Hamper or helper? The role of language in learning mathematics
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Published in:

Publication date:
2004

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
Mathematics and Language

Edited by
Christer Bergsten and Barbro Grevholm

Proceedings of MADIFF 4
The 4th Swedish Mathematics Education Research Seminar*
Malmö, January 21-22, 2004

*Fjärde MatematikDidaktiska Forskningsseminariet med SMDF

Skrifter från Svensk Förening för MatematikDidaktisk forskning, Nr 3
Linköping 2004
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Hamper or Helper: The Role of Language in Learning Mathematics

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Introduction
This paper reports a part of a study of how ten successful high school pupils from Denmark and England explain that they learn a mathematical concept that is new to them. It focuses on what the pupils tell about the role of language. The analysis uses various theories of learning to get a greater understanding of the pupils' explanations.

Methods
Metacognition can be understood both as knowledge about and regulation of cognition. This knowledge develops with age and there is a positive correlation between performance on many tasks and the degree of metacognition. In relation to regulation of cognition, this is related to the planning of activities prior to problem solving, the monitoring as one goes along, and the checking of the outcome. The “presence of such behavior has a positive impact on intellectual performance ... its absence can have a strong negative effect” (Schoenfeld, 1985, p. 138). One might therefore assume that successful pupils know how they learn mathematics. I therefore asked five teachers to pick some of their best pupils in the classes that studied mathematics at the highest high school level. In Denmark it is Niveau A at the Gymnasium and in England it is the AS (Advanced Subsidiary) Level Mathematics. I named the Danish pupils: Z, Å, Ø, å and the English pupils: A, B, C, D, E, F. The Danish pupils were interviewed as one group and the English pupils were interviewed in pairs. Pupils D, Z, Å, Ø, å were girls; A, B, C, E, F were boys. The English pupils were given a sheet with some knot theory to initiate a discussion, but otherwise all the interviews were explorative.

Theories of learning that focus on language
The opinions of the role of language are divided. To some, language is a necessary thinking-tool; to others it obstructs thinking.

Language as helper
According to Hadamard, Polya said that “the decisive idea which brings the solution of a problem is rather often connected with a well-turned word or sentence. The word or the sentence enlightens the situation, gives things, as you say, a physiognomy” (Hadamard, 1945, p. 84). Also Russell talked in a positive way about language: “Language serves not only to express thoughts, but to make possible thoughts which could not exist without it. ... I hold that there can be thought, and even true and false belief, without language. But however that may be, it cannot be denied that all fairly elaborate thoughts require words” (Russell, 1948, p. 74). I will call this a moderate positive view since it holds that words not only express or “mirror” thoughts but also more actively help thinking. However, not all thought requires words, only higher-order thinking.

Another positive view is seen in Muller who stated that “no thought is possible without words” (Hadamard, 1945, p. 66). This view is connected with Vygotsky’s description of language as the logical and analytical thinking-tool (Vygotsky, 1962, p. viii) and that thoughts are not just expressed in words but come into existence through the words (ibid., p. 125). Vygotsky also said that “Language does not of necessity depend on sound” (ibid., p. 38), and that: “Thought development is determined by language, i.e., by the linguistic tools of thought and by the socio-cultural experience of the child” (ibid., p. 51). Thus, to Vygotsky, thoughts develop from social interaction and what we learn is inherently a product of human communication, and it would not exist for us if we were not part of the human community. These strong positive arguments are basically that no thought or learning can take place without the use of language and that language is an indispensable thinking-tool that makes thoughts come into existence.

Language as hamper
Berkeley argued that “words are the great impediment to thought” (Hadamard, 1945, p. 68). Galton explained that results can be perfectly clear to himself but “when I try to express them in language I feel that I must begin by putting myself upon quite another intellectual plane. I have to translate my thoughts into a language that does not run very evenly with them” (Hadamard, 1945, p. 69). Hadamard stated that “thoughts die the moment they are embodied by words” (ibid., p. 75), and that a thought “can be accompanied by concrete representations other than words. Aristotle admitted that we cannot think without images” (ibid., p. 71). Furthermore, “the more complicated and difficult a question is, the more we distrust words, the more we feel we must control that dangerous ally and its sometimes treacherous precision” (ibid., p. 96). But Hadamard acknowledged that “signs are necessary support of thought” (ibid., p. 96). Wittgenstein argued that a main source in our lack of understanding is that we do not have an overview of the use of our words, the grammar is

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1 Full name: Bettina Dahl Soeeggaard; alternative spelling: Soendergaard.
2 The study reports parts of a Ph.D. study (Dahl, 2002); Other findings are reported in Dahl (in press).
confusing. Philosophy is a battle against the bewitchment of our mind by means of the language (Wittgenstein, 1983, §109 & §122-126). The views quoted here all centre on language as either obstructing or confusing thinking, that words are not always necessary for thoughts, or that language is not always able to express thoughts. Piaget (1970, pp. 18-19) stated that "This, in fact, is our hypothesis: that the roots of logical thought are not to be found in language alone, even though language coordinations are important, but are to be found more generally in the coordination of actions, which are the basis of reflective abstraction". Here Piaget seemed to disagree with Vygotsky who stated that thoughts develop from social interaction where the language is the thinking-tool. To Piaget, knowing an object does not mean to copy it, but to act upon it: "Knowing reality means constructing systems of transformations that correspond, more or less adequately, to reality" (ibid., p. 15). An abstraction is "drawn not from the object that is acted upon, but from the action itself. It seems to me that this is the basis of logical and mathematical abstraction" (ibid., p. 16). Hence, actions perform the basis of mathematical thinking, not language or interaction.

### A dual nature of language

Vygotsky criticised Piaget around the concept of egocentrism¹ and egocentric speech.² Vygotsky stated that egocentric speech is not just accompanying the child’s activity but the child uses it as a means of expression and to release tension, but it soon becomes an instrument of thought that the child uses to, for instance, plan the problem-solving (Vygotsky, 1962, p. 16). Thus, to Vygotsky egocentric speech, besides its communicative role, is an important thinking-tool and as a tool to solve problems. “Egocentric speech emerges when the child transfers social, collaborative forms of behavior to the sphere of inner-personal psychic functions” (ibid., p. 19). Vygotsky then continued and stated that: “Thus our schema of development – first social, then egocentric, then inner speech³ – contrast both with the traditional behaviorist schema – vocal speech, whisper, inner speech – and with Piaget’s sequence – from nonverbal autistic thought through egocentric thought and speech to socialized speech and logical thinking” (ibid., p. 19-20). To Vygotsky, the order of the development of thinking is from the social to the individual. This internalisation process has two levels, the social and the individual: “first between people (interpsychological), and then inside the child (intrapsychological)” (Vygotsky, 1978, pp. 56-57). As a response to Vygotsky’s views and critique, Piaget referred to Vygotsky’s propose that:

> egocentric speech is the point of departure for the development of inner speech, which is found at a later stage of development, and that this internalised language can serve both autistic ends and logical thinking. I find myself in complete agreement with these hypotheses. On the other hand, what I think Vygotsky still failed to appreciate fully is egocentrism itself as the main obstacle to the co-ordination of viewpoints and to co-operation. ... In brief, when Vygotsky concludes that the early function of language must be that of global communication and that later speech becomes differentiated into egocentric and communicative proper, I believe I agree with him. But when he maintains that these two linguistic forms are equally socialized and differ only in function, I cannot go along with him because the word socialization becomes ambiguous in this context: if an individual A mistakenly believes that an individual B thinks the way A does, and if he does not manage to understand the difference between the two points of view, this is, to be sure, social behavior in the sense that there is contact between the two, but I call such behavior unadapted from the point of view of intellectual co-operation. (Piaget, 1962, pp. 7-8)

Vygotsky emphasises that language is not just a means of expression; it is also an instrument of thought. Piaget agrees, but found that Vygotsky failed to understand that egocentrism itself could be a main obstacle for reaching understanding through the use of language.

### Analysis

Below I will discuss three examples from the interviews.

### Hamper AND helper

This example is from an interview with two English pupils, D and E. ‘They’ refers to the authors of the book about knot theory.

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¹ The notion of egocentrism in Piaget’s work is “quite unrelated to the common meaning of the terms, hyperrhaphy of the consciousness of self. Cognitive egocentrism, as I have tried to make clear, stems from a lack of differentiation between one’s own point of view and the other possible ones, and not at all from an individualism that precedes relations with others” (Piaget, 1962, p. 4).

² Piaget stated that “I have never spoken of speech ‘not meant for others’; this would have been misleading, for I have always recognized that the child thinks he is talking to others and is making himself understood. My view is simply that in egocentric speech the child talks for himself” (Piaget, 1962, pp. 7-8). This was Piaget’s response when Vygotsky wrote that Piaget’s view was that “In egocentric speech, the child talks only about himself, takes no interest in his interlocutor, does not try to communicate” (Vygotsky, 1962, pp. 14-15).

³ To Vygotsky, inner speech “is not the interior aspect of external speech – it is a function in itself. It still remains speech, i.e., thought connected with words. But while in external speech thought is embodied in words, in inner speech words die as they bring forth thought. Inner speech is to a large extent thinking in pure meanings” (Vygotsky, 1962, p. 149).
Pupil D and E seem to agree that language can be a hindrance to learning but at the same time at least Pupil E argues that one cannot learn mathematics without using language, but he finds that one can use different language, for instance simple language. Pupil D seems to argue that one indeed can learn mathematics without the use of language. Elsewhere in the interview she argues that one can learn mathematics without language, at least when the mathematics is easy. She explains that it took her a long time to understand what the authors wrote but it was easy as soon as she translated it. It can therefore be interpreted that to Pupil D, language can obstruct thinking, but it is also a necessary translation-tool, and that simple language, after being translated from a difficult language, seems to be a thinking-tool. Pupil E also expresses that the appropriate (for instance simple) language is a thinking-tool. This might seem odd as they also state that language/notation confuses the meaning, which could be an example of the double nature of language that Piaget argued for. Pupil E does not himself directly argue for a dual nature of language whereas Pupil D directly states that one has to “do the two together, you have to translate while you’re trying to understand”.

Written language
This example is from an interview with two English boys, Pupil A and C.

A If I am revising from my notes, I find that helps if I actually write the notes out again, just copying them, out from you know [C coughs] the previously. Because, if you just sort sit down to revise, you read through, even if you read it aloud, you pick up some things, remember some things. But if you write it out, you sort of read again and then write again (I: mmm) and it sort of reinforces it. And I definitely found er, if it is something where I have to memorize (1 sec silence), you know sort of examples, sort of methods, equations, how things work, I definitely find it easier if I write, er, as an aid to memory (2 sec silence) (I: mmm) er, [1-2 sec silence] Yea.

I Do you know other tricks or do you know other people have other tricks you know about?

[2-3 second where C and A talk at the same time].

C Remembering or learning?

I Er. Why is? Is there a difference between remembering and learning?

A Yea, well [interrupted].

C Memorizing something, then you need to know the (1 sec silence) set number of points (I: mmm) so you need to write them out, and, you know, find some sort of sequence, in to remember it. But if you are just learning, then it’s about understanding, you don’t need to remember (A: Yea) the detail, just (I: mmm) need to know the overall, you know, concept (I: mmm) [A tries to break in] you know, principals.

A Yea, it’s important to understand how things (C: Before) how you GET the answer (C: mmm). Because [2 sec silence] YEA, it’s in math, it’s going back to the same thing. It’s fairly easy to learn a general formula for loads of different things like trigonometry and things like that, you can just LEARN the general formula (I: mmm), and every time you get a question, you can just sit and use it and get the answer out. But then if you come up against a problem which is SLIGHTLY different to the general [inaudible] you’ve learn, if you just memorized it and you don’t understand how it’s got there, so you’ve got no change to work back, and work it out for yourself (I: mmm) whereas if you understand it [1 sec silence], you can see where it’s comes from and see where you need to change it to fit what you’ve got. (I: mmm) Got to try and do.

Pupil A argues that written language is an aid to memory, which does not mean that he sees written language as the thinking-tool. Vygotsky’s view was that thoughts come into existence through the words after the learner has participated in a social interaction where he has internalised aspects of the knowledge. To aid the memory is according to Pupil A not real learning. Real learning is to
know the overall concepts and principles. Furthermore, elsewhere in the interviews there is a discussion about whether words come first or not, and here Pupil A says that he is not sure but it probably depends on the problem. If it is a visual problem where one has to think it through in 3D, he finds that it is probably better to first have the picture or a graph, but with a linear algebra problem, it might be better to have the words first and then the pictures to help one understand, because he finds that it is the words one is trying to understand. This means that although words and language have a great value they are by no means the way to get to understand all branches of mathematics.

**Ping-pong between language and images**

Below are pieces from the interview with four Danish girls, here mainly emphasising on Pupil Z’s explanations. The original Danish text can be found in Dahl (2002, pp. 356, 359-361).

<table>
<thead>
<tr>
<th>Z</th>
<th>If you for instance sit down and then say, all right, what is a triangle, and then line up what kind of concepts we are working with, it is so very important to have the concepts lined up. What is the problem really about, and what kind of area are we in, and then you can begin to work with it. And I think that it is very important to have some terms and have them worked through.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Yes OK, this sounded like you talk about a kind of bricks built up on top of each other (someone else says yes yes). [Interrupted by Z]</td>
</tr>
<tr>
<td>Z</td>
<td>I am a pupil-teacher in mathematics and I have been sitting with a pupil who get so caught in what is f. (Someone else: mmm) and it is idiotically to sit and wonder about this, it is not what it is all about. I want to get to the point. But this is what is relevant when one does not understand what it is that is really relevant (Someone else: mmm). And where one is going.</td>
</tr>
<tr>
<td>I</td>
<td>It's like the teacher speaks a different language than us (Someone else: mmm). It's a little like if you learn Italian through Danish then you would also sit and get stuck in what the teacher said instead of the meaning of it, right (I: mmm). The teacher comes with this big mathematics language and then we become so stuck with what the things are called instead of what it means.</td>
</tr>
<tr>
<td>Z</td>
<td>This is perhaps a bit philosophical?</td>
</tr>
<tr>
<td>Z</td>
<td>I think it is difficult. I think that it is very difficult.</td>
</tr>
<tr>
<td>Ø</td>
<td>I also think so. Then you would need a kind of flair for it, or a kind of feeling of in what direction it goes. Because it is almost hopeless if you do not have the ehh basics and then to understand what it is that you are suppose to reach (Someone else: mmm).</td>
</tr>
</tbody>
</table>

Pupil Z explains that it is important to have some terms and have them worked through. For instance in geometry it is very important to line up the concepts one works with to be clear about what the problem is about. Pupil Z also expresses that it is difficult (but she did not say impossible) to learn without the language and as an example she mentions spatial geometry. She explains this further by saying that if one knows the notation, it is simply logical. According to her, one can perhaps learn mathematics even if one has not understood the notation. The notation is what seems relevant when one does not understand what it is that is really relevant as one becomes stocked with what the things are called, the mathematics language, instead of what it means. Learning is ping-pong between images and the language. She seems to argue that language/notations are necessary tools in learning, but that they in themselves are not mathematics and that an unsuitable use of language can block learning. Notions can hamper learning if one does not know them, especially if one does not know them in the beginning of the learning process. It may be possible to learn mathematics even if one has not understood the notation. This seems to some extent to contradict what Pupil Z says when she states that one cannot find meaning without language, but it could also reflect that, to her, the (right) language is necessary but not sufficient.

**Discussion of typologies**

The ten pupils fall into various groups. Pupils C, Å, and Æ do not seem to express much here, but otherwise I will also include information about pupils not quoted above. See Dahl (2002) for full transcriptions of all the interviews.
I: Difficult to learn outside language \{A, Z, \emptyset\}.
II: Language is the thinking-tool \{B, D, E, F\}.
III: Language can hamper learning but language can also help learning \{B, D, Z\}.
IV: Language can hamper learning \{E, F, \emptyset\}.

The main difference between Group I and II is whether learning is only "difficult" outside language or if it is "impossible" without language. In Group I, knowing the notation is essential to be able to learn the mathematics. Language can function as a thinking-tool and it is difficult to find the meaning outside the language. They do not, however, say that it is impossible to learn outside of language. This seems similar to the moderate positive view of language. Group II has a strong positive view of the nature of language. Language is not just a means of translation, but language is a necessary thinking-tool for learning. Different languages facilitate learning. Some pupils argue that simple language is the best. In Group III the pupils describe in their own words a dual nature of language in learning. This could be interpreted to be in line with Piaget's description of that language, due to egocentrism, can both hamper and help learning. Members of Group IV have either a more "negative" view of the role of language for learning or they elsewhere describe the dual nature of language, but without themselves reflecting directly on this. They merely at one place in the interview say one thing, at another place they say something else.

**Conclusions**

There seem to be various views of language in relation to how the pupils explain that they learn a mathematical concept that is new to them. Some pupils say that language is the main thinking-tool, others that it hampers thinking, and again others think that language has a dual nature as it both facilitates learning and hampers learning. Some directly indicate this through expressions such as "you have to do the two together" or agrees that there is a "ping-pong" between language and images. This also depends on the kind of language, for instance simple language or written language. The members of these groups are criss-crossing nations and gender. Referring to Piaget's critique of Vygotsky, Piaget declared himself to be very much in line with Vygotsky about the positive role of language, but Piaget also argued that Vygotsky failed to acknowledge the obstacles language can give rise to. Thus, based on what the pupils here have explained, Piaget seemed to be right in his critique. But the quotes shown here do not reveal whether thoughts develop from social interaction and internalisation or from personal activities and construction. However, in Dahl (in press) it is discussed that the pupils are divided on this issue. The pupils do, however, in the quotes above seem to argue that thoughts (in line with Hadamard) can be accompanied by other representations than words, for instance images or, as Pupil Z puts it, by having a "flair" for it. The latter might be connected with the notion of inner speech which is thinking in pure meaning. They do not seem to argue that language is not always able to express thoughts. Hence, the pupils investigated here seem in various degrees to argue for a dual role of language, namely as language both being a hamper and a helper.

**References**


