Pitch Estimation and Tracking with Harmonic Emphasis on the Acoustic Spectrum

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ML Pitch Estimation

Proposed Method

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Harmonic Signal Model:

\[ s(n) = \sum_{l=1}^{L(n)} \alpha_l e^{j(\omega_l(n)n + \phi_l)}, \]  

where \( \omega_l(n) = l\omega_0(n) \) for \( l = 1, \ldots, L(n), \)

- \( L(n) \): number of sinusoids
- \( \alpha_l \): real magnitudes
- \( \omega_0 \): fundamental frequency
- \( \phi_l \): phases of harmonics
Signal Model
Additive noise

The observed signal can be written as a sum of a desired signal \( s(n) \) and a noise signal \( v(n) \), i.e.,

\[
x(n) = s(n) + v(n)
\]

\[
= \sum_{l=1}^{L} \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n).
\]
At a high narrowband SNR, the harmonic frequency $\omega_l$ is perturbed with a real-valued phase-noise [S.Tretter 1985], which has a normal distribution with zero mean and the variance

$$E\{\Delta \omega_l^2(n)\} = \frac{\sigma^2}{2\alpha_l^2} \tag{3}$$

We can approximate $x(n) = \sum_{l=1}^{L} \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n)$ like

$$x(n) \approx \sum_{l=1}^{L} \alpha_l e^{j(\omega_l n + \Delta \omega_l(n) + \varphi_l)} \tag{4}$$
Signal Model
Unconstrained frequency estimates (UFE)

Unconstrained frequency estimates (UFE) of the constrained frequencies:

\[ \hat{\Omega}(n) = [\hat{\omega}_1(n), \hat{\omega}_2(n), \ldots, \hat{\omega}_L(n)]^T \quad (5) \]
\[ = d_L(n) \omega_0(n) + \Delta \Omega(n), \quad (6) \]

where

\[ d_L(n) = [1, 2, \ldots, L(n)]^T \quad (7) \]
\[ \Delta \Omega(n), = [\Delta \omega_1(n), \Delta \omega_2(n), \ldots, \Delta \omega_L(n)]^T, \quad (8) \]

and

\[ R_{\Delta \Omega}(n) = E\{\Delta \Omega(n) \Delta \Omega^T(n)\} \quad (9) \]
\[ = \sigma^2/2 \diag\left\{ \frac{1}{\alpha_1^2}, \frac{1}{\alpha_2^2}, \ldots, \frac{1}{\alpha_L^2} \right\}. \]
Max. Likelihood (ML) Pitch Estimator

For the time-frame $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-M-1)]^T$, the PDF of the UFE is

$$P(\hat{\Omega}(n)|\omega_0(n)) \sim \mathcal{N}(\mathbf{d}_L(n)\omega_0(n), \mathbf{R}_{\Delta\Omega}(n)). \quad (10)$$

The ML pitch estimator:

$$\hat{\omega}_0(n) = \arg \max_{\omega_0(n)} \log P(\hat{\Omega}(n)|\omega_0(n)) \quad (11)$$

$$= \left[ \mathbf{d}_L^T(n)\mathbf{R}_{\Delta\Omega}^{-1}(n)\mathbf{d}_L(n) \right]^{-1}\mathbf{d}_L^T(n)\mathbf{R}_{\Delta\Omega}^{-1}(n)\hat{\Omega}(n) \quad (12)$$
The ML Estimators are statistically efficient, e.g., the non-linear least-squares (NLS), and the weighted least squares (WLS) [H.Li, et al. 2000], but the minimum variance is limited by the number of samples.

Consecutive pitch values are estimated independently.
Pitch values are usually correlated in a sequence, i.e.,

$$P(\omega_0(n)|\omega_0(n-1), \omega_0(n-2), \cdots),$$  \hspace{1cm} (13)

that motivate Bayesian methods to minimize an error incorporating prior distributions.

State-of-the-art methods mostly track pitch estimates in a sequential process without concerning noise statistics.
1- Jointly estimate and track pitch incorporating both the harmonic constraints and noise characteristics.

2- Estimate the state \( \omega_0(n) \) through a series of noisy observations:

\[
P(\omega_0(n)|\hat{\Omega}(n), \hat{\Omega}(n-1), \cdots) \tag{14}
\]

3- Recursively update the prior distribution of the pitch value.
Bayesian Pitch Estimator
Discrete state-space (HMM)

\( \omega_0(n) \): Discrete random variable (Hidden states)
\[ P(\omega_0(n) | \omega_0(n-1)) : \text{Transition probability in a 1st-order Markov model,} \]
\[ \text{i.e.,} \sum_{\omega_0(n)} P(\omega_0(n) | \omega_0(n-1)) = 1 \]

\[ \hat{\omega}_0(n) = \arg \max_{\omega_0(n)} \log P(\omega_0(n) | \hat{\Omega}(n), \hat{\Omega}(n-1), \cdots) \]
\[ = \arg \max_{\omega_0(n)} \log P(\hat{\Omega}(n) | \omega_0(n)) + \log P(\omega_0(n) | \hat{\Omega}(n-1), \cdots). \] (15)

The priori distribution is defined recursively like
\[ P(\omega_0(n) | \hat{\Omega}(n-1), \hat{\Omega}(n-2), \cdots) = \]
\[ \sum_{\omega_0(n-1)} P(\omega_0(n) | \omega_0(n-1)) P(\omega_0(n-1) | \hat{\Omega}(n-1), \cdots), \] (16)

where \( P(\omega_0(n-1) | \hat{\Omega}(n-1), \cdots) \) is the past estimate.
Bayesian Pitch Estimator
state-space representation of the pitch continuity

Continuous state-space:

\[ \omega_0(n) = \omega_0(n-1) + \delta(n) \]
\[ \hat{\Omega}(n) = d_L(n) \omega_0(n) + \Delta \Omega(n), \]

where \( \delta(n) \sim \mathcal{N}(0, \sigma_i^2) \) and \( \Delta \Omega(n) \sim \mathcal{N}(0, R_{\Delta \Omega}(n)) \) are the state evolution and observation noise, respectively.
Bayesian Pitch Estimator
Continuous state-space (Kalman filter)

First, a pitch estimate is predicted using the past estimates as
\[ \hat{\omega}_0(n|n-1) = \hat{\omega}_0(n-1|n-1) \]  
(17)

with the variance

\[ \sigma^2_K(n|n-1) = \sigma^2_K(n-1|n-1) + \sigma_t^2. \]  
(18)

Second, the pitch estimate is updated with the error of

\[ e(n) = \hat{\Omega}(n) - d_L(n)\hat{\omega}_0(n|n-1). \]  
(19)

Then, the predicted estimate is updated:

\[ \hat{\omega}_0(n|n) = \hat{\omega}_0(n|n-1) + h_k(n)e(n) \]  
(20)

\[ h_k(n) = \sigma^2_K(n|n-1)d_L^T(n)\left[ \Pi_L(n)\sigma^2_K(n|n-1) + R_{\Delta\Omega}(n) \right]^{-1}, \]  
(21)

where \( \Pi_L(n) = d_L(n)d_L^T(n) \), and update

\[ \sigma^2_K(n|n) = \left[ 1 - h_k(n)d_L(n) \right] \sigma^2_K(n|n-1). \]  
(22)
The ML estimator of the covariance matrix among \( N \) estimates:

\[
R_{\Delta \Omega}(n) = \mathbb{E}\{\Delta \Omega(n)\Delta \Omega^T(n)\}
\]

\[
= \frac{1}{N} \sum_{i=n-N+1}^{n} \Delta \Omega(i)\Delta \Omega^T(i), \quad (23)
\]

where \( \Delta \Omega(n) = \hat{\Omega}(n) - \hat{\mu}(n) \), and \( \mu(n) = \mathbb{E}\{\hat{\Omega}(n)\} \).

Exponential moving average:

\[
\hat{\mu}(n) = \lambda \hat{\Omega}(n) + (1 - \lambda) \hat{\mu}(n-1) \quad (24)
\]

The forgetting factor \( 0 < \lambda < 1 \) recursively updates the time-varying mean value.
A linear chirp signal \((r = 100 \text{ Hz/s})\) with \(L = 5\) harmonics, random phases, and identical amplitudes during 0.1 s.

\[
M = 80, \quad \omega_0(1) = \frac{400\pi}{f_s}, \quad f_s = 8.0 \text{ kHz}, \quad \sigma_t = \sqrt{2\pi r/f_s^2}, \quad \text{and for the HMM-based pitch estimator, the frequency range } \omega \in [150, 280] \times (2\pi/f_s) \text{ was discretized into } N_d = 1000 \text{ samples.}
\]
Numerical Results

Real signal

Speech signal + Car noise at SNR = 5 dB.

The MAP order estimation [Djuric 1998], $M = 240$, $\lambda = 0.9$, and $N = 150$. 
Conclusion

- For pitch estimation, we have formulated the ML estimate from the UFE.
- For pitch estimation and tracking, we have proposed HMM- and KF-based methods.
- Experimental results showed that both HMM- and KF-based methods outperform the corresponding ML pitch estimators.
- The KF-based method statistically performs better than the HMM-based method, while the it tracks pitch changes more accurate than the KF-based method.
Thank you!