ABSTRACT
In this paper the assessment of reliability profiles for reinforced concrete bridges exposed to chloride attack is discussed. Three stochastic models for chloride induced corrosion of reinforced concrete slabs are presented. The service lifetime of a reinforced concrete bridge is defined as initiation of corrosion of the reinforcement. On these assumptions the stochastic service lifetime of an existing structure is estimated. An example on the assessment of the lifetime of an existing bridge including sensitivity analysis is included.

1. INTRODUCTION
Corrosion of the reinforcement is the main reason for deterioration of concrete bridges in many countries. Therefore, modelling of the corrosion process is an important aspect of the estimation of the service life time. Service life time is in this paper defined as the initiation time of corrosion of the reinforcement. The assessment of the reliability profile and the service lifetime of an existing bridge are shown.

2. CORROSION MODELLING
In this paper only one deterioration mechanism is considered namely chloride induced corrosion of the reinforcement. When concrete is exposed to chloride it has become normal practice to describe the response of the concrete to the chloride exposure by its chloride profile, i.e. the distribution of the chloride content of the concrete in its near-
to-surface layer or by the concentration-distance curve.

Estimation of the chloride profile is a very uncertain matter since it is controlled by a number of factors which are difficult to model. The controlling parameters with regard to the corrosion initiation time are the initial chloride content \( C_i \), the chloride content at the surface \( C_0 \), and the chloride diffusion coefficient \( D_c \). After corrosion has been initiated then the controlling parameter is the rate of corrosion \( i_{corr} \).

Corrosion of the reinforcement is supposed to take place when the chloride concentration at the site of the reinforcement reaches a critical level \( C_{cr} \). The corrosion due to chloride ingress will usually be pitting corrosion, which is a localised corrosion of the reinforcement. However, pitting corrosion is very difficult to model. For structures where ductility is needed the initiation stage of corrosion can be taken as the service lifetime.

For a reinforced concrete slab bridge pitting corrosion of a single rebar or a few rebars will not drastically change the ductility due to the "parallel" behaviour of the rebars. Therefore, it is considered acceptable to model the corrosion as a uniform corrosion of the rebars to avoid the difficult task of including pitting corrosion.

The rate of chloride penetration into concrete is often modelled by Fick's law of diffusion

\[
\frac{\partial c(x,t)}{\partial t} = D_c \frac{\partial^2 c(x,t)}{\partial x^2}
\]

where \( D_c \) is the chloride diffusion coefficient, \( x \) is the distance from the surface and \( t \) is the time. The solution of the equation (2.1) is

\[
C(x,t) = C_0 \left[ 1 - \text{erf} \left( \frac{x}{2 \sqrt{D_c t}} \right) \right]
\]

where \( C(x,t) \) is the chloride content in the distance \( x \) from the surface and at time \( t \). \( C_0 \) is the chloride content on the surface. The corrosion initiation period can then be estimated as

\[
T_i = \frac{(d_i - D_i/2)^2}{4D_c} \left( \text{erf}^{-1} \left( \frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^2
\]

where \( C_i \) is the initial chloride concentration, \( C_{cr} \) is the critical chloride concentration, and \( d_i - D_i/2 \) is the concrete cover.

The diameter \( D_i(t) \) of the reinforcement bars at time \( t \) after initiation of corrosion can as a first approximation be modelled by

\[
D_i(t) = D_1 - C_{corr} i_{corr} t
\]

where \( D_1 \) is the initial diameter, \( C_{corr} \) is a corrosion coefficient, and \( i_{corr} \) is the rate of corrosion.

Based on a literature survey the following modelling for chloride penetration is proposed for wet areas (the initial chloride concentration is assumed to be zero):
General model:

- Diffusion coefficient $D_C : N(30.0, 5.0)$ [mm$^2$/year]
- Chloride conc., surface $C_0 : N(0.65, 0.075)$ [%]
- Corrosion density $i_{corr} : \text{Uniform}[3.0, 4.0]$ [$\mu$A/cm$^2$]

Simulation of realisations of the general model is shown in Figure 1. Observe the very wide spreading of the realisations.

![Figure 1: Normalised reinforcement area $A/A_0$ as function of time.](image)

(Cover on reinforcement is set to $x_d : N(40.0, 4.0)$ [mm]).

Based on this general deterioration model three levels of deterioration are proposed: low deterioration, medium deterioration and high deterioration.

**Low deterioration:**
- Diffusion coefficient $D_C : N(25.0, 5.0)$ [mm$^2$/year]
- Chloride conc., surface $C_0 : N(0.575, 0.038)$ [%]
- Corrosion density $i_{corr} : \text{Uniform}[2.0, 3.0]$ [$\mu$A/cm$^2$]

**Medium deterioration:**
- Diffusion coefficient $D_C : N(30.0, 5.0)$ [mm$^2$/year]
- Chloride conc., surface $C_0 : N(0.650, 0.038)$ [%]
- Corrosion density $i_{corr} : \text{Uniform}[3.0, 4.0]$ [$\mu$A/cm$^2$]

**High deterioration:**
- Diffusion coefficient $D_C : N(35.0, 5.0)$ [mm$^2$/year]
- Chloride conc., surface $C_0 : N(0.725, 0.038)$ [%]
- Corrosion density $i_{corr} : \text{Uniform}[4.0, 5.0]$ [$\mu$A/cm$^2$]

Realisations of these three models are shown in figure 2.
3. RELIABILITY PROFILES

3.1 Modelling of Failure

Only ultimate limit states are used reliability analysis namely bending failure and shear failure. Two modes of bending failure are considered namely compression failure in concrete and yielding failure in the reinforcement. Compression failure is the dominating failure mode for overreinforced concrete sections whereas yielding failure is dominating for underreinforced concrete sections.

The following safety margin is used for yielding (yield line) failure

\[ M_1 = Z_1 E_D - W_D \]  

where \( E_D \) is the energy dissipated in the yield lines (or plastic zones) and \( W_D \) is the work done by the applied load. \( Z_1 \) is a model uncertainty variable related to the calculation of the energy dissipation.

The following safety margin is used for shear failure

\[ M_2 = Z_2 V_{j,\text{ult}} - V_j \]  

where \( V_{j,\text{ult}} \) is the ultimate shear strength at section \( j \), \( V_j \) is the shear force at section \( j \) and \( Z_2 \) is a model uncertainty variable related to ultimate shear strength.

3.2 Reliability modelling

The critical failure modes used in the estimation of the reliability profiles are the critical failure modes identified at time \( t = 0 \). After corrosion is initiated the critical failure may change due to the reduced bending strength. However, in this study homogeneous corrosion is assumed, so it is unlikely that the critical failure modes (yield patterns) will change significantly.

The probability of failure for a slab bridge is estimated as a series system consisting of the two failure modes - one for bending failure and one for shear failure. However, the correlations between these failure modes are usually very close to one so the series system reliability index \( \beta^{sys} \) may be approximated by the smallest of the element reliability indices.
3.3 Software Programs

The computer program COBRAS developed by (C. Middleton 1995) is used for yield-line analysis of slab bridges. Elastic analysis is done using finite element programs (STAAD-III 1995). Reliability analysis of individual components is done using (RELIAB01 1994) and reliability analysis of systems by (RELIAB02 1994). The determination of optimal yield line patterns in COBRAS has been done using (OPTIM01 1994). The deterioration is estimated using (CORROSION 1995).

3.4 Example

An existing UK concrete slab bridge is used for illustration of the reliability profile estimation. The bridge was constructed in 1965-66, the span is 9.86 m and the width is 37.0 m with a skew of 9 degrees, see figure 3. The bridge is designed to carry 45 units of HB loading. The motorway has two separate carriageways. The width of each carriageway is 14.5 m and each has 3 marked lanes + a hard strip. There is a 2 m verge on both sides and 4 m central reserve.

The stochastic variables and their distributions are indicated in table 1. The stochastic variables and their distributions are indicated in table 1.

Table 1. Stochastic variables.

<table>
<thead>
<tr>
<th>No.</th>
<th>Stochastic variable</th>
<th>Distrb. Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Depth of slab</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>Cube strength of concrete</td>
<td>LogNormal</td>
</tr>
<tr>
<td>3</td>
<td>Density of concrete</td>
<td>Normal</td>
</tr>
<tr>
<td>4</td>
<td>Yield strength: long. reinf.</td>
<td>LogNormal</td>
</tr>
<tr>
<td>5</td>
<td>Depth of long. reinforcement</td>
<td>Normal</td>
</tr>
<tr>
<td>6</td>
<td>Yield strength: transv. reinf.</td>
<td>LogNormal</td>
</tr>
<tr>
<td>7</td>
<td>Depth of transv. reinforc.</td>
<td>Normal</td>
</tr>
<tr>
<td>8</td>
<td>Initial area of long. reinforc.</td>
<td>Fixed</td>
</tr>
<tr>
<td>9</td>
<td>Initial area of transv. reinforc.</td>
<td>Fixed</td>
</tr>
<tr>
<td>10</td>
<td>F&amp;N static load factor</td>
<td>Gumbel</td>
</tr>
<tr>
<td>11</td>
<td>F&amp;N dynamic load factor</td>
<td>Normal</td>
</tr>
<tr>
<td>12</td>
<td>Model uncertainty variable</td>
<td>Normal</td>
</tr>
<tr>
<td>13</td>
<td>Carriageway surfacing</td>
<td>Normal</td>
</tr>
<tr>
<td>14</td>
<td>Chloride conc. on surface</td>
<td>Normal</td>
</tr>
<tr>
<td>15</td>
<td>Initial chloride concentration</td>
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<tr>
<td>16</td>
<td>Diffusion coefficient</td>
<td>Normal</td>
</tr>
<tr>
<td>17</td>
<td>Critical chloride concentration</td>
<td>Normal</td>
</tr>
<tr>
<td>18</td>
<td>Corrosion parameter (19)</td>
<td>Uniform</td>
</tr>
</tbody>
</table>

Figure 3. Concrete slab bridge.
Figure 4. Reliability profiles for low, medium, and high deterioration.

3.5 Sensitivity Analysis

Sensitivity analysis is a very useful tool in connection with data collection and bridge management. Sensitivity analysis can be used to decide whether a parameter can be modelled by a deterministic variable or whether a stochastic modelling is needed. It also gives information on the importance of the single parameters with regard to the reliability estimation, so that requirements to the data collection can be defined.

During the estimation of the reliability index the most central point on the failure surface in the standardised Gaussian space called the design point is determined. Let \( \bar{\alpha} = (\alpha_1, \ldots, \alpha_n) \) be the normal unit vector to the failure surface at the design point. The element \( \alpha_i \) is then called the sensitivity factor of stochastic variable \( i \). \( \alpha_i^2 \) is the fraction of the variance of the safety margin that originates from stochastic variable \( X_i \) if the stochastic variables are uncorrelated. If the stochastic variables are mutually dependent then \( \alpha_i^2 \) is only an approximation of the variance.

The reliability elasticity coefficient associated with a parameter \( p \) (e.g. the mean value or the standard deviation of a stochastic variable or a constant in the failure function) is defined by

\[
e_p = \frac{\frac{d \beta}{dp}}{\beta} \tag{7}
\]

It follows from this definition that if the parameter \( p \) is changed 1 % then the reliability index \( \beta \) is changed \( e_p \) %.

The third sensitivity measure is called the omission sensitivity factor. It is introduced by (Madsen 1988). The omission sensitivity factor \( \varsigma_i \) is for a linear failure function and normally distributed stochastic variables defined by

\[
\varsigma_i = \frac{1 - \alpha_i u_i^0}{\sqrt{1 - \alpha_i^2}} \tag{8}
\]
This factor gives the relative importance on the reliability index $\beta$ assuming that the stochastic variable no. $i$ is fixed on the value $u_i^0$, i.e. if it is considered as a deterministic parameter.

3.6 Sensitivity analysis of the bridge in section 3.4

The numbering of the stochastic variables used in the sensitivity analysis is shown in table 1. The failure mode considered is bending failure.

The $\alpha_i^2$-values for the time $t = 0$ years (no deterioration of reinforcement), and for $t = 200$ years (high deterioration) are shown in figure 5.

![Figure 5. $\alpha_i^2$-values at $t = 0$ and $t = 200$ years.](image)

For $t = 0$ years it is noted that the concrete density, the area and the depth of transverse reinforcement have almost no influence on the variance of the safety margin. The concrete density may for this failure mode and for most other failure modes be modelled as a deterministic variable. The reason is the small standard deviation (2 %) for the concrete density. The transverse reinforcement has very little influence. The transverse reinforcement is included in the failure function for bending failure, but the depth makes the contribution to the strength negligible.

The important stochastic variables are the (yield strength of longitudinal reinforcement and the loading. The deterioration variables have of course no influence at $t = 0$ years. The change observed when going to $t = 200$ years is that the deterioration variables become important and that the depth of the reinforcement becomes more significant. At $t = 200$ years the significant stochastic are the deterioration and the depth of the reinforcement becomes dominating since it has a double influence on the strength and on the corrosion of reinforcement.

The changes of $\alpha_i$ values during the analysed time period [0; 200 years] are shown in figure 6. The stochastic variables have been divided into four groups: “strength”, “geometry”, “load” and “deterioration”. The change of $\alpha_i$ is as expected. It is seen that when stochastic variables related to deterioration becomes significant (after the expected initiation of corrosion) then the load variables become less significant. It is also noted that the sign for “depth of rebar” changes when deterioration starts. Before deterioration starts a lower “depth of rebar” will give a more secure bridge (due the larger internal arm). When deterioration has started a larger “depth of rebar” will give a more secure bridge since a larger cover on reinforcement bars will give less expected deterioration of reinforcement bars.
Figure 6. The change in time of $\alpha_j$.
Figure 7 shows the reliability elasticity factors with regard to mean values and standard deviations. The full lines indicate the elasticities at time $t = 0$ years and the dotted lines the elasticities at time $t = 140$ years. These charts correspond well with the alpha-vector analysis. It is noted that the elasticity of the depth of the reinforcement changes sign after deterioration has started (going from a negative elasticity to a positive elasticity coefficient).

Omission sensitivity factors are shown in table 2. The highest omission sensitivity factors at $t = 0$ years are obtained for the yield strength of longitudinal reinforcement, the static load factor, the dynamic load factor, and the model uncertainty variable. The omission sensitivity factor for the yield strength of longitudinal reinforcement is 1.1304 at $t = 0$ years, i.e. the error in the reliability index $\beta$ is approximately 13% by assuming the yield strength of longitudinal reinforcement deterministic. The omission sensitivity factor at $t = 140$ years is increased to 1.3093 so the error by modelling the yield strength deterministic is increased to 31% (assuming the deterministic value is chosen as the mean value).

For the failure mode (bending) considered it can be concluded that the yield strength of the longitudinal reinforcement, the depth of the longitudinal reinforcement, the static load factor, the dynamic load factor, and the model uncertainty variable are the important stochastic variables before the corrosion is initiated. After corrosion is initiated the following significant stochastic variables are added: the critical chloride concentration, and the corrosion parameter.
4. DEFINITION OF SERVICE LIFETIME

The service life time of a reinforced concrete bridge is in this paper defined as the initiation time $T_I$ for corrosion of the reinforcement see (Thoft-Christensen 1997). This is a rational definition from a life-cycle cost of view since repair of corroded reinforced elements is a major contributor to the life-cycle cost. It is relatively inexpensive to repair a structural element by replacing some part of the concrete instead of waiting until corrosion has taken place.

On basis of equation (3) outcomes of the corrosion initiation time $T_I$ has been performed on basis of the following data by simple Monte Carlo simulation (1000 simulations) (CORROSION 1995):

- Initial chloride concentration: 0%
- Surface chloride concentration: Normal (0.65 ;0.038)
- Diffusion coefficient: Normal (30;5)
- Critical concentration: Normal (0.3;0.05)
- Cover: Normal (40;8).

It is shown in (Thoft-Christensen 1997) that a Weibull distribution $W(x; \mu, k, \varepsilon)$, where $\mu = 63.67$, $k=1.81$ and $\varepsilon=4.79$ can be used to approximate the distribution of the simulated data. It follows from these data that there is about 90% probability that corrosion is initiated before 100 years in the considered case. Such a design seems to be unacceptable.

The design corresponding to the data above can be improved in different ways e.g. by increasing the cover $d$ or reducing the diffusion coefficient $D$.

Consider a design serviceability limit of the form

$$M = T_I - T_D$$

(9)
where $T_D$ is the design serviceability life time. The serviceability failure probability is then defined by

$$P_f = P(T_i - T_D \leq 0) \quad (10)$$

The serviceability failure probability $P_f$ as function of $E[d]$ (in mm) and $E[D]$ (in mm$^2$/year) is illustrated in figure 8. As expected $P_f$ decreases with decreasing values of $E[D]$, and decreased with increasing values of $E[d]$.

![Serviceability failure probability](image)

Figure 8. Serviceability failure probability $P_f$ as function of $E[d]$ and $E[D]$.

5. CONCLUSIONS

Corrosion modelling based on Fick’s law of diffusion is used to derive an expression for the corrosion initiation time. Further three models for low, medium, and high deterioration is defined.

Based on these models for deterioration reliability profiles for reinforced concrete slab bridges are derived.

The service life time of a reinforced concrete bridge is defined as the initiation time of corrosion of the reinforcement. Using the diffusion modelling of corrosion simulation data show that the service life time can be modelled by a Weibull distribution. The influence of the cover and the diffusion coefficient on the service life time is illustrated.

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