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A Fast Tool for Assessing the Power Performance of Large WEC arrays

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Abstract: In the present work, a tool for computing wave energy converter array hydrodynamic forces and power performance is developed. The tool leads to a significant reduction on computation time compared with standard boundary element method based codes while keeping similar levels of accuracy. This makes it suitable for array layout optimization, where large numbers of simulations are required. Furthermore, the tool is developed within an open-source environment such as Python 2.7 so that it is fully accessible to anyone willing to make use of it.

1. INTRODUCTION

A successful industrialization of wave energy converters (WECs) will require them to be deployed in arrays. Therefore, nowadays, R&D aims to achieve a better understanding on how and how much WECs interact each other.

Interactions among WECs arise from the action of water waves and so they are usually called hydrodynamic interactions. Briefly, incident waves cause WECs' motion (excitation) while WECs generate disturbances around them by scattering of the incident wave (scattered wave) and generating (radiating) waves upon their own motion (radiated waves).

The Boundary Element Method (BEM) has been extensively used to compute hydrodynamic forces among WECs. This method has the advantage of being able to cope with a 3D formulation of the hydrodynamic problem, which allows the calculation for any WEC geometry. However, it becomes too computationally expensive when dealing with many interacting WECs. On the other hand, Kagemoto and Yue [1] came up with a direct matrix method approach, comparable to BEM codes in terms of capabilities and accuracy on the results and which significantly reduces the computation time. This approach has been successfully implemented in [2] for array layout optimization of cylinders, in [3] to develop an analytical method for solving the hydrodynamic problem of an array of cylinders and in [4] where a combination with conventional BEM codes was presented for solving the hydrodynamic problem for any WEC geometry.

The present work is focused on developing a tool, based on [6] and [9], capable of producing WEC array hydrodynamic forces within a significantly reduced computation time than standard BEM codes while keeping similar levels of accuracy. Besides, the tool functionality is extended to produce the power performance of WEC arrays as a usual post-process required for WEC array layout optimization. Ultimately, a comparison in computation time for different numbers of WECs between a commercially available BEM code and the developed tool is carried out.

2. METHODOLOGY

First of all, the numerical model used for tool development is presented. Here, a formal definition of the array power performance is given, based on current regular practices in array layout optimization processes. Thereafter, the assumptions and formulation to calculate such performance is detailed.

Secondly, the hydrodynamic model to compute hydrodynamic forces is presented. These were already identified within the previous section as required inputs for array power performance calculation. Therefore, it is hereafter referred as hydrodynamic submodel.

2.1 Numerical Model

A common practice in array layout optimization is to maximize the array power performance, which is usually quantified by means of the so called q—factor:

$$q = \frac{P}{NP^*} \tag{1}$$

Where N is the number of WECs and P^* is the power absorbed by an isolated WEC.

In order to produce q, a numerical model is developed under the assumptions of linearized potential flow and harmonic time variation. Hence, all time dependent quantities $x(t)$ are herein studied in the domain of frequencies $\tilde{X}(\omega)$ through:

$$x(t) = \Re\left(\tilde{X}(\omega)e^{i\omega t}\right) \tag{2}$$

Where $\Re(Z)$ returns the real part of the complex number Z, ω is the angular frequency, t is time and $i = \sqrt{-1}$.

Then, the equation of motion of an array of N interacting WECs with the only external force caused by the action of the power take off (PTO) system can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}_{PTO}\dot{\boldsymbol{x}} + \boldsymbol{K}_{PTO}\boldsymbol{x} = \boldsymbol{F}_{ex} - \boldsymbol{F}_{rad} - \boldsymbol{F}_H \tag{3}$$

Where:

- $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}$
- $\boldsymbol{M} = \begin{pmatrix} \mathfrak{m}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathfrak{m}_N \end{pmatrix}$ and $\mathfrak{m}_j$ is the mass matrix of WEC$_j$.
- $\boldsymbol{C}_{PTO} = \begin{pmatrix} \boldsymbol{c}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \boldsymbol{c}_N \end{pmatrix}$ and $\boldsymbol{c}_j$ is the damping matrix due to the action of the PTO system on WEC$_j$.
- $\boldsymbol{K}_{PTO} = \begin{pmatrix} \boldsymbol{k}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \boldsymbol{k}_N \end{pmatrix}$ and $\boldsymbol{k}_j$ is the stiffness matrix due to the action of the PTO system on WEC$_j$.
- $\boldsymbol{F}_{ex}$ is the excitation force column vector.
- $\boldsymbol{F}_{rad}$ is the radiation force column vector which is usually decomposed into the added mass (proportional to the acceleration) and the radiation damping (proportional to the velocity) so that: $\widetilde{\boldsymbol{F}}_{rad} = (-\omega^2\boldsymbol{M}_{add} + i\omega\boldsymbol{C}_{rad})\widetilde{\boldsymbol{X}}$.
- $\boldsymbol{F}_H$ is the hydrostatic force column vector which is usually rewritten into the hydrostatic stiffness (proportional to the displacement) so that: $\widetilde{\boldsymbol{F}}_H = \boldsymbol{K}_H\widetilde{\boldsymbol{X}}$.

Finally, the mean wave power absorbed by the array of WECs equipped with linear dampers can be expressed as:

$$P = \frac{1}{2}\dot{\boldsymbol{x}}^T\boldsymbol{C}_{PTO}\dot{\boldsymbol{x}} \tag{4}$$

Where the superscript ^{T} indicates matrix transpose.

$\mathfrak{m}_j$, $\boldsymbol{c}_j$, $\boldsymbol{k}_j$ and $\boldsymbol{K}_H$ are known from experiments, analytically or by extrapolation from numerical simulations.

2.2 Hydrodynamic Submodel

Let's consider the local cylindrical coordinate system (r_j, θ_j, z_j), Fig. 1, for each single WEC of the array ($j = 1, \dots, N$) and assume that

$$\tilde{\phi}_j^S(r_j, \theta_j, z_j) = (\boldsymbol{a}_j^S)^T\boldsymbol{f}_j^S \tag{5}$$

is solution of the linear wave problem in such local cylindrical coordinates, i.e. it fulfils the Laplace's equation, the bottom boundary condition and the linearized free surface boundary condition; written as a linear combination of wave modes, $\boldsymbol{f}_j^S$, and amplitude coefficients, $\boldsymbol{a}_j^S$.

Leaving the amplitude coefficients to be determined and taking (5) as the scattered wave field around WEC$_j$, one may write the incident wave field to WEC$_k$ as the contribution from the ambient wave and the waves scattered by the other WECs. To this end, let's assume before that a linear operator $\boldsymbol{T}_{jk}$ exists, which transforms waves scattered by WEC$_j$ into incident waves to WEC$_k$, such as:

$$\boldsymbol{f}_j^S = \boldsymbol{T}_{jk}(L_{jk}, \alpha_{jk})\boldsymbol{f}_k^I \tag{6}$$

Where $\boldsymbol{f}^I$ contains incident wave modes, L_{jk} is the distance between WEC$_j$ and WEC$_k$, and α_{jk} is the azimuth of θ_j relative to θ_k.Then, the incident wave field to WEC$_k$ is:

$$\tilde{\phi}_k^I = (\boldsymbol{a}_k^I)^T\boldsymbol{f}_k^I = \left((\boldsymbol{a}_k^A)^T + \sum_{\substack{j=1 \\ j\neq k}}^{N} (\boldsymbol{a}_j^S)^T\boldsymbol{T}_{jk} \right)\boldsymbol{f}_k^I \tag{7}$$

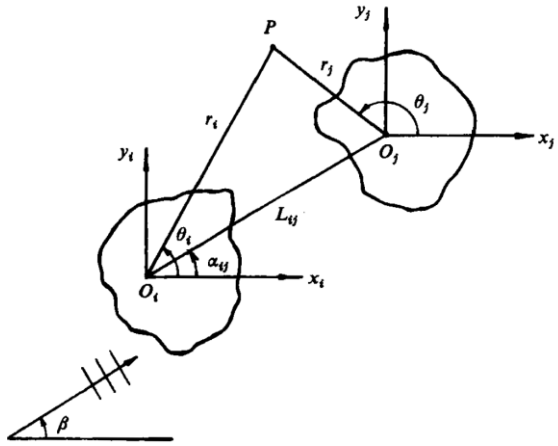

Fig. 1. Illustration of the local coordinate system and the geometrical relationships between two generic wave energy converters within the array. The ambient wave direction (planar wave) is shown as β.

Where $\boldsymbol{a}^I$ and $\boldsymbol{a}^A$ contain respectively incident and ambient wave amplitude coefficients.

From (7) and considering another linear operator, the so called diffraction transfer matrix, $\boldsymbol{B}_k$, which transforms incident wave amplitude coefficients to WEC$_k$ into scattered wave amplitude coefficients of WEC$_k$, it turns out:

$$\boldsymbol{a}_k^S = \boldsymbol{B}_k\left(\boldsymbol{a}_k^A + \sum_{\substack{j=1 \\ j\neq k}}^{N} \boldsymbol{T}_{jk}^T\boldsymbol{a}_j^S \right) \quad k = 1, \dots, N \tag{8}$$

Then, if $\boldsymbol{f}_k^S$, $\boldsymbol{T}_{jk}$, $\boldsymbol{a}_k^A$ and $\boldsymbol{B}_k$ are known, the scattered and radiated amplitude coefficients for each scattering and radiation problem can be calculated through (8). In this

regard, the only difference between solving scattering and radiation problems through (8) is the considered ambient wave. For scattering problems they are plane waves whereas for radiation problems they are radiated waves.

Once the scattered and radiated amplitude coefficients are known, the total scattering, $\tilde{\phi}^S$, and radiation, $\tilde{\phi}^R$, potentials can be calculated as the contribution from the potentials obtained for each WEC through (7).

Finally, taking into account the linearized Bernoulli equation:

$$gz + \frac{p}{\rho} + \frac{\partial \phi}{\partial t} = 0 \tag{9}$$

where g is the gravitational acceleration and ρ is the water density; the pressure field, p, might be calculated on the WECs surface accordingly. Hence, the excitation and radiation forces on WEC_k can be deduced through direct integration of the pressure field throughout its surface, $(S_b)_k$, with unitary normal vector, $\boldsymbol{n}_k$:

$$\left(\tilde{\boldsymbol{F}}_{ex}\right)_k = -i\omega\rho \iint_{(S_b)_k} \tilde{\phi}^S \boldsymbol{n}_k dS \tag{10}$$

$$\left(\tilde{\boldsymbol{F}}_{rad}\right)_k = -i\omega\rho \iint_{(S_b)_k} \tilde{\phi}^R \boldsymbol{n}_k dS \tag{11}$$

Where the hydrostatic term z has been omitted as it is already considered within the equation of motion as $\tilde{\boldsymbol{F}}_H$. However, the radiation potential is well accepted to be expressed as $\tilde{\phi}^R = i\omega \sum_q \tilde{X}_q \tilde{\varphi}_q$, with q the degrees of freedom of the whole array, so that (11) yields:

$$\left(\tilde{\boldsymbol{F}}_{rad}\right)_{kq} = \left(\omega^2 \rho \iint_{(S_b)_k} \tilde{\varphi}_q \boldsymbol{n}_k dS\right) \tilde{X}_q \tag{12}$$

3. RESULTS

The numerical model presented before was implemented in Python 2.7 environment and so it provides an open-source tool.

In order to check the performance of the open-source tool, simulations are carried out with a commercially available BEM code and the developed tool (DT) for increasing N. The computation time and the power performance from both tools are plotted below.

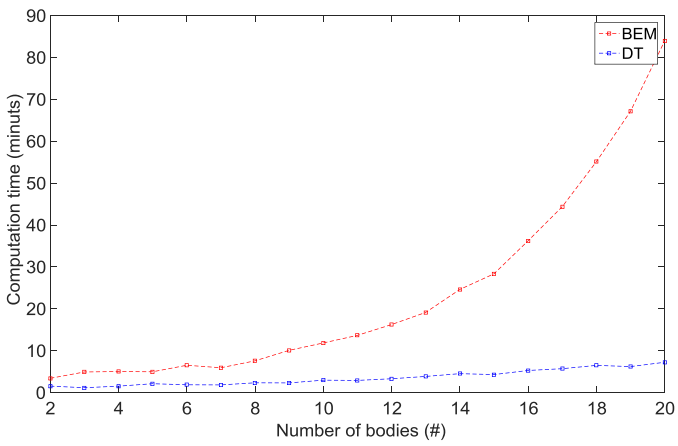

Fig. 2. Comparison of required computation time for increasing number of bodies between a BEM code and the developed tool (DT).

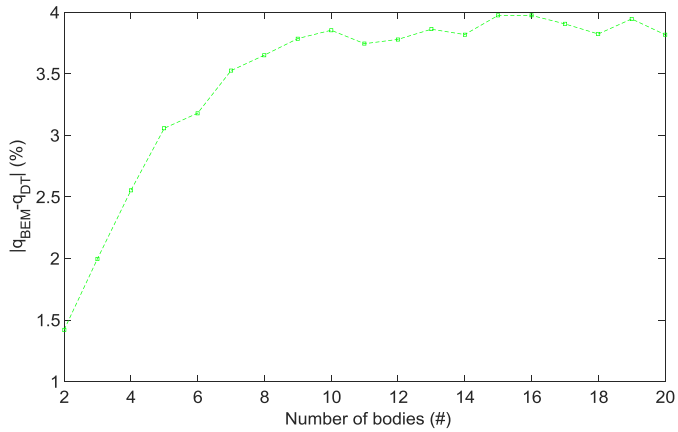

Fig. 3. Absolute difference between the q—factor computed by a BEM code and the developed tool (DT).

4. CONCLUSIONS & DISCUSSIONS

A tool to compute the power performance of any WEC array has been successfully developed. Results show that the computation time is reduced exponentially with increasing N by using DT and compared with the BEM code. Moreover, assuming the BEM code to be more accurate than DT, the absolute error made by using DT appears to be asymptotically limited by 4% in array performance which in turn is very satisfactory.

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