How do gifted students become successful? A study in learning styles
Søndergaard, Bettina Dahl

Published in:
<em>TSG 4 (Topic Study Group 4): Activities and programmes for gifted students</em>

Publication date:
2004

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
The 10'th International Congress on Mathematical Education

PROCEEDINGS

July 4-11, 2004,
Copenhagen, Denmark

Editors:
Edward Barbeau
Hyunyong Shin
Emiliya Velikova
Alex Friedlander
Shailesh Shirali
Agnis Andžāns
PROCEEDINGS OF
The Topic Study Group 4:
ACTIVITIES AND PROGRAMS FOR GIFTED STUDENTS

THE 10’TH INTERNATIONAL CONGRESS ON
MATHEMATICAL EDUCATION

Editors: Edward Barbeau, Hyunyong Shin, Emiliya Velikova, Alex Friedlander, Shailesh Shirali, Agnis Andžāns
Riga, University of Latvia, University of Rousse (Bulgaria) 2004–205 pp.

The monograph contains papers/abstracts accepted by the International Programme Committee of TSG4 for presentation at the ICME-10.

The electronic version of this volume is used within Latvian Education Informatization System.

The volume was prepared technically by Ms. Emily Velikova, Ms. Dace Bonka, Ms. Lasma Strazdina and Ms. Inese Berzina. The cover was designed by Ms. Valentina Vojnohovska.

ISBN 9984-770-17-6
Copyright © 2004, University of Latvia, Riga

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or translations in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without permission of the publisher.

Printed in Riga. Latvia

Reg. apl. No 2-0266

Lespiests SIA “Macibu gramata”, Raina Bulv. 19, Riga, LV-1586, tal/fax : 7325322
HOW DO GIFTED STUDENTS BECOME SUCCESSFUL?  
A STUDY IN LEARNING STYLES

Bettina Dahl

Abstract: The purpose of this paper is to argue that gifted students need special programmes to avoid, for instance, psychological disturbances and/or being turned off school and furthermore the paper argues that successful students learn in a qualitative differently way from less successful students and that even among the successful there are differences in how they learn.

Keywords: Gifted, successful, analysing, generalising, problem-solving procedures, memory, learning theories, learning strategies.

INTRODUCTION

In this paper I discuss if gifted students can take care of themselves, how successful students learn mathematics compared to lower-achieving students, and if there are similarities or differences in how successful high school students learn mathematics.

1. CAN GIFTED STUDENTS TAKE CARE OF THEMSELVES?

There are many synonymous for ‘gifted’ such as ‘talented’, ‘able’, ‘successful’, ‘capable’ ‘high-achieving’ etc. Basically these terms fall into two groups. Some describe the state of actually being “good” (yet another expression) namely: ‘successful’ and ‘high-achieving’ while others more describe a person who has the potential for being ‘successful’ or ‘high-achieving’. These terms are: ‘gifted’, ‘talented’, ‘able’, and ‘capable’. By introducing this distinction I at the same time argue that there is not necessarily a direct link between having the potential to become successful and actually being or becoming successful in either daily life or the classroom. Some might argue that gifted students can take care of themselves and helping gifted students is perceived as taking resources from weaker students. I will argue that it is a question of equality of opportunity to provide for the needs of the gifted. The focus of the education system should be on meeting every student where he is and help him to reach his full potential. Special emphasis on the needs of the gifted is for instance seen in the United Kingdom where Ofsted (Office for Standards in Education) considers the needs of able students as part of equality of opportunity. The DfEE (Department for Education and Employment) has furthermore in two Circulars (14/94 & 15/94) in 1994 recommended that in primary and secondary schools all School Prospectus should include details of arrangements to identify and provide for exceptionally able students ([2], pp. 16-17). One can argue as follows:

If we accept that it is the duty of society ... to provide educational opportunities for all children appropriate to their individual abilities and aptitudes, and if one further accepts that some children are exceptional ... then the issue is settled. ... For children to receive specialized educational treatment in such circumstances is not for them to get more than their fair share; they are simply receiving what, in their individual circumstances, is appropriate. ([6], p. 4)
UNESCO’s Salamanca Statement, 1994, declares that “The guiding principle that informs this Framework is that schools should accommodate all children regardless of their physical, intellectual, social, emotional, linguistic or other condition. This should include disabled and gifted children, street and working children, children from remote or nomadic populations, children from linguistic, ethnic or cultural minorities and children from other disadvantaged or marginalized areas or groups”([10], p. 6) and further: “every child has unique characteristics, interests, abilities and learning needs; education systems should be designed and educational programmes implemented to take into account the wide diversity of these characteristics and needs ([10], p. viii). Hence, it is not “un-just” to help successful and gifted students; they too have a right to receive what fits them. Furthermore: “all children are born as unique individuals, each different from the other, and in developing them we need to make them more equal by overcoming whatever inabilities they may have and more different from one another by developing their abilities and propensities” ([12], p. 31). In that sense, special education (for both weak and strong students) both improves inabilities and develops the person’s talent(s).

No student can progress towards the limit of his capacity unless he has an opportunity to learn: “Mozart might have had an extraordinary aptitude for music, but this could hardly have been realized unless his parents possessed a piano. It is at best inefficient to rely on nature or chance to develop talents, while for potentially gifted children in homes with limited cultural horizons it borders on neglect” ([6], p. 5). Studies have furthermore shown that some gifted students are underachieving and sometimes suffer psychological disturbances including poor concentration, exaggerated conformity, excessively inhibited behaviour, anxiety, social isolation and aggressiveness, or the opposite such as extreme passivity ([6], p. 6). Other studies have shown that if gifted students are held back or bored in school, some of them will be ‘turned off’ by school, achieve far below the level of which they are capable, drop out, fail, or even become delinquent ([6], p. 14). Another study showed that children who could read before beginning in school do not develop new competencies if they are just being taught what they already know, and many of the early readers later loose interest in reading. These students therefore need special attention and need to be challenged ([5], p. 6). It is further stated that some gifted students deliberately hold themselves back:

Some able students receive a shock when they move on to university. The leisurely study habits which had ensured reasonable grades in the mixed ability classes in secondary schools prove to be inadequate for the more intellectually demanding environment of the university. … there are too many students of high ability who wastefully drop out. … it is very probable that many gifted children ‘learn to be average’ or deliberately hold themselves back in order to have a quite life in school: this is the phenomena of ‘faking bad’. ([6], pp. 14-15)

Gifted students therefore need adequate stimulation. Studies suggest that association with other students of high ability raises a student’s level of performance. One study showed that the “overall intellectual level within a group had an effect on the development of the level of individuals within the group - contact with clever people tended to raise the level of ability of the less clever” ([6], p. 13). Another study showed that “down to an IQ of about 65, mentally retarded students taught with normal peers achieved better than those who were taught in self-contained classes” ([6], p. 13). And further “that students of high ability were penalized academically by being taught with students of lesser ability” ([6], pp. 13-14). Hence, it might seem as a Catch-22 situation: when each student seems to do better when taught together with more gifted students, and suffer from being with less gifted...
students, there will always be a “looser” in the “game”. However, if does not have to be this way if it is the teachers’ duty to stimulate the students according to their abilities, which is also what is argued below:

Refusing to make special provision for the unusually able, on the grounds that they are necessary for the optimal development of the other children, means that adults shrug off the task of promoting the development of less gifted youngsters onto the shoulders of clever children. Naturally, educators should be looking at the needs of the less gifted, but not at the expense of the gifted and talented. ([6], p. 14)

2. GIFTED STUDENTS COMPARED WITH OTHER STUDENTS

Analysing

When gifted students work on a mathematical problem they perceive the mathematics of it analytically, which means that they isolate and assess the different elements in its structure, systematise them, and determine their ‘hierarchy’. At the same time they perceive the mathematical material synthetically, and here combine the elements into complexes and investigate the mathematical relationships ([4], pp. 227-228; [9], p. 15). Gifted students perceive problems as a composite whole, while average students see a problem in its separate mathematical elements. It is only through analysing the problem that the average students are able to find the connections of the mathematical elements. Lower-achieving students have great difficulties in establishing these connections, even when they achieved help. The speed of the analytical-synthetic process in the gifted student is so fast that they see its ‘skeleton’ at once. It is often impossible to trace the process. The fast grasping of a problem’s structure has been observed to be the result of exercises, but gifted students need only a minimal number of exercises to make the analytical-synthetical perception arise ‘on the spot’ ([4], pp. 228-232).

Example 1 ([4], p. 230)

A 6th grade class gets the following problem:

A jar of honey weighs 500 g, and the same jar, filled with kerosene, weighs 350 g. How much does the empty jar weigh?

A gifted 3rd grader (V.L.) answers (E is the experimenter):

V.L.: And then?
E: That’s the whole problem.
V.L.: No, that isn’t all. I still must know how much heavier honey is than kerosene.
E: Why?
V.L.: Without that, there could be many solutions. There are two unequal quantities, connected by the fact that some of their parts are equal. There could be very many of these parts. To limit their number, we must introduce one more quantity, characterizing the ‘remainder’.

A less gifted 6th grader was not able to solve this problem, even when he got the hint: “honey is twice as heavy as kerosene”.
Generalising

The ability to ‘grasp’ structural relationships in a generalised form is a central feature for the productive thinking ([4], p. 234). The gifted students do this on the spot whereas lower-achieving students need a lot of practice and exercises covering all possible cases and levels before an elementary level of generalization is possible ([4], pp. 240-242). Gifted students can analyse one phenomenon and generalise from this by separating the essential features from inessential. Their method is to infer “the features’ generality from their essentiality. … to be essential means to be necessary and, consequently, it should be common to a number of phenomena of this type, that is, it should inevitably be repeated” ([4], p. 259). Lower-achieving students perceive the generality of features by contrast.

Example 2 ([4], p. 241)

A gifted student, O.V., had previously solved just a single example using the formula of the square of a sum: \((a+b)^2 = a^2 + b^2 + 2ab\). Then he got the problem: \((C+D+E) \times (E+C+D)\). (E is the experimenter.)

O.V.: What’s this? Here it’s not by the formula – we must simply multiply the polynomials. ... But that will be 9 terms. That’s a lot. But we can use the formula – that is a square [quickly writes: \((C+D+E)^2\)]. Right. Now any two terms can be combined [writes: \((C+(D+E))^2\)].
E: But can you do that? The formula applies only to the square of a binomial, but didn’t you have a trinomial?
O.V.: As soon as I combined D and E into one term, I got a binomial – look [shows]. A ‘term’ can be any expression. ... [Solves it, repeating the formula aloud. Writes: \(C^2 + 2C(D+E) + (D+E)^2 = C^2 + 2CD + 2 CE + D^2 + 2DE + E^2\)]

Procedures for problems-solving

The trials for problem-solving for lower-achieving students are blind, unmotivated, and unsystematic. On the contrary gifted students have an organised plan of searching ([4], p. 292). Gifted students switch easily from one mental operation and method to another, they have great flexibility and mobility in their mental processes in solving mathematical problems, and it is therefore easy for them to reconstruct established thought patterns. For average students it is much harder to switch to a new method of problem-solving. Lower-achieving students experience even greater difficulties in that ([4], pp. 278-282). For the gifted students the trials are a way to thoroughly investigate the problem through extracting information from each trial. Without having finished the trial, gifted students seem to know if they are on the right track. This is owing to the existence of an acceptor, which is a psychological control-appraisal mechanism, where ‘line-of-communication’ is received from each mathematical operation. Under this acceptor lies a generalised and concentrated system of past mathematical experience ([4], p. 293). The gifted students thoroughly investigate the problem, which may suggest that they enjoy working with mathematics. The emotional factor is seen in that they often try to solve the problem in a more simple way or improve the solution and they show satisfaction when the solution was economical, rational, and elegant ([4], p. 285), which is seen in the example below.
Example 3 ([4], p. 279)

G. Kh., a capable seventh grader, solved problem XIII-A-7: "Four liters of water at room temperature (15°) were added to 3 liters of water at a temperature of 36°. What temperature was established in the container?" At once, without thinking, he gave this solution:

"Three liters of water gave 108° in sum."
"Four liters of water gave 60° in sum."
"A total of 168° for 7 liters = 24°."

Without stopping, G. Kh. gave the following "visual" solution as well:

\[
\begin{array}{c}
36° \\
\hline
\hline
\hline
x° \\
\hline
\hline
15°
\end{array}
\]

Here is his clarification: "The average temperature will not be in the middle, but moved toward 15° (because there are more of the 15° liters). The ratio will be 4:3. We divide the difference of 21° in such a relation (21:7 = 3), and then either add 9 to 15, or subtract 12 from 36."

Afterward G. Kh., having thought for about 10 seconds, applied the formula he knew for the weighted arithmetical average, making the following notation:
\[
\frac{(36 \cdot 3 + 15 \cdot 4)}{(3 + 4)} = 24°.
\]

He took a total of 1 minute 8 seconds to solve the problem in all of its variants. No incapable pupil could find two solutions for this problem. The majority of them could find one solution only with help from the experimenter.

Memory

Gifted students do not have a "better" memory than lower-achievers, but gifted students usually remember the general character of a problem-solving operation and not the problem's specific data. On the contrary, lower-achieving students usually only remember the problem's specific facts. The mathematical memory of gifted students is selective and only keeps the mathematical information that represents generalised and curtained structures. This means that the brain is not loaded with extra information which makes it possible to retain the information longer and use it more easy ([4], pp. 299-300).

Example 4 ([4], pp. 298-299)

A lower-achieving student, I.G., solved the problem: \(113^2 - 112^2\) with the experimenter's help. After one week she had forgotten the mathematical relationship (difference of squares) but remembered that the problem had used the numbers 112 and 113.

The figure below ([4], p 297) shows the forgetting-curve of generalized relations, concrete data, and unnecessary data for gifted students.
3. SIMILARITIES AND DIFFERENCES IN HOW SUCCESSFUL HIGH SCHOOL STUDENTS LEARN MATHEMATICS

I have previously done research ([1]) in how ten successful high school students (aged 17-20) explain that they come to understand a mathematical concept that is new to them. Four students were Danish (Z, Æ, Ø, Å) and six were English (A, B, C, D, E, F). They were interviewed in pairs and fours: Z-Æ-Ø-Å, A-C, D-E, B-F. All studied mathematics at the highest level possible in each of their school system and their teachers selected them as successful. The study rests on the assumption that successful students have a metacognition, which means that they have knowledge about and regulation of their cognition. Knowledge of own learning means that one has relatively stable information about own learning processes. This knowledge develops with age and there is a positive correlation between the degree of one’s insight into own learning and one’s performances on many tasks. Regulation of own learning is the planning before one begins to solve a problem and the ongoing evaluation and control while one learns something new or solves a problem ([8], pp. 138-141). I asked general explorative questions to not be leading. In their own words, the students, among other things, describe the relationship between visualization and verbalization and the individual and the social side of learning. I used the learning theories of Ernest, von Glasersfeld, Hadamard, Krutetskii, Mason, Piaget, Polya, Sfard, Skemp, and Vygotsky in the analysis. The ten students fall in different groups regarding their preference for learning style. For language reasons the examples below are from the English interviews.

Visualization and verbalization

Regarding verbalization, Student A, C, Z, and Æ tell that an oral explanation helps the one that is talking. For instance Student Æ tells that very often if she tries to explain the mathematics to a person, then when she is explaining it, she understands it herself. Student C adds that verbalization (saying things out loud) helps the visualization:
If you just read it in your head, I just read it and I don’t understand. If you come across a phrase which sounds really awkward, like the one here: “in the plane whose interior intersects the diagram in one of the configurations involved”, you just read it over, but you don’t think about it, but if you read it out, then you think about the disc, and then you visualise the disc, and then you visualise the plane.

If one focuses on visualization, Student Z describes that she does not need to see pictures but to see things in writing. Another group consists of the students being moderately positive to visualization (A, D, E, F, Å, Å). They describe that pictures sometimes makes it easier. The two examples below are from two interviews, but both students talk about some knot theory that I had given them to work on, to initiate a discussion:

F: The first thing to do [when explaining to classmates] would be to draw some knots and then show the respective oriented diagrams whatever they are called, these graphs.

E: Actually the graphics are a big help, I know I was relieved when I got to this first graphic [D & E laugh], it showed, it kind of showed what they were.

A third group (C, Ø) perceive visualization as something that is very important for learning. Student Ø says for instance that visualisation can be a great advantage for instance in terms of cosine equation. Student A, C, and Å argue that the relationship between visualisation and verbalisation depends on the mathematics. For instance Student A:

A: Sometimes if you just READ a sentence like the first here “A link is a finite collection of mutually disjoint disjointed simple closed curves”, if you just take that as the words it is difficult for you to see any other way that it can work. If you start to visualize what it is talking about then you can SEE all the different ways in which it can happen.

I: So is it important that the words come first and then you can see what the words are about, or do you want to see it first and then be explained and then get the words?

A: Well. I don’t know really. It probably depends on the sort of problem. If it is a very visual problem where you have to think it through maybe in 3 dimensions, I think it is probably better to have pictures first and maybe dealing with graphs as well. If you are doing vectors it may be better to have the picture first, and then the words explaining how it works. But if it is more a linear methodological process it might be better to have the words first and then pictures to help you understand, cause it is the words you are trying to understand. But where there is something visual like this or graphs or vectors it’s the pictures that you are trying to understand so the one supports the other depending on what it is you are trying to learn, I think.

The students group as below. They do not all express something in this connection.

1. Oral explanation helps the one that is talking: Student: A, C, Z, Å
2. Mainly verbal: Student Z
3. Relatively visual: Student A, D, E, F, Å, Å
4. Mainly visual: Student C, Ø
5. Depends on circumstances, for instance the type of mathematics: Student A, C, Å

**Individual or social learning**

It seems that the students argue that learning has both a social and an individual side. The value of the social side is mainly when the students experience problems with self-
learning. After input from the outside, they can continue on their own. Particularly Student C, D, E, F, Z, Æ, Ø, and Å argue this. Conversely, Student A and B express that they learn more through the discussions than through self-activity. I will now investigate some of these students further. First an example from the interview with Student D and E who both emphasize the individual side:

E: I know people do er differently but it's all very individual even if you work on something together er and you're both aiming to solve the problem, I mean you'll do it completely differently from someone else and quite oftenly I find I don't like other people's styles, you know, you always get your own.

D: Yea, but that's the whole point in learning why you do something rather than how, because a lot of the times there are some things I do my own little way, I don't necessarily follow the textbook example and it, that doesn't matter cause I understand what I'm doing, what I want to achieve by doing it, and I can get the right answer by doing it a different way to the textbook, but, you know, it just works for me.

Student Z tells that basically the teacher cannot help her learn mathematics. She has to work for herself, she has to take the responsibility for her learning.

The social side seems to be most important to Student A and B. They tell that they learn more through the discussions that by sitting for themselves. Student A tells that it is better to work for oneself through looking at examples than being told “this is wrong, try again”. This might sound like support of individually aimed theories such as Piaget’s ([7], pp. 16-19), who argued that the students through own activities in interaction with the surrounding world construct their knowledge. In that case, it would be a contrast to what Student A also tells, namely that he learns best through the discussions. However, what Student A actually says is that it is better to “work for himself” instead of “being told”. To “be told” is not necessarily the same as a discussion, perhaps rather the opposite. Student A therefore priorities as follows: 1. discussion, 2. work for himself, 3. be told. Student A is therefore probably more a “Vygotsky-student”. Vygotsky ([11], pp. 56-57) argued that internalisation happens through activity and communication in social interaction. Student B supports this (I is the interviewer):

B: I think it certainly helps if you can discuss it with someone else. Two brains are better than one.
I: Why?
B: Er, one person can have one idea which should trigger another idea in the other person's head which the first person wouldn't have had, and then the second person having said that thing, and then one thing leads to another if you got two people to think.

Student C and Æ do on own initiative use words such as “combination” or “two-way thing” to describe the relationship between the individual and the social side of learning, however still with emphasis on the individual side. An example:

C: It all boils down to the teaching method and the teacher. It's a two-way thing you see, it's more about you learning, you being able, no, you learning as well you being taught properly. If you are taught in a way that you can fit in, you know, then it is good.

Student Æ and Z tell that one can learn through discussions with oneself, perhaps even better than in groups. The students group as follows:
1. Individual: Student C, D, E, F, Z, Æ, Ø, Å
2. Social: Student A, B
3. Combination (emphasis on the individual side): C, Æ
4. Learn from discussing with oneself: Student Æ, Z

Relating to the discussion above about if it is best to work with students on the same level or not, Students A, C, and Z argue that if they have problems learning, it is best to discuss with someone who also has not understood. The reason is that they do not want to be bullied. Student Å says that discussion with “equals” is good in any case, and Student B argues that it is more helpful to discuss with someone who knows.

The students’ learning styles

It seems that the students divide themselves into different groups that either support individual or social learning, or support visualization or verbalization with a preference for one, or describe a kind of combination between these factors. The preferences are independently of nationality or gender (D, Z, Æ, Ø, Å are girls; A, B, C, E, F are boys). The students therefore learn in different ways. These students constitute a rather homogeneous group of gifted students, which might suggest that if one looks at the whole spectrum of students, even more learning strategies or preferences might emerge. It is therefore vital with variation in the teaching. Teaching after a one-sided pedagogical theory or idea will lead to that some students are lost.

Furthermore Student A says that the learning strategies one uses are connected with the ways one has been taught to do things. Other students explains the following in two of the interviews (I is the interviewer):

D: When I first came here [to the new school], the first couple of weeks I found math very difficult because it is kind of hard to adapt to a different teaching style.

I: I noticed when you talked about presenting it to the class you wanted to give them examples and you also mention, while you were talking, that it would be nice with examples. Why?

B: That’s because the way that we’ve always been taught is using examples thoroughly to explain, so that’s the way we think the people in our class will understand it easiest, explain through examples.

This phenomena might be explained by that the teaching methods must be part of, what I would express as a zone of proximal teaching (ZPT), inspired by Vygotsky’s ZPD. According to Vygotsky, the potential for learning is limited to the “zone of proximal development (ZPD)”, where ZPD is the area between the tasks a student can do without assistance, and those, which require help ([11], p. 86). Hence, if a (new) teacher uses teaching methods that are too “far away” from teaching styles the students are used to, the students might have difficulties learning.

4. CONCLUSIONS
Gifted students need special programmes not just to make sure that they, for instance, avoid psychological disturbances, but there are also major differences in how successful and less successful students learn mathematics and even among the successful students there are some differences in learning styles. Also Hadamard wrote about different mathematical minds and “even among men who are born mathematicians, important mental differences may exist” ([3], p. 11). Successful students furthermore experience problems when facing a new teaching style but they do nevertheless seem to overcome these difficulties. This is to some extent supported by [4]'s investigations that showed that successful students switch easily from one mental operation and method to another.

REFERENCES


ABOUT THE AUTHOR

Bettina Dahl (Soendergaard), Ph.D.
Norwegian Centre for Mathematics Education
Norwegian University of Science and Technology (NTNU)
Realfagbygget A4, 7491 Trondheim
NORWAY
E-mail: bdahts@math.ntnu.no