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## CHAPTER 148

### SOME OBSERVATIONS ON THE SUBSET SIMULATION RELATED TO THE WIND TURBINE MECHANICS<sup>1</sup>

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#### ABSTRACT

The subset simulation method is considered to be one of the most powerful methods among the variance reduction Monte Carlo techniques. Potential shortcomings of the method are the bias in its estimations and potential challenges in finding important directions in high dimensional nonlinear problems. The important directions in the  $n$ -dimensional space of the problem are those toward which the failure region extends i.e. by moving in those directions the simulation will fall into the safe domain. It is clear that finding these important directions becomes increasingly difficult as the number of the basic random variables of the problem increases. Moreover when the failure domain of the problem is not a simply connected domain, e.g. failure islands, finding the correct direction, or island, becomes even more difficult. This case occurs frequently in time variant dynamic reliability analysis of nonlinear systems. It is interesting to determine applicability of the Subset Simulation (SS) techniques, as a powerful representative of Variance Reduction Monte Carlo (VRMC) methods, on the wind turbine systems specifically with an active controller. Hence in this paper we apply and discuss these methods on a benchmark wind turbine model and analyze the results in view of their applicability.

#### 1. INTRODUCTION

In order to estimate the return period of the wind turbines (IEC 2005a) it is necessary to estimate the first passage probability, alternatively called failure probability, of these systems. For this aim the IEC standard recommends fitting one of the extreme value distributions, i.e. reversed Weibull or the Gumbel distribution, to the peaks extracted from six epochs of 10min. duration, of the wind turbine data (IEC 2005b).

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Unfortunately the choice of Extreme Value Distribution (EVD) combined with the part of the data used to find EVD parameters result in considerably different extrapolated design values. Alternative to extreme value curve fitting and extrapolation, simulation techniques may be used for estimating these return periods. The natural choice for this purpose is the Standard Monte Carlo (SMC) simulation. The computation cost of this approach is however far beyond reach of the available computers' power even on a modern machine. This is since wind turbines have rather complicated dynamic models which ends up in a high dimensional nonlinear Limit State Function (LSF). The nonlinearities coupled with high dimensions of LSF is enough to make the problem very difficult to solve (Valdebenito, Pradlwarter, & Schueller 2010; Katafygiotis & Zuev 2008).

Therefore a method that can handle these problems with reasonable effort could be of great interest in this area. The Variance Reduction Monte Carlo (VRMC) methods are an alternative choice which are able to produce estimations similar to SMC but with less variance. The Subset Simulation (SS), introduced by Au & Beck (Au & Beck 2001), for estimation of small probabilities of high dimensional systems is reportedly one of the most powerful techniques in the field of structural dynamics. An advantage of SS over the other VRMC methods, such as IS, is its capability in handling high dimensional and complicated problems within reasonable effort (Schueller & Pradlwarter 2007).

Recently two new algorithms have been proposed to increase the efficiency of the subset simulation (Santoso, Phoon, & Quek 2011; Zuev & Katafygiotis 2011). These algorithms tackle the problem by the way they generate the conditional samples of the problem. The first algorithm (Santoso, Phoon, & Quek 2011) aims at increasing the quality of the estimates by decreasing the correlation of the chains which are generated during simulation e.g. Markov Chains. The other algorithm (Zuev & Katafygiotis 2011) improves the performance by delaying the rejection of the generated Markov Chain in the Limit State Function evaluation.

In this article a nonlinear model of a wind turbine according to (Sichani, Nielsen, & Bucher 2011; Sichani, Nielsen, & Naess 2011) is used as a benchmark for analysis of the applicability of these methods. The discussed algorithms are applied to this benchmark problem and the results in terms of their bias and capability of estimating low probabilities are studied.

## 2. SUBSET SIMULATION

Assume that the LSF is defined as  $G(\mathbf{X})$  where  $\mathbf{X}$  consists of the random variables of the problem. The barrier level  $b_j$  which corresponds to a sample of  $\mathbf{X}$ , i.e.  $\mathbf{x}_j$ , is then given by  $b_j = G(\mathbf{x}_j)$ . In view of structural dynamics  $\mathbf{X}$  can be recognized as the stochastic excitation within a given time duration and  $b$  as the maximum of the magnitude of the response to the given excitation.

The strategy of the SS is to obtain samples of  $b$  which have low probability of occurrence, starting by that which can be accurately estimated with low number of simulations e.g.  $b^{(1)}$ . Next, this barrier level will be increased gradually until the highest(required) barrier level or probability is estimated with desired accuracy. This is done by defining intermediate probability levels  $p_f = p_f^{(m)} < p_f^{(m-1)} < \dots < p_f^{(1)}$  corresponding to the intermediate barrier levels  $b = b^{(m)} > b^{(m-1)} > \dots > b^{(1)}$ . Using this property taken from the fact that the first passage probability cannot increase as the barrier level increases, the required first passage probability pf can be written as

$$\text{pf}(b^{(2)}|b^{(1)}) =$$

$$p_f(b^{(2)}|b^{(1)}) = \frac{p_f(b^{(2)} \cap b^{(1)})}{p_f(b^{(1)})} = \frac{p_f(b^{(2)})}{p_f(b^{(1)})} \quad (1)$$

$p_f(b^{(2)}|b^{(1)})$  is the conditional probability of exceeding  $b^{(2)}$  on the condition that  $b^{(1)}$  is exceeded.

Using Equation (1) the final first passage probability, i.e. the lowest first passage probability required, may be written as the following product

$$p_f(b) = p_f(b^{(1)}) \prod_{i=1}^{m-1} p_f(b^{(i+1)}|b^{(i)}) \quad (2)$$

The SS method aims at estimating each of the  $m$  terms on the right hand side of Equation (2) by some type of Monte Carlo simulation. Therefore, it is beneficial to let the barrier level be chosen after simulation of each stage is performed and fix the intermediate first passage probabilities associated with them. All the terms in the product are chosen large enough so that they can be estimated with low number of samples i.e.  $p_0 = 0.1$  in conjunction with Equation (3).

$$\left. \begin{aligned} p_f(b^{(1)}) &= p_0 \\ p_f(b^{(i+1)}|b^{(i)}) &= p_0 \quad , \quad i = 1, \dots, m-1 \end{aligned} \right\} \quad (3)$$

$b^{(1)}$  with its probability  $p_f(b^{(1)}) = p_0$ , is determined by performing SMC with low number of samples, i.e.  $N_{sim} = 500$ , as the  $(p_0 \times N_{sim})^{\text{th}}$  barrier level when all simulated barrier levels are sorted in descending order. The conditional probability terms on the right hand side of Equation (1) cannot be estimated by SMC and need a technique which is capable of generating samples conditioned on the previous samples. For this reason those samples of  $\mathbf{X}^{(1)}$  which correspond to the barrier levels higher than  $b^{(1)}$ , the so-called seeds, are saved for simulating next generation of the excitation. This consists of  $N_{seed} = p_0 \times N_{sim}$  seeds to be kept in memory. In the next section the original Metropolis-Hastings algorithm is described which is the basis for the conditional sampling.

In the following sections the proposed modifications are explained with emphasis on their difference with the original algorithm.

### 2.1 Metropolis-Hastings algorithm

Let  $x(k)$  denote the samples of a discrete stochastic load process  $X(k)$  at the instants of time  $k = 1, \dots, N$  - in the reliability analysis these refer to the basic random variables used to generate the loads -. The random variables  $X(k)$  are assumed to be mutual independent and identical distributed with the Probability Density Function(PDF)  $\pi(x(k))$ .  $X(k)$  is assembled in the random vector  $\mathbf{X}$  with the Joint Probability Density Function(JPDF)  $\pi(\mathbf{x})$ . Due to the iid components  $\pi(\mathbf{x})$  becomes as indicated in equation (4). It should be noted that in our context the random variables  $X(k)$  refer to neither the time samples of the turbulence process nor the response of the wind turbine.

These are the basic random variables which will be transformed into the turbulence process (Sichani 2011). Correspondingly the samples are stored in the vector  $\mathbf{x}$ . Let  $\mathbf{X}^{(i)}$  and  $\mathbf{X}^{(i+1)}$  denote stochastic vectors representing (or transformed into) the load process, when changing from barrier level  $b^{(i)}$  to barrier level  $b^{(i+1)}$ . Both of these vectors are identical distributed, but not independent. The transition kernel, or alternatively called the *proposal distribution*, that  $\mathbf{X}^{(i)} = \mathbf{x}$  moves to  $\mathbf{X}^{(i+1)}$  is shown by

$p(\mathbf{X}^{(i+1)}|\mathbf{X}^{(i)})$ . Due to the independence and identical distribution of the components within  $\mathbf{X}^{(i+1)}$  and  $\mathbf{X}^{(i)}$ , respectively, this may be written as

$$\left. \begin{aligned} \pi(\mathbf{x}^{(i)}) &= \prod_{k=1}^N \pi(x^{(i)}(k)) \\ p(\mathbf{x}^{(i+1)}|\mathbf{x}^{(i)}) &= \prod_{k=1}^N p(x^{(i+1)}(k)|x^{(i)}(k)) \end{aligned} \right\} \quad (4)$$

where  $\pi(\cdot)$  and  $p(\cdot)$  are the one dimensional PDFs of the discrete components  $X(k)$ .

Consider “ $N_{sim}$ ” samples  $\{\mathbf{x}_1^{(i)}, \dots, \mathbf{x}_{N_{sim}}^{(i)}\}$  of  $\mathbf{X}^{(i)}$  from the  $i^{\text{th}}$  simulation level. The transition kernel that  $\mathbf{X}^{(i)} = \mathbf{x}_j^{(i)}$  moves to a state in  $\mathbf{X}^{(i+1)}$ , e.g.  $p(\mathbf{X}^{(i+1)}|\mathbf{X}^{(i)} = \mathbf{x}_j^{(i)})$ , can be chosen with the mean value  $\mathbf{x}_j^{(i)}$ , but can otherwise be arbitrarily chosen (Santoso, Phoon, & Quek 2011). For instance a uniform or Gaussian distribution with an arbitrary standard deviation, such as the sample standard deviation of the seeds (Au, Cao, & Wang 2010), and its mean value at the value of  $\mathbf{x}_j^{(i)}(k)$  for  $k = 1, \dots, N$ . Initially a candidate  $\xi_j$  for  $\mathbf{x}_j^{(i+1)}$ ,  $j = 1, \dots, N_{sim}$  is drawn from  $p(\cdot|\mathbf{x}_j^{(i)})$ .

In order to ensure that samples of  $\mathbf{X}_j^{(i+1)}$  generated by MH will also be distributed with distribution  $\pi(\cdot)$ , it is necessary that the so-called “reversibility condition”, which states that  $\pi(\xi_j) p(\mathbf{x}_j^{(i)}|\xi) = \pi(\mathbf{x}_j^{(i)}) p(\xi|\mathbf{x}_j^{(i)})$ , be satisfied for all samples of  $\mathbf{x}_j^{(i)}$  and  $\xi_j$  (Chib & Greenberg 1995; Santoso, Phoon, & Quek 2011). For this reason Equation (5) is used as the probability of accepting candidate samples.

$$a(\mathbf{x}_j^{(i)}, \xi_j) = \min \left\{ 1, \frac{\pi(\xi_j) p(\mathbf{x}_j^{(i)}|\xi_j)}{\pi(\mathbf{x}_j^{(i)}) p(\xi_j|\mathbf{x}_j^{(i)})} \right\} \quad (5)$$

Next  $\xi_j$  is accepted as the next sample, e.g.  $\mathbf{x}_j^{(i+1)} = \xi_j$  with probability

$$\mathbf{x}_j^{(i+1)} = \begin{cases} \xi_j & \text{w.p. } a(\mathbf{x}_j^{(i)}, \xi_j) \\ \mathbf{x}_j^{(i)} & \text{w.p. } 1 - a(\mathbf{x}_j^{(i)}, \xi_j) \end{cases} \quad (6)$$

where the term w.p. means “with probability”. Therefore, after generation of a candidate sample  $\xi_j$  a random number is drawn from a uniform distribution between 0 and 1 e.g.  $U(0, 1)$ . If this number is less than  $a(\mathbf{x}_j^{(i)}, \xi_j)$  of equation (5),  $\xi_j$  will be accepted as the next sample; else will be rejected and replaced by the seed  $\mathbf{x}_j^{(i)}$ .

This procedure guarantees that the distribution of the samples will not be changed as barrier levels increase (Hoff 2009). In case the proposal distribution is chosen to be symmetric, i.e.  $p(\mathbf{B}|\mathbf{A}) = p(\mathbf{A}|\mathbf{B})$ , it is called the *random walk Metropolis Hastings* and equation (5) reduces to

$$a(\mathbf{x}_j^{(i)}, \xi_j) = \min \left\{ 1, \frac{\pi(\xi_j)}{\pi(\mathbf{x}_j^{(i)})} \right\} \quad (7)$$

## 2.2 Conditional probability estimation

The method follows the procedure described in section 2 that started by a SMC and defining the first barrier level  $b^{(1)}$ . In the next step(s)  $N_{sim}$  candidate samples for  $\mathbf{x}_j^{(i+1)}$ ,  $j = 1, \dots, N_{sim}$  will be generated using a conditional sampler, i.e. MH. The next generation of excitations is conditioned on a randomly chosen seed of the previous simulation. If  $\xi_j$  is accepted according to equation (6) using either equation (5) or equation (7), the second accept/reject test will be performed as

$$\mathbf{x}_j^{(i+1)} = \begin{cases} \xi_j & \text{if } \xi_j \in \mathfrak{F}^{(i)} \\ \mathbf{x}_j^{(i)} & \text{if } \xi_j \notin \mathfrak{F}^{(i)} \end{cases} \quad (8)$$

where  $F^{(i)}$  denotes the failure domain of the  $i^{\text{th}}$  level e.g.  $F^{(i)} = \{ \xi_j | G(\xi_j) > b^{(i)} \}$ . Equation (8) means that  $\xi_j$  is accepted (after being accepted in the accept/reject test of the MH) only if it increases the barrier level to higher than  $b^{(i)}$ , else is rejected and replaced with its seed. This step provides the estimation for the conditional terms in equation (2) and will be repeated  $m-1$  times, c.f. Equation (2). The same strategy that was described in section 2 for choosing barrier levels and seeds will be used in all  $m-1$  stages of the simulation.

This result in

$$p_f^i = \frac{p_0^{i-1}}{N_{sim}} \sum_{j=1}^{N_{sim}} I_{\mathfrak{F}^{(i)}}(\mathbf{x}_j^{(i)}) \quad , \quad i = 1, \dots, m \quad (9)$$

$p_f^i$  represents the minimum failure/first passage probability calculated in the  $i^{\text{th}}$  step of the simulation.  $p_0^{i-1}$  means  $p_0$  raised to power “ $i-1$ ”.  $I_{\mathfrak{F}^{(i)}}(\mathbf{x}_j^{(i)})$  is the indicator function which will be one if the response to  $\mathbf{x}_j^{(i)}$  lies in the  $i^{\text{th}}$  intermediate failure domain and is zero otherwise.

### 2.3 Modified Metropolis-Hastings algorithm

The MH algorithm as presented in section 2.1 breaks down in high dimensional problems. This is since the probability of moving from  $\mathbf{x}^{(i)}$  to  $\mathbf{x}_j$ , defined as Equation (5), decreases exponentially as the number of basic variables - dimension of the problem - increases (Au & Beck 2001). Therefore, Markov chains do not move so frequently from their current state to the next state and get stocked where they are. This problem can be solved by taking advantage of independency between candidate coordinates (components) and breaking the  $N$ -dimensional JPDFs  $\pi(\mathbf{x}_j^{(i)})$  and  $p(\cdot | \mathbf{x}_j^{(i)})$  into their corresponding  $N$  independent one dimensional PDFs  $\pi(x_j^{(i)}(k))$  and  $p(\cdot | x_j^{(i)}(k))$  respectively. Accordingly probability of accepting the next state for each sample is defined independently as

$$a(x_j^{(i)}(k), \xi_j(k)) = \min \left\{ 1, \frac{\pi(\xi_j(k)) p(x_j^{(i)}(k) | \xi_j(k))}{\pi(x_j^{(i)}(k)) p(\xi_j(k) | x_j^{(i)}(k))} \right\} \quad (10)$$

which in case of symmetric proposal distribution  $p(B|A) = p(A|B)$  reduces to

$$a(x_j^{(i)}(k), \xi_j(k)) = \min \left\{ 1, \frac{\pi(\xi_j(k))}{\pi(x_j^{(i)}(k))} \right\} \quad (11)$$

Next, the accept/reject test will be performed for each component of each realization. So for  $k = 1, \dots, N$

$$x_j^{(i+1)}(k) = \begin{cases} \xi_j(k) & \text{w.p. } a(x_j^{(i)}(k), \xi_j(k)) \\ x_j^{(i)}(k) & \text{w.p. } 1 - a(x_j^{(i)}(k), \xi_j(k)) \end{cases} \quad (12)$$

This process will be repeated for  $j = 1, \dots, N_{sim}$  times to generate the next set of excitations i.e.  $\mathbf{X}^{(i+1)} = \{\mathbf{x}^{(i+1)}_1, \dots, \mathbf{x}^{(i+1)}_{N_{sim}}\}$ . This modification is proposed in (Au & Beck 2001) and is called the *Modified Metropolis-Hastings* (MMH). Hereafter when this sampling scheme is used for the SS, it is invoked by the term SS-MMH.

### 2.4 Modified Metropolis Hastings with Reduced Chain Correlation

The Modified Metropolis-Hastings with Reduced chain Correlation (MMHRC) is recently proposed by (Santoso, Phoon, & Quek 2011) which aims in reducing the correlation between the Markov chains in the MMH. In view of the sample generation MMHRC follows the original MH algorithm based on Equation (5), Equation (6) and Equation (7), i.e. the N-dimensional JPDF is used. However every time the generated  $\underline{j}$  is rejected according to Equation (6), a new sample is generated conditioned on the same seed. This process is repeated as many times as needed to let the generated candidate be accepted by Equation (6). Clearly this modification takes more time for sample generation compared to MMH. SS-MMHRC shows good performance for low to medium dimensional problems i.e.  $N \leq 100$  (Santoso, Phoon, & Quek 2011).

However, on the numerical simulation performed in this study, see section 3, the Markov chains generated by MMHRC have high tendency to stay in the initial state, i.e.  $a(\mathbf{x}^{(i)}, \xi_j) = 0$ . This means that the barrier level is rarely increased. This is due to the high dimensions of the problem which is the same problem that causes breaking down of the original MH algorithm in high dimensions (Au & Beck 2001).

### 2.5 Modified Metropolis Hastings with Delayed Rejection

Following the idea of (Tierney & Mira 1999) the so-called Modified Metropolis-Hastings with Delayed Rejection (MMHDR) is proposed by (Zuev & Katafygiotis 2011). Here the MMH approach is followed for generation of the conditional samples. Although in case a candidate sample does not belong to the failure region, i.e.  $\xi_j \notin \delta^{(i)}$  in Equation (8), it will not be rejected and will be given a second chance. In such a case the components of the candidate  $\xi_j$ , e.g.  $\xi_j(k)$ , are divided into two non-overlapping sets. Set  $T = \{k | x_j^{(i)}(k) = \xi_j(k)\}$  which consists of the set of coordinates which have evolved to new states; and its complementary set  $\bar{T}$  which includes the rest of the components.

Next, the coordinates which belong to  $T$  will be given another chance to evolve to a new state  $\xi^{(2)}$ . The proposal density of moving to  $\xi^{(2)}(k)$ , e.g.  $q(\cdot | x_j^{(i)}(k), \xi_j(k))$ , can in general be chosen different from the proposal density of moving from  $x_j^{(i)}(k)$  to  $\xi_j(k)$ , e.g.  $p(\cdot | x_j^{(i)}(k))$ . It should be noted that the candidate samples for  $\xi^{(2)}$  are again generated around the original seed  $x_j^{(i)}(k)$ , and not  $\xi_j(k)$  i.e.  $q(\cdot | x_j^{(i)}(k), \xi_j(k)) = q(\cdot | x_j^{(i)}(k))$  (Zuev & Katafygiotis 2011).

The probability of accepting the new samples conditioned on the two previous samples is defined as (Tierney & Mira 1999)

$$a^*(x_j^{(i)}(k), \xi_j(k), \xi_j^{(2)}(k)) = \left. \begin{aligned} & \min \left\{ 1, \frac{\pi(\xi_j^{(2)}(k)) p(\xi_j(k) | \xi_j^{(2)}(k))}{\pi(x_j^{(i)}(k)) p(\xi_j(k) | x_j^{(i)}(k))} \times \right. \\ & \left. \frac{q(x_j^{(i)}(k) | \xi_j^{(2)}(k), \xi_j(k)) a(\xi_j^{(2)}(k), \xi_j(k))}{q(\xi_j^{(2)}(k) | x_j^{(i)}(k), \xi_j(k)) a(x_j^{(i)}(k), \xi_j(k))} \right\} \end{aligned} \right\} \quad (13)$$

where  $a(\xi_j(k), \xi_j^{(2)}(k))$  determines the probability of moving from  $\xi_j(k)$  to  $\xi_j^{(2)}(k)$  in the same manner as defined in Equation (10). In case that both transition kernels are chosen symmetry Equation (13) reduces to

$$a^*(x_j^{(i)}(k), \xi_j(k), \xi_j^{(2)}(k)) = \min \left\{ 1, \frac{p(\xi_j(k)|\xi_j^{(2)}(k)) \min \{ \pi(\xi_j^{(2)}(k)), \pi(\xi_j(k)) \}}{p(\xi_j(k)|x_j^{(i)}(k)) \min \{ \pi(x_j^{(i)}(k)), \pi(\xi_j(k)) \}} \right\} \quad (14)$$

where in Equation (14) the equality  $a \times \min \{1, b/a\} = b \times \min \{1, a/b\}$  is used which is true for any positive pair  $\{a, b\}$ .

### 3. SUBSET SIMULATION ON WIND TURBINE

The time duration for simulation is chosen 800[sec] where the first 200s are discarded to take into account the effect of the transient phase of the system response. The rest simulates a 10min. interval which is prescribed in design codes for extraction of probabilistic behavior of the turbines (IEC 2005b). The resolution of the time integrator is set to  $\Delta t = 0.2s$ . Turbulent wind field is simulated on 31 nodes, one on the hub and others at  $0.8L$  radial distance from hub on an equidistance angular grid.

The mean wind is set to  $V_r = 15[m/s]$  and the cut-in and cut-out speeds are set to  $V_i = 5[m/s]$  and  $V_o = 25[m/s]$  respectively. The limit state function is defined as the first passage of the magnitude of the tower displacement from the threshold (barrier) level  $b$  i.e.  $p_f(b) = \text{Prob} \{ \max_{t \in [0, T]} |z_4(t)| \geq b \}$  with  $T = 600[s]$  of simulation. Discarding the transient simulation time, the LSF is defined as a function of 93000 stochastic variables. Failure probability of the model is estimated by SS compared to the SMC with  $4.95 \times 10^5$  samples.

A practical issue is the very high number of the basic random variables, e.g. the iid Gaussian random numbers which will pass through the turbulence filter, needed to be stored in the memory for the next stage of the simulation. These consists of seeds for two consecutive simulation levels which contains  $2N_{seed}$  sets of basic random variables requiring approximately 12MB of disc space for only one simulation. Therefore a simulation with 500 initial samples and  $p_0 = 0.1$  requires approximately 1.2GB memory (or disc space) to save  $2N_{seed} = 100$  seeds for two simulation levels. The proposal distributions are chosen uniform centered at the sample seed with spread equal to 2 times standard deviation of the seeds of the previous level.

Figures 1a and 1b show estimates of the first passage probability of the fixed speed wind turbine with SS-MMH and SSMMHDR respectively. In both figures number of samples is  $N_{sim} = 500$  and probability increment is set to  $p_0 = 0.1$ . Each figure shows 10 estimates of the first passage probability with SS together with the SMC results. The thick solid lines the figures show the SMC simulation results. The figures show that both methods are successful in increasing barrier levels and their estimates are close to that of the SMC. However SS-MMHDR results suffer from small over estimation of the first passage probability at high barrier levels.

Figures 2a and 2b show the estimates of the first passage probability of the variable speed wind turbine. The figures show that presence of the controller has considerable effect on first passage probability estimation. The controller not only changes the range of barrier levels but also makes the estimation of the first passage



probability a more difficult task. As seen in figures 2a and 2b both methods have rather poor estimates of the first passage probabilities of order  $10^{-7}$  for the variable speed wind turbine case. For the fixed speed wind turbine the value  $p_0 = 0.1$  is shown to be a good choice and both methods overcome the difficulties faced by high dimensions of the model. However the variable speed model has difficulties in estimating very low probabilities.

#### 4. CONCLUSIONS

The low first passage probability of a reduced order wind turbine model is estimated based on the Markov Chain Monte Carlo. A well-known method for this aim, e.g. SS-MMH,

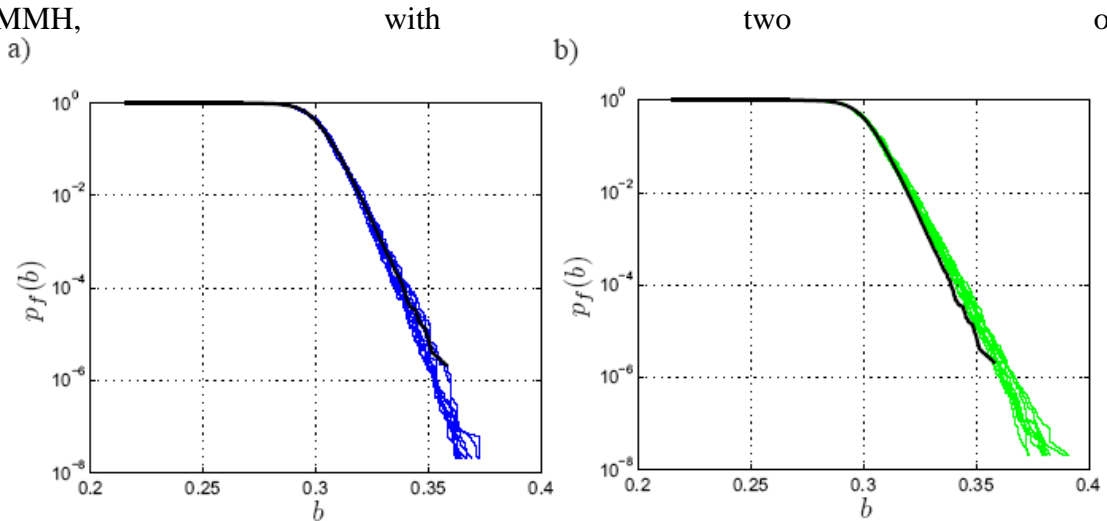


Figure 1: First passage probability estimation of the fixed speed wind turbine; 6 stages with  $p_0 = 0.1$ , a) Subset simulation with MMH sampling scheme b) Subset simulation with MMHDR sampling scheme

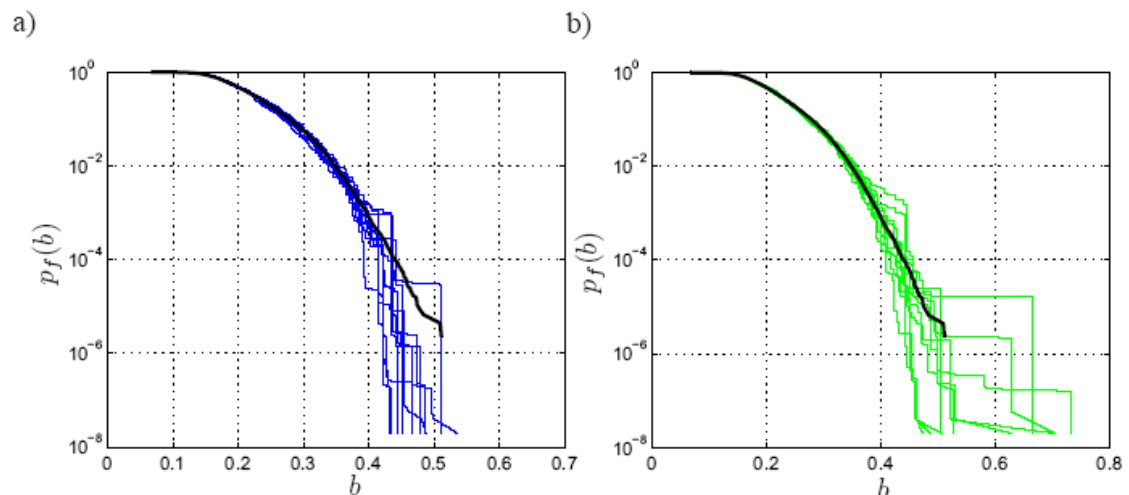


Figure 2: First passage probability estimation of the variable speed wind turbine; 6 stages with  $p_0 = 0.1$ , a) Subset simulation with MMH sampling scheme b) Subset simulation with MMHDR sampling scheme

the most recent modifications to the original algorithm have been implemented and compared to the original method on the wind turbine model. The estimated first

passage probability of the fixed speed wind turbine with SS-MMH is in good agreement with SMC. On the other hand SSMMHDR results have small over estimations in their predictions. Nevertheless estimations of the first passage probability of the variable speed wind turbine is more difficult. The results show that in high dimensions the chains constructed by SS algorithms do not move to the next state often, and tend to stay in their initial state.

In most nonlinear problems the failure domain of the problem is not a continuous region but a set of the so-called failure islands surrounded by the safe domain. Therefore scaling up an excitation which belongs to the failure domain, may end in the safe domain. In case these islands are not small and distant from each other, i.e. the fixed speed model, Markov chains have a fair chance to increase the barrier level as they evolve. However finding the directions toward which (the islands in which) the barrier level increases becomes increasingly difficult as the islands shrink or their distance increases. This seems to be the case in the variable speed wind turbine model. The aforementioned reasons seem to be responsible for deterioration of the performance of the method in estimating the small failure probabilities of the variable speed model.

Results of this study show that subset simulation will suffer from presence of too many basic random variables in dynamic reliability analysis of wind turbines. Hence further research should take direction toward decreasing the number of the basic random variables to as few as possible. This is possible by using reduction schemes such as Karhunen-Loève expansion or stochastic harmonic functions. Reducing the number of basic random variables however may affect the shape of the LSF of the problem and make it more complicated. These changes may be more design points, irregularities or discontinuities. It is already suspicious that the wind turbine model has discontinuous LSF; so reduction of the number of random variables may make it worse.

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