Design of $LLCL$-filter for Grid-Connected Converter to Improve Stability and Robustness

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Abstract — The $LLCL$-filter has recently emerged into grid-connected converters due to the improved filtering capability which ensuring a smaller physical size. An $LLCL$-based grid-connected converter has almost the same frequency-response characteristic as that with the traditional $LCL$-filter within half of the switching frequency range. The resonance frequencies of the $LLCL$-filters based grid-connected converters are sensitive to the grid impedance as well as cable capacitance, which may influence the stability of the overall system. This paper proposes a new parameter design method for $LLCL$-filter from the point of stability and robustness of the overall system, when the grid-side current control is used. Based on this design method, the system can be stable without damping and also robust to the grid parameters variation. The influence of delay and parameter variations is analyzed in details. Last, a design example for $LLCL$-filter is given. Both simulations and experimental results are provided through a 5 kW, 380V/50 Hz grid-connected inverter model to validate the theoretical analysis in this paper.

Keywords — $LLCL$-filter; robustness; stability; delay; impedance variation; filter design

I. INTRODUCTION

Most renewable energy sources and distributed generation (DG) resources are connected to the power grid through a grid-connected converter [1]. However, the use of Pulse Width Modulation (PWM) scheme introduces undesirable harmonics that may disturb other sensitive loads/equipment on the grid and also result in extra power losses. Hence, a low-pass power filter is required between the voltage source inverter (VSI) and the grid to attenuate the high-frequency PWM harmonics to limit the harmonic content of the grid-injected current [2].

Fig. 1 shows four different filter structures. Typically, a simple series inductor $L$ is used as the filter interface between power converters in the renewable energy system. But it only has $20 dB/dec$ attenuation around the switching frequency, so a high value of inductance needs to be adopted to reduce the current harmonics, which would lead to a poor dynamic response of the system and a higher power loss. In contrast to the typical $L$-filter, the high order $LCL$-filter can achieve a $60 dB/dec$ harmonic attenuation performance with less total inductance, significantly smaller size and cost, especially for applications above several kilowatts [3]. Recently, the trap filter which is also called $LLCL$-filter is becoming attractive for industrial applications [4]-[7], as shown in Fig. 1(c). Compared to the $LCL$ filter, a small inductor is inserted in the branch loop of the capacitor, composing an $L_f-C_f$ series resonant circuit at the switching frequency to eliminate this major harmonic component. Hence, the total inductance or capacitance of the filter can be reduced. In order to further reduce the size of the filter, a multi-tuned filter was proposed [8], but it brings the complexity to the circuit and has possible parallel resonances between the multi-tuned traps.

Similarly to the $LCL$-filter, the $LLCL$-filter resonance is challenging the stability of the grid-connected VSI. Hence, the high order resonance should be properly damped either passively or actively [9]-[15]. The $LLCL$-filter does not bring any extra control difficulties because an $LLCL$-based grid-connected converter has almost the same frequency-response characteristic as that with the traditional $LCL$-filter within half of the switching frequency range. In digital-controlled systems, sampling and transport delays caused by controller and the PWM modulation will affect the system stability and should be taken into account [9]-[11]. The stability of the $LLCL$-filter considering the delay effect was studied in [7]. Ref. [7] comes to the conclusion that one-sixth of the sampling frequency ($f_s/6$) is regarded as a critical $LLCL$-filter resonance frequency and the system can be stable, if the resonance frequency is higher than $f_s/6$ without damping method.

However, long distance distribution lines will introduce inductive impedance to the grid. The resonance frequency of the $LLCL$-filter is sensitive to the grid impedance. What’s more, distribution cables and interaction between multi-converters also influence the stability of the system with the multiple resonance frequencies [16]-[19]. A system is robust if
it is not very sensitive to grid and converter parameter variation. For this reason, a novel parameter design method of the LLCL filter for stabilizing the system operation and improving system robustness is proposed and validated. In section II the LLCL-filter based grid-connected inverter is modelled using the Norton equivalent model with grid current control. Section III analyzes the stability and robustness of the system considering the transmission cables. Then, a novel LLCL-filter design method is proposed and parameters variation is analyzed in section IV. Finally, the experiments of a 5 kW grid-connected converter are carried out to verify the theoretical analysis.

II. MODELING OF THE LLCL-FILTER BASED GRID-CONNECTED INVERTER

A. System Description

Fig. 2 illustrates a LLCL-filter based grid-connected voltage source inverter with single-loop control. The inverter output voltage and current are represented as \( u_i \) and \( i_i \), and the grid voltage and current are represented as \( u_g \) and \( i_g \). The voltage \( u_{pcc} \) is the voltage at the Point of Common Coupling (PCC). \( Z_g \) is the grid impedance, which can be inductive or capacitive. \( U_{dc} \) is the DC voltage. Table I shows the main parameters of the system which is used as an example for this study.

When the grid current is controlled, the corresponding control block diagram is shown in Fig. 3. \( G_i \) is the implemented current controller using a Proportional Resonant (PR) controller and harmonic compensator, expressed as:

\[
G_i(s) = K_p + \sum_{h=1,3,5} \frac{K_{h} s}{s^2 + (\omega_h h)^2} \quad (1)
\]

where \( \omega_h = 2\pi f_h \) is the fundamental angular frequency, \( K_p \) is the proportional gain, and \( K_{h} \) is the integral gain of the individual resonant frequency \( h \).

\( G_i(s) \) is the computational and Pulse Width Modulation (PWM) delays.

![Fig. 2. LLCL-filter based grid-connected voltage source inverter with single-loop control.](image)

![Fig. 3. Block diagram of grid current control of grid-connected converter.](image)

![Fig. 4. Norton equivalent model of grid-connected converter with grid current control.](image)

TABLE I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{dc} )</td>
<td>DC link voltage</td>
<td>650 V</td>
</tr>
<tr>
<td>( U_g )</td>
<td>Grid phase voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>( f_o )</td>
<td>Grid frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>( T_s )</td>
<td>Sampling period</td>
<td>100 ( \mu s )</td>
</tr>
</tbody>
</table>

where \( T_s \) is the sampling period of control system and \( \lambda T_s \) is delay time, \( K_{PWM} \) is the transfer function of the inverter. \( Z_{L1} \) is the impedance of the inverter-side inductor. \( Z_{L2} \) is the impedance of the grid-side inductor. \( i_g^* \) is the reference grid current.

B. Norton Equivalent Model

Fig. 4 shows the Norton equivalent model of grid-connected converter with grid current control [19]. The dotted block is the cable capacitance \( C_g \) and line impedance \( L_g \). The derivations of the terminal behavior of the grid current control are shown below.

The open loop transfer functions from \( i_g \) to \( u_i \) and \( i_g \) to \( u_{pcc} \) are expressed in (3) and (4), respectively.

\[
G_1 = \left. \frac{i_g}{u_i} \right|_{u_{pcc}} = \frac{Z_{CL}}{Z_{L1}Z_{L2} + Z_{L1}Z_{G} + Z_{L2}Z_{C}} \quad (3)
\]

\[
G_2 = \left. \frac{i_g}{u_{pcc}} \right|_{u_i} = \frac{Z_{L1}Z_{G} + Z_{L2}}{Z_{L1}Z_{L2} + Z_{L1}Z_{G} + Z_{L2}Z_{C}} \quad (4)
\]

\( T \) and \( G_{CL} \) are the open-loop and closed-loop gains of the grid current control loop, which are expressed as:

\[
T = K_{PWM} \cdot G_2 \cdot G_1 \quad (5)
\]

\[
G_{CL} = \frac{T}{1 + T} \quad (6)
\]

The closed-loop output admittance \( G_{CL} \) can be derived as:

\[
G_{CL} = \frac{G_2}{1 + T} = 1/(1 + T) \cdot \frac{1}{G_2} \cdot G_1 \quad (7)
\]

Hence, the closed loop expression of the grid current \( i_g \) is expressed as:

\[
i_g = G_{CL} \cdot i_g^* - G_{CL} \cdot u_{pcc} \quad (8)
\]

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The grid voltage is seen as the disturbance term in the design of the current loop controller. The resonance at the grid side will also influence the stability of the whole system. The resonance frequency of the LLCL-filter \( \omega_r \) is derived as:

\[
\omega_r = \frac{1}{\sqrt{L_1(L_2 + L_3) + L_r}} C_f
\]  

(9)

III. STABILITY AND ROBUSTNESS ANALYSIS

A. Concept of Passivity of the System

Given a linear and continuous system \( G(s) \), two requirements should be met in order to obtain the passivity [20]:

1) \( G(s) \) has no Right Half Plane poles.

2) \( \text{Re}\{G(j\omega)\} \geq 0 \Leftrightarrow \arg \{G(j\omega)\} \in [-90^\circ, 90^\circ], \forall \omega > 0 \).

For the grid-connected converter system, the cables and LLCL-filters are passive if the closed-loop output admittance \( G_{c2} \) has non-negative real parts the interactions among the current control and the resonant grids will be stable [19]. But due to the presence of the delay time in the sampling and updating of PWM, a negative part could be introduced in \( G_{c2} \). According to (3) – (6), the part of \( G_{c2} \) containing a delay can be expressed as:

\[
G_{c2} = \frac{G_c}{T} = \frac{1-(L_1 + L_f)C_f \omega^2}{K_{psb}K_r(1-C_f, \omega^2)} e^{j\Delta \omega}
\]  

(10)

It can be clearly seen from (10) that a negative part is probably be presented in the closed-loop output admittance \( G_{c2} \). Hence, the passivity of the system is dependent on the filter parameters and the delay time.

According to (10), the frequency boundaries of the positive or negative are expressed in (11), (12)

\[
f_{rs} = \frac{1}{2\pi \sqrt{(L_1 + L_f)C_f}}
\]  

(11)

\[
f_{re} = 1/(2\pi \sqrt{L_f/C_f})
\]  

(12)

\[
f_{rd} = \frac{f_s}{4\lambda}
\]  

(13)

where \( f_{rs} \) is the switching frequency and \((1-C_f\omega^2)\) is always larger than zero before switching frequency. \( f_c \) is the resonance frequency of \( L_1, L_f, \) and \( C_f \). Hence, it can be deduced that:

When \( 0 < f_{re} < f_c / (4\lambda) \), the system has a negative real part in the frequency interval \([f_c, f_c / (4\lambda)]\).

When \( f_c / (4\lambda) < f_c < f_r \), the system has negative real part in the frequency interval \([f_c / (4\lambda), f_c]\).

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the LLCL-filter the current control may be destabilized. So it is better to design the \( L_1, L_2 \) and \( C_f \) to make \( f_c \) to be chosen to \( f_s/6 \) in order to obtain the best robustness and passivity.

IV. LLCL-FILTER PARAMETERS DESIGN PROCEDURE

A. Conventional Design Constraints

When designing a power filter, the base impedance of the system should be known. The base values of the impedance, the inductance and the capacitance are referred to as:

\[
Z_b = \frac{U_g}{P_0}, \quad \omega_b = \frac{1}{\omega_0 Z_b}, \quad I_b = \frac{Z_b}{\omega_0}
\]

where

\( U_g \) the line-to-line RMS voltage;
\( \omega_b \) the grid frequency;
\( P_0 \) rated active power.

The following aspects of the design guideline should be satisfied [3], [21] and [22]:

1) Limit the total inductance \((L_1+L_2)\). The upper limit for the total inductance should be less than 0.1 pu in order to limit the dc-link voltage on operation. A higher dc-link voltage will result in higher switching losses and thereby lower efficiency.

2) The value of the inverter-side inductor \((L_1)\). This inductor works with high frequency ripple current and it is constrained by the maximum ripple current.

3) Maximum harmonic distortion of the grid current. The lower limit of the filter inductance is determined by the harmonic requirement of the grid-injected current according to IEEE 519-1992 [23], as specified in Table II. \( I_{gs} \) is the nominal grid-side fundamental current. \( I_{GC} \) is the short circuit current of the power system. The harmonic currents can be calculated by the corresponding harmonic voltage amplitudes at different harmonic frequencies.

4) Design of the filter capacitor. Large capacitance can provide a better high frequency harmonics attenuation but it consumes more reactive power. For low voltage converter, it is considered that the maximum power factor variation at rated power is less than 5%, as it is expressed by the value of the base impedance of the system \( C_f \leq 5\%C_b \).

5) Resonance frequency of the filter. The resonance frequency is assumed to be in a range between ten times the line frequency to avoid the major low frequency harmonics and one-half of the switching frequency to avoid resonance problems.

B. Design Procedure

According to the analysis before, the parameter choices of the LLCL-filter are very important to the system stability and robustness. The basic design guideline is given as [24] - [26]:

1) Design of inverter-side inductor \( L_1 \). Due to the PWM, the output voltage of the inverter has high frequency harmonics as shown Fig. 7. In order to smooth the inverter side current inverter-side inductor \( L_1 \) should meet a specific current ripple requirement. The inductance can be calculated from equation \((11)\): \( L_1 = \frac{I_{gs}}{\omega_0} \), \( I_{gs} \) is the rated reference peak current, \( \omega_0 \) is the inverter-side current ripple ratio, which generally is lower than 40% of the rated reference current [4], [6];

2) Design of capacitor \( C_f \). According to the analysis, \( C_f \) should be designed by the boundary frequency \( f_c \). It can be seen from (11) the switching frequency \( L/C_f \) is fixed at the switching frequency. When the value of \( L_1 \) is fixed \( C_f \) can be chosen according to the delay time and sampling frequency. At the same time, the capacitor value should meet the reactive power requirements. Fig. 8 shows the value of the capacitance in pu changes with different switching frequencies and delays. It can be seen from the figure that the capacitance increases with the switching frequency and the delay coefficient \( \lambda \) increasing. The capacitance should satisfied the constrain that \( C_f \leq 5\%C_b \).
3) **Design of inductor** $L_f$ of $L_fC_f$ circuit. If $C_f$ is selected $L_f$ can be chosen based on the switching frequency to attenuate the dominant harmonics. The attenuation characteristics are influenced by the $L_f-C_f$ circuit quality. The $L_f-C_f$ series resonant circuit quality factor can be taken as:

$$Q = \frac{1}{R_f} \sqrt{\frac{L_f}{C_f}}$$

$$n = \frac{L_f}{C_f}$$

where $R_f$ means the sum of the equivalent series inductor resistance and the equivalent series capacitor resistance. It is a parasitic resistance and no external resistance is added to the circuit. Fig. 9 shows the characteristic impedance of the $LC$ trap filter with different $L_f/C_f$. It can be seen from Fig. 9 that the trap range is wider with the Q-factor increasing. When the range of side-band harmonics around the specific frequency is relatively wide, it should be considered to obtain a larger Q-factor of the $LC$ trap filter branch to get better harmonic attenuation. Fig. 10 shows that characteristic impedance of the $LC$ trap filter with different $R_f$. $R_f$ will reduce the depth of the $LC$ trap filter. These factors tend to lower the quality factor. Normally, the Q-factor can be $10 \leq Q \leq 50$ [27].

4) **Selection of the grid-side inductor** $L_2$. When $L_1$ and $C_f$ are designed, $L_2$ should be designed to reduce the harmonic around the double of the switching frequencies down to 0.3% [6], as shown in (5).

$$\frac{4U_{dc}}{3\sqrt{3}\pi} \times \max \left\{ J_1(\pi M) \left| J_1(\pi M) \right| \right\} \times \left| G_s(j2\omega) \right| \leq 0.3\%$$

where $J_1(\pi M)$ and $J_5(\pi M)$ are the Bessel functions corresponding to the 1$^{st}$ and 5$^{th}$ sideband harmonics at the double of the switching frequency. Because the dominant harmonics around the switching frequency are already attenuated by the $LC$ trap circuit, the value of $L_2$ can be selected relatively low. The resonance frequency will be higher than $f_s/6$. 

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**Fig. 9.** Characteristic impedance of the $LC$ trap filter with different $L_f/C_f$.

**Fig. 10.** Characteristic impedance of the $LC$ trap filter with different $R_f$.

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**Fig. 11.** The relationship between capacitance, switching frequency and total inductance.

**Fig. 12.** Flow chart of the parameter design procedure of $LLCL$-filter.
5) Check of the total inductance ($L_1+L_2$). Fig. 11 shows the value of the total inductance increases with the switching frequency and the capacitance decreasing. It can be seen from the Fig. 11 that the total inductance increases with the switching frequency and the capacitance is reduced. The value of the total inductance should be less than 0.1pu in order to limit the ac voltage drop during operation and also lower the high dc-link voltage.

6) Component tolerance. The influence of the variation of inductance and capacitance of the $\text{LLCL}$-filter will be discussed. The parameter drift of $L_1$, $L_f$, or $C_f$ may result in the resonance frequency $f_{\text{rc}}$ variation. Generally, for industrial filters the tolerances are: Capacitors: 5% and no negative tolerance; Inductors: 2% [28]. Considering the variation of the capacitor and inductor, $f'_{\text{rc}}$ is between [96.9% $f_{\text{rc}}$ - 103% $f_{\text{rc}}$].

C. Design Example

A step-by-step procedure has been proposed to obtain the parameter values of the $\text{LLCL}$-filter. The system is specified in Table I. The rated power is 5 kW. Hence, the base impedance $Z_b$ is 29.0 Ω, the base capacitance $C_b$ is 109.6 μF and the base inductance is $L_b$ 92.5 mH. A design example is shown in Table III with following steps:

1) Based on the constraint of the total inductor and inverter-side current ripple, a 30% current ripple can be obtained to design the inverter inductor $L_1$. Then the inverter-side inductor is selected to be 2.2 mH.

2) Considering 1.5$T_d$ delay and 10 kHz switching frequency the capacitor value is designed as 4 μF in order to make the frequency $f_{\text{rc}}$ is $f_s/6$ and meet the constraint of 5% reactive power.

3) For the $L_f$-$C_f$ resonant circuit, $L_f$ can be chosen based on the chosen $C_f$ and the switching frequency. It is calculated to be 64 μH.

4) The grid-side inductor value of $L_2$ can be calculated by the injected grid current harmonics standard. $L_2$ is selected to be 1.8 mH.

5) Check the total inductance and the possible variation of real value of $f_{\text{rc}}$.

V. EXPERIMENTAL RESULTS

The experimental setup consists of a 5 kW Danfoss FC302 converter connected to the grid through an isolating transformer and the DC-link supplied by Delta Elektronika power sources. The control algorithm has been implemented in a dSPACE DS1103 real time system. TABLE I shows the experimental parameters.

Fig. 13(a) shows the steady state waveforms of the grid current and the $L_f$-$C_f$ trap voltage for the designed parameters when the grid impedance is neglected, $L_g$ = 0 mH. Fig. 13(b) shows the grid current spectrum. It can be seen from the spectrum that the dominant harmonics occur around the double of the switching frequency because the switching harmonics have been attenuated by the trap circuit.

![Fig. 13. Experimental results of the designed $\text{LLCL}$-filter parameters.](image)

![Fig. 14. Transient experimental results when the grid current reference is changed. (a) $L_g = 0$. (b) $L_g = 4.8$ mH.](image)
1. It can be seen that: A step by step filter design method is proposed based on the analysis. Parameters variation also influences the system robustness and accuracy of the design.

4. A 5 kW grid-connected converter system is implemented to verify the theoretical analysis on the LLCL-filter. It can be found that the experimental results match the theoretical analysis results well.

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