid current, \( Y_{ds} \) is the open-loop output admittance. Thus, the terminal behavior of grid current control is derived as

\[
Y_{ds} = \frac{Y_o}{1+T_i} = \frac{1}{Y_{ip}} + \frac{1}{G_2}, \quad Y_{id} = \frac{Z_{22}}{G_1 G_2} \tag{8}
\]

where \( T_i \) and \( G_{cl} \) are the open-loop and closed-loop gains of grid current control loop, respectively. \( Y_{ds} \) is the closed-loop output admittance, which is equivalent to (4), is equivalent to a series connection of the open-loop output admittance and an admittance introduced by the control loop \( Y_{id} \).

B. Passivity of Current Control

Given a linear, continuous system \( G(s) \), the passivity in the frequency-domain is defined as follows [15]:

1) \( G(s) \) has no Right Half-Plane (RHP) poles.

2) \( \Re\{G(j\omega)\} \geq 0 \implies \arg\{G(j\omega)\} \in [-\pi, \pi], \forall \omega > 0 \).

This is an important property of a dynamical system, which has been applied to the design of current controller for grid-connected converters [6], and the stability assessment of the cascaded DC-DC converters [16]. It has been shown that if each subsystem in an interconnected system is passive, the whole system will be stable [15]. Hence, if the closed-loop output admittances \( Y_{ds} \) and \( Y_{id} \) have non-negative real parts, the interactions among the current control and the resonant grids will be stable, since cables and LCL filters are passive.

However, from (4) and (8), it can be found that the time delay in digital control system, denoted as \( G_d \), will introduce a negative conductance into the admittances \( Y_{ds} \) and \( Y_{id} \) and consequently the closed-loop output admittances \( Y_{ds} \) and \( Y_{id} \). They are illustrated as follows

\[
Y_{ds} = \frac{1}{k_p} e^{j \omega_m \tau} \cos(1.5T, \omega) + j \sin(1.5T, \omega) \tag{9}
\]

\[
Y_{id} = \frac{1-LC_i \omega^2}{k_p} e^{j \omega_m \tau} [\cos(1.5T, \omega) + j \sin(1.5T, \omega)] \tag{10}
\]

where (1) and (2) are substituted into the admittances \( Y_{ds} \) and \( Y_{ds} \) and the resonant controller in (1) is nullified since it only takes effect at the fundamental frequency. Based on (9), it is seen that \( Y_{id} \) yields a negative conductance between the one sixth of the sampling frequency and the Nyquist frequency \( (f_s/6, f_s/2) \). From (10), it can be derived that \( Y_{id} \) possesses a negative conductance between the resonance frequency of \( L_1 \) and \( C_f \) and \( f_s/6 \).

C. Impedance-Based Stability Analysis

To illustrate first the basic principle of impedance-based stability analysis, the stability of converter current control is
analyzed in the following based on Fig. 4 (a). By rearranging (5), the closed-loop response of converter current \(i_1\) can be expressed as

\[
i_1 = \frac{1}{1 + \frac{1}{Y_{L2C}} G_{cl,ref} T_1} \frac{V_{ref}}{Z_{L2}} Y_{1c} \bigg( 1 + \frac{1}{Y_{L2C}} \bigg) \]

(11)

where \(Y_{L2C}\) is the admittance of paralleling filter capacitor \(C_f\) and grid-side inductor \(L_2\), \(Y_{L2C} = (Z_{L2} + Z_{Cf})/(Z_{L2} Z_{Cf})\). Thus, the stability of converter current control is dependent on a minor feedback loop composed by the admittance ratio \(Y_{1c}/Y_{L2C}\), in addition to the closed-loop gain \(G_{cl,ref}\).

Fig. 5 shows the frequency responses for the open-loop gain \(T_1\), and the admittances \(Y_{1c}\) and \(Y_{L2C}\). The proportional gain of current controller is designed with the 45° of phase margin, as shown in Fig. 5 (a). Hence, \(G_{cl,ref}\) is stable and the system stability is determined by the minor feedback loop. Fig. 5 (b) plots the frequency responses of the admittances \(Y_{1c}\) and \(Y_{L2C}\), where the phase difference at the intersection points of their magnitude responses give the phase margin of the minor feedback loop. A negative phase margin is resulted if the phase difference is higher than 180°. It is seen from Fig. 5 (b) that the phase of \(Y_{1c}\) is out of \([-90°, 90°]\) between \(f_s/6\) and \(f_s/2\), which verifies (9). Moreover, an intersection point falls in this frequency range, where the phase difference indicates a negative phase margin of converter current control loop. Hence, the frequency range, within which the phase of the closed-loop output admittance has a negative conductance, is the region that the system may be destabilized.

Then, substituting the Norton equivalent models for grid converters in Fig. 1, the equivalent circuits of the distribution feeder can be drawn in Fig. 6. Since converter current control is unstable at the PoC, only the interaction among the grid current control of converters and cables shown in Fig. 6 (b) is analyzed with the impedance-based ratio derived in (12).

\[
T_{2m} = Y_{1c}/Y_{2load}
\]

(12)

where \(Y_{2load}\) is the equivalent load admittance derived at the PoC of the converter which is farthest from grid. \(T_{2m}\) is the open-loop gain of the minor feedback loop.

Fig. 7 plots the frequency responses for the open-loop gain \(T_2\) and the closed-loop output admittances \(Y_{1c}\). A stable terminal behavior at the PoC is observed in Fig. 7 (a). This is due to the inherent damping effect of time delay in the digital
region that $Y_{221}$ has a negative real part, multiple intersections of their magnitude responses result in unstable oscillations at different frequencies. Hence, instead of the single resonance like in the LCL filter itself, as shown in Fig. 5, the resonant interactions of paralleled converters and cables may lead to unstable oscillations at multiple frequencies.

IV. PROPOSED CONTROL APPROACH

A. Control Structure

Fig. 9 illustrates the block diagrams of the proportional derivative based stabilizing control methods in the discrete z-domain. The approach is based on the single current control loop, where the basic idea is to use the derivative controller to compensate the phase lag induced by the computation and PWM delays, and thus restrict the frequency region where the closed-loop output admittance has negative conductance.

Fig. 9 (a) shows the proposed controller for the converter current control loop, where an additional zero compensation is introduced with the backward Euler derivative controller, in order to improve the passivity property of the control loop. Fig. 9 (b) depicts the proposed controller for the grid current control loop, where the backward Euler derivative controller is used with a positive feedback.

Consequently, the open-loop gains of the current control loops can be derived in the following

$$T_{1d}(z) = [k_y + (k_{pd} - k_{dd} z^{-1})(1-z^{-1})] z^{-1} \text{ZOH} \{ Y_{r1}(s) \}$$

(13)

$$T_{2d}(z) = [k_y - k_{dd} (1-z^{-1})] z^{-1} \text{ZOH} \{ Y_{r2}(s) \}$$

(14)

where $T_{1d}(z)$ and $T_{2d}(z)$ are the open-loop gains of converter and grid current control loops with the derivative controllers respectively. The resonant controller gain $k_i$ is neglected. The PWM delay is included by performing the Zero-Order Hold (ZOH) transformation on the “plants” of control loops.

control system [8]. Fig. 7 (b) shows that the phase of $Y_{22}$ is out of $[-90, 90]$ between the resonance frequency of $L_1$ and $1/f_6$, which agrees with the analysis in (10). This range is therefore the frequency region that the current control may be destabilized when it interacts with the resonant grid.

Fig. 8 compares the frequency responses for the closed-loop output admittance $Y_{22}$ and the load admittance $Y_{22 \text{load}}$. It is seen that multiple resonance frequencies are introduced in the load admittance $Y_{22 \text{load}}$ due to the interactions among the other three converters and cables. Moreover, in the frequency
B. Passivity-Based Design

The passivity-based design of the proposed controllers is illustrated in the following:

1) The root locus analyses based on (13) and (14) are used to design for a stable terminal behavior, i.e., $Y_{1c}$ and $Y_{2c}$ have no RHP poles.

2) Substituting $e^{-Ts}$ for $z^{-1}$ in Fig. 9, and replacing $k_d$ in (9) and (10) by proposed controllers, then selecting controller parameters for passivity.

Fig. 10 shows the closed-loop pole trajectory of converter current control loop with the different controller. The ratio $k_d/k_{pd}$ is swept from 0.2 to 2 with a step of 0.2. The proportional gain of the PR controller $k_p$ is the same as Fig. 5. It is shown that the converter current control seen from the filter capacitor can be designed stable with the derivative controller. The stable limit of $k_{dd}$ can be designed stable with the same as Fig. 5. It is shown that the converter current control. Three different ratios of $k_d/k_{pd}$ will enlarge the frequency region of the non-negative Re{$k_d$}, while the high $k_{dd}$/kp will cause the increase of $k_{dd}/k_{pd}$, and the maximum $k_{dd}$ for $k_{dd}/k_{pd}=2$ is corresponding to 10.4.

Then, replacing $k_p$ in (9) by the derivative controller, the passivity of control loop can be analyzed based on (15).

\[ \text{Re} \left( \frac{1}{Y_{2d}} \right) = \left( k_p + k_{pd} \right) \cos(1.5T \omega) - \left( k_{pd} + k_{dd} \right) \cos(2.5T \omega) + k_{dd} \cos(3.5T \omega) \]

(15)

Fig. 11 shows the influence of $k_d/k_{pd}$ on the passivity of the converter current control. Three different ratios of $k_d/k_{pd}$ are compared. It is seen that at low $k_d/k_{pd}$, two frequency regions with negative Re{$Y_{1c}$} are caused by increasing $k_{dd}/k_{pd}$, while at the high $k_d/k_{pd}$, only the high frequency region with the negative Re{$Y_{1c}$} is left, but the frequency region with non-negative Re{$Y_{1c}$} is reduced to be below 0.3s. Thus, there is a compromise in selecting $k_d/k_{pd}$. Moreover, the increase of $k_{dd}/k_{pd}$ will enlarge the frequency region of the non-negative Re{$Y_{1c}$}, but it also reduces the allowed $k_{dd}/k_{pd}$ for stability, as illustrated in Fig. 10. Hence, a trade-off is made by selecting $k_{dd}/k_{pd} = 1$, and $k_{dd}/k_{pd} = 1.4$ in this work, for which the corresponding closed-loop poles are specified in Fig. 10.

The root locus analysis of grid current control loop based on (14) is shown in Fig. 12. The ratio $k_d/k_p$ is swept from 0 to 2.1 with a step of 0.3. A stable grid current control loop with the derivative controller in a positive feedback can be observed. Similarly to (15), $k_p$ in (10) is substituted by the proportional derivative controller, which is then given by

\[ \text{Re} \left( \frac{1}{Y_{2d}} \right) = \frac{f(k_d/k_p)}{1 - L C \omega^2} \]

(16)

Unlike the converter control current, the passivity of grid current control is not only dependent on the ratio $k_d/k_p$, but affected by the resonance frequency between $L_1$ and $C_f$, $f_c(L_1 C_f)$. Hence, to enlarge the frequency region of passivity, $k_d/k_p$ should be designed yields the positive $f(k_d/k_p)$ for the frequencies lower than $f_c(L_1 C_f)$, and the negative $f(k_d/k_p)$ for the frequencies above $f_c(L_1 C_f)$.

Fig. 13 plots $f(k_d/k_p)$ in terms of $k_d/k_p$. It is seen that the frequency region of $f(k_d/k_p) \leq 0$ is shift to the left as the increase of $k_d/k_p$. This implies that the frequency region with negative Re{$Y_{2c}$} in Fig. 8 can be eliminated, provided that the lowest frequency of $f(k_d/k_p) \leq 0$ is higher than $f_c(L_1 C_f)$. On the other hand, the highest frequency of Re{$Y_{2c}$} $\geq 0$ is also reduced. Hence, $k_d/k_p$ is selected as 0.9 according to Fig. 13 for a good compromise, while $k_d$ is selected as 9 for a proper damping of transient response based on Fig. 12.
C. Stabilizing Effect of Proposed Controller

Substituting $e^{-Ts}$ for $z^{-1}$ in Fig. 9, the stabilizing effect of proposed controllers can then be assessed by the impedance-based analysis. Fig. 14 plots the frequency responses of the closed-loop output admittances of converters and equivalent load admittances after including the proposed controllers. It is shown that the system keeps stable for both converter and grid current control with the proposed controllers. From Fig. 14 (a), it is seen that $Y_{1c}$ has a phase out of $[-90^\circ, 90^\circ]$ at the frequencies closed to $0.3f_s$, which matches with Fig. 11 (b). Compared to Fig. 8, Fig. 14 (b) gives an enlarged frequency region of passivity, which stabilizes the system.

V. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

To confirm the theoretical analysis and the performance of the proposed controllers, time-domain simulations for the system shown in Fig. 1 are carried out within Simulink and Plecs blockset.

Fig. 15 shows the simulated grid currents of converters with converter current control, where the proposed controller is disabled at the time instant of 0.26 s. It can be seen that the system is destabilized by the interactions among converters.

Fig. 16 shows the simulated grid currents of converters with grid current control. Similarly, the derivative controller is disabled at the time instant of 0.26 s, and the system turns into unstable thereafter.

B. Experimental Results

The laboratory tests on two paralleled grid converters are implemented, where a California Instruments MX-series AC power supply is used for grid emulation. The control system is implemented with a dSPACE DS1006 system.

![Fig. 12. Closed-loop pole trajectory of grid current control loop.](image1)

![Fig. 13. Influence of $k_d/k_p$ on the nominator of (16), denoted as $f(k_d/k_p)$.](image2)

![Fig. 14. Frequency responses of the converters output admittances $Y_{1c}$ and $Y_{2c}$, and the equivalent load impedances $Y_{1load}$ and $Y_{2load}$ with the proposed controllers. (a) Converter current control. (b) Grid current control.](image3)

![Fig. 15. Simulated grid currents of converters with converter current control.](image4)
shown that the phase lag induced by system delays result in negative conductance at the PoC of converters, which may destabilize the system. Two proportional derivative control schemes have been designed for the passivity of converter and grid current control loops respectively. Simulations and experimental results have been shown that the phase lead of derivative control action improves the passivity of control systems, and thus stabilize the system.

REFERENCES


