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Enhancing the Frequency Adaptability of Periodic Current Controllers with a Fixed Sampling Rate for Grid-Connected Power Converters

Yongheng Yang, Member, IEEE, Keliang Zhou, Senior Member, IEEE, and Frede Blaabjerg, Fellow, IEEE

Abstract—Grid-connected power converters should employ advanced current controllers, e.g., Proportional Resonant (PR) and Repetitive Controllers (RC), in order to produce high-quality feed-in currents that are required to be synchronized with the grid. The synchronization is actually to detect the instantaneous grid information (e.g., frequency and phase of the grid voltage) for the current control, which is commonly performed by a Phase-Locked-Loop (PLL) system. Hence, harmonics and deviations in the estimated frequency by the PLL could lead to current tracking performance degradation, especially for the periodic signal controllers (e.g., PR and RC) with a fixed sampling rate. In this paper, the impacts of frequency deviations induced by the PLL and/or the grid disturbances on the selected current controllers are investigated by analyzing the frequency adaptability of these current controllers. Subsequently, strategies to enhance the frequency adaptability of the current controllers are proposed for the power converters to produce high quality feed-in currents even in the presence of grid frequency deviations. Specifically, by feeding back the PLL estimated frequency to update the center frequencies of the resonant controllers and by approximating the fractional delay using a Lagrange interpolating polynomial for the RC, respectively, the frequency-variation-immunity of these periodic current controllers with a fixed sampling rate is improved. Experiments on a single-phase grid-connected system are presented, which have verified the discussions and the effectiveness of the frequency adaptive current controllers.

Index Terms—Frequency adaptive; frequency variations; resonant controller (RES); repetitive controller (RC); fractional order filter; Lagrange interpolating polynomial; grid-connected power converters

I. INTRODUCTION

POWER electronics converters have been widely used in grid-connected renewable energy systems like wind turbine systems and PhotoV voltaic (PV) systems [1]–[3], and increasingly stringent requirements have been imposed on the grid-connected power converters [3], [4]. Due to their non-linearity and also the intermittency, harmonic challenges are also associated with the power electronics interfaced renewable energy systems, which have to be dealt with by employing advanced control strategies according to demands [5], [6]. Commonly, a two-cascaded control system is adopted in the grid-connected power converters [7]. Since the inner current controller of the cascaded loops is responsible for shaping the current (i.e., power quality issues), great efforts have been devoted to the control of the feed-in grid current, which is also required to be synchronized with the grid voltage. Phase Locked Loop (PLL) systems are widely used in the grid-connected inverters for synchronization [8]–[15]. Hence, the information (especially the grid frequency) provided by a PLL system is of importance for the current controllers, and it is extensively used at different levels of the entire control system as well as for condition monitoring [7], [16], [17]. For instance, the grid voltage phase estimated by a PLL system has been widely used for the reference frame transformation in [7]; while in [17], a PLL synchronization algorithm has been employed to detect the grid voltage fault for low voltage ride-through operation.

In regards to the current control loop, typically, it can be implemented in the rotating reference frame ($dq$), the stationary reference frame ($\alpha\beta$), or the three-phase natural reference frame ($abc$) [7], [16], [18]. Taking the control in the $dq$-frame for an example, Park and/or Clarke transforms enable the employment of Proportional Integrator (PI) controllers, where the PLL estimated grid frequency is a must for the transforms as aforementioned. Consequently, either frequency variations in the grid itself or the frequency estimation error by a PLL system will result in the control degradation when using PI controllers. On the other hand, in order to simplify the control, periodic signal controllers like Repetitive Controller (RC) [19]–[25] and Proportional Resonant (PR) controller with parallel RESonant (RES) based harmonic compensators [7], [22], [26]–[28] are developed in either the $\alpha\beta$- or the $abc$-frame. It has been demonstrated that the periodic RES controller offers a selective harmonic control scheme, while the periodic RC compensator enables the mitigation of a wide range of harmonics [7], [16], [29]. In those cases, the control accuracy of both the PR with RES or RC controllers is inevitably affected by the designed center frequency of the resonant controller [20], [26]. Basically, the center frequency (e.g., the fundamental frequency ~ 50 Hz) should be placed at which the control gain can approach infinite, and a constant value is selected for the sake of simplicity. As a consequence, any frequency deviation (either induced by the grid frequency changes and/or the PLL estimation errors) will inevitably lead to a finite control gain at the resonant frequencies of the periodic current controllers.
In order to enhance the control performance of such periodic signal controllers, frequency adaptive schemes were developed in the prior-art work [20], [21], [29]–[39]. In general, those solutions are accomplished by taking the PLL estimated frequency as a feedback of the controllers, and thus the center frequency can be updated online. For instance, in [28], the center frequencies of the RES controller are continuously adjusted according to the PLL estimated frequency. In contrast, since the RC compensator is implemented in digital controllers using \( N \) unity delays (\( N = f_s/f_0 \) with \( f_s \) and \( f_0 \) being the sampling and grid fundamental frequencies, respectively), the adaptive solutions are directed to ensure an integer of \( N \). An intuitive possibility is to online change the sampling frequency according to the grid frequency estimated with PLL systems [21], [34], [38]–[40], where the total cost and implementation complexity are increased, and also the stability may be challenged [39]. Moreover, this solution is suitable for the digital control systems with the multi-rates control capability [34], [41], or with a high precision of the controller clock [38], in such a way to update the sampling frequency online. Alternatively, properly approximating the delay number \( N \) seems to be a cost-effective scheme as discussed in [20], [37], [42], where a Lagrange interpolating polynomial based filter has been used and however the implementation is not well addressed. Thus, this paper further investigates the above promising solution due to its cost, simplicity, portability (with less modifications), and effectiveness, especially in the digital controllers with a fixed sampling rate.

Notably, the PLL estimated frequency is essential to enhance the frequency adaptability of the periodic current controllers. However, the grid voltage as the input of the PLL systems cannot always be maintained as “constant” in terms of amplitude, frequency, and/or phase, due to multiple eventualities like continuous connection and disconnection of loads and fault to ground because of lightning strikes [43], [44]. That is why the grid codes also demand that the power converters should be able to operate within a specified frequency range or even regulate the frequency [6], [45]. Together with background distortions in the grid voltage, a large obstacle has been posed for the PLL systems. As a result, the PLL system presents inaccuracy in the estimated frequency. Subsequently, the current controllers in the \( \alpha\beta \)- or the \( abc \)-frame will suffer from frequency deviations either due to the PLL errors or the grid disturbances [19], [20], [40], resulting in a possibility for the feed-in current to reach the Total Harmonic Distortions (THD) limits [5], [6]. This impact is inevitable when seeing from the aforementioned, but it still lacks of a discussion on how the frequency deviation will affect the current controllers. It thus should be emphasized that advanced synchronizations (e.g., PLL systems) are desirable in order to ensure a reliable and satisfactory control of the grid current.

In light of the above issues, in this paper, the frequency adaptability of the selected periodic current controllers (i.e., PR, RES, and RC) is explored in the consideration of the PLL estimated frequency variations owing to either the PLL inherent errors or the grid disturbances. In § II, a brief description of the dual-loop control method for single-phase grid-connected inverters is presented. Then, the frequency adaptability of the periodic current controllers is focused on. More important, solutions to enhance the frequency adaptability of these current controllers are discussed, being the frequency adaptive current controllers, where the implementation is also emphasized. The discussions and the effectiveness of the frequency adaptive current controllers are verified by experiments in § IV before the conclusion.

II. FREQUENCY ADAPTABILITY ANALYSIS

A. Control of Single-Phase Grid-Connected Converters

Fig. 1 shows a typical configuration of a single-phase grid-connected system and its overall cascaded dual-loop control structure, where an LCL-filter is used considering the power quality issues [7]. It is shown in Fig. 1 that the PLL estimated grid frequency (\( \omega_{ppl} \)) is feeding back to the current controller as aforementioned in order to improve the control performance. Especially, the frequency \( \omega_{ppl} \) is used to transform AC quantities (i.e., the grid current \( i_g \) and voltage \( v_g \)) to DC quantities (i.e., \( i_d \) and \( v_d \)) for PI controllers in the \( dq \)-frame or reversely (\( dq \rightarrow \alpha\beta \)). Yet for simplicity, in the case of the current control in either the \( \alpha\beta \)- or the \( abc \)-frame, a fixed constant frequency (i.e., the nominal grid frequency \( \omega_0 \)) is designed for the periodic current harmonic controllers in practice (especially, when implemented in a digital signal processor), as it is shown in Fig. 2. In both cases, the current controller performance will be affected by the PLL estimated frequency, which is used to generate the grid current reference according to Fig. 1. Notably, other current controllers like the Dead-Beat (DB) control can also be used as the fundamental-frequency current controller [46], [47].

B. Frequency Sensitivity Analysis of the Current Controllers

In practice, it is difficult to attain an acceptable feed-in current even with high-order grid filters (e.g., an LCL-filter) because of the always existing background distortions in the grid voltage. Moreover, the grid-side filter should be cost- and size-effective in commercial applications, e.g., using an LC filter. Thus, harmonic compensators are typically incorporated in the current control loop in order to improve the current quality, as it is shown in Fig. 2, where the fundamental-frequency current controller (i.e., \( G_{PR}(s) \)) can be given as

\[
G_{PR}(s) = k_p + \frac{k_i s}{s^2 + \omega_0^2} \quad (1)
\]
in which $k_p$ and $k_i$ are the control gains. It can be seen in Fig. 2 that the harmonic compensator embraces either a paralleled multi-resonant controller $G_{RES}(s)$ or a repetitive controller $G_{RC}(s)$, which exhibits good performance for controlling periodic signals (i.e., the grid current $i_g$) [7], [16], [20], [22]. Accordingly, the harmonic compensators can be expressed as

$$G_{RES}(s) = \sum_{h=3,5,7,\ldots} G^h_{RES}(s)$$

(2)

$$G_{RC}(s) = \frac{k_{rc} e^{-2\pi s/\omega_0}}{1 - e^{-2\pi s/\omega_0}}$$

(3)

where $G^h_{RES}(s)$ is the $h$th-order resonant controller with $h$ being the harmonic order and $k_{rc}$ is the control gain of the RC harmonic compensator. Furthermore, the individual resonant controller can be given as

$$G^h_{RES}(s) = \frac{k^h s}{s^2 + (h\omega_0)^2}$$

(4)

in which $k^h$ is the control gain of the corresponding $h$th-order resonant controller. In addition, the RC based harmonic controller can further be expanded into [47]

$$G_{RC}(s) = k_{rc} \left[ -\frac{1}{2} + \frac{\omega_0}{2\pi s} + \frac{\omega_0}{s} \sum_{k} \frac{s}{s^2 + (k\omega_0)^2} \right]$$

(5)

with $k = 1, 2, 3, \ldots$. Eq. (5) indicates the inherent resonant characteristic of the RC controller with an identical resonant gain (i.e., $k_{rc}\omega_0/\pi$), and it also shows that the internal models of the DC signal and all harmonics are incorporated in the harmonic compensator $G_{RC}(s)$.

According to Fig. 2, the error rejection transfer function $G_e(s)$ can be given as

$$G_e(s) = \frac{E_i(s)}{I^*_g(s)} = \frac{1}{1 + [G_{CC}(s) + G_{HC}(s)] G_P(s)}$$

(6)

with $G_{CC}(s)$ being the fundamental-frequency current controller (e.g., PR or DB controllers), $G_{HC}(s)$ being the harmonic compensators (e.g., RES or RC controllers), and $G_P(s)$ being the plant model. When $s \rightarrow jh\omega_0$, it can be seen from (1)-(5) that the magnitude response of these controllers will theoretically approach to infinite (i.e., $|G_{CC}(j\omega_0) + G_{HC}(j\omega_0)| \rightarrow \infty$), as illustrated in Fig. 3. Consequently, the tracking error $e_i(t) (E_i(s)$ in (6)) will be zero at the frequencies of interest (i.e., $h\omega_0$). In other words, the RES controller enables a selective harmonic compensation, while the RC controller can eliminate all harmonics below the Nyquist frequency theoretically, being a good alternative for harmonic control [20]-[22], [48].

However, in practical applications, the grid frequency is not exactly the nominal one $\omega_0$ but a time-varying element of the grid voltage with small deviations. Hence, in most grid standards [43], [44], it is also demanded that the grid-connected devices should be able to operate within a certain frequency range. Under short-term abnormal grid conditions (e.g., a frequency sag), the inverter-based systems are even required to ride through such events [6], [49]. Nonetheless, in those cases, infinite magnitudes of those current controllers can not always be maintained when $s \rightarrow jk\omega_0$, leading to reduced tracking performance and thus a poor THD of the feed-in current. Even with an advanced PLL system, the frequency deviations can not be completely eliminated. In general, the PLL estimated frequency $\omega_{PLL}$ can be expressed as

$$\omega_{PLL} = \omega_0 + \Delta \omega$$

(7)

in which $\Delta \omega = \Delta \omega_p + \Delta \omega_{PLL}$ represents the estimated angular frequency deviations. It consists of the grid frequency disturbances $\Delta \omega_p = \omega_p - \omega_0$ with $\omega_p$ being the instantaneous grid frequency and/or the PLL tracking errors $\Delta \omega_{PLL}$. As discussed above, (1)-(5) and (7) imply that a small frequency variation (i.e., $\Delta \omega$) induced by the grid frequency changes and/or PLL estimation errors can contribute to a degradation of the error rejection capability for those current controllers, which are supposed to approach to infinite at the targeted frequencies (i.e., $h\omega_0$). This impact is referred to as the frequency adaptability, which is illustrated as the following.

According to (4) and (7), the magnitude response (i.e., $s = jh\omega_{PLL}$) of an individual resonant controller $G^h_{RES}(s)$ at the corresponding frequency ($h\omega_{PLL}$) can be obtained as

$$|G^h_{RES}(jh\omega_{PLL})| = \frac{k^h}{h^2 \omega^2_{PLL} + h^2 \omega^2_0} = \frac{k^h}{h\omega_0} \left| \frac{\delta + 1}{\delta^2 + 2\delta} \right|$$

(8)
with $\delta = \Delta \omega / \omega_0$, and Eq. (8) indicates that the gain will not be infinite unless $\delta = 0$ (i.e., $\Delta \omega = 0$). The control gain reduction of the resonant controllers due to the frequency variations $\Delta \omega$ is illustrated in Fig. 4, where it can be observed that even a small frequency variation of $\pm 0.2\%$ can result in a significant performance degradation of the resonant controllers (e.g., the magnitude decreases from $\infty$ dB to 48.5 dB). It demonstrates that the RES based harmonic compensator (and also the resonant controller of the PR controller with $h = 1$) is sensitive to frequency variations. In other words, the RES controller in (4) has a poor frequency-variation-immunity.

In the same manner, substituting $s = j\omega_{PLL}$ into (3) gives the magnitude response of the RC controller $G_{RC}$ as

$$|G_{RC}(j\omega_{PLL})| = \left| \frac{k_{rc}}{\sqrt{2 - 2 \cos(2\pi h)}} \right|$$

(9)

According to the Euler’s formula, the following is obtained

$$|G_{RC}(j\omega_{PLL})| = \frac{k_{rc}}{\sqrt{2 - 2 \cos(2\pi h)}}$$

(10)

which implies that the RC controller no longer can approach infinite control gain when there is a frequency tracking error from the PLL system (and/or grid frequency changes), i.e., $\delta \neq 0$ and $\Delta \omega \neq 0$. Fig. 5 further illustrates the effect of a frequency deviation on the current control error rejection ability of the RC harmonic compensator. As it can be observed in Fig. 5, a remarkable gain drop (e.g., the magnitude decreases from $\infty$ dB to 28.5 dB) occurs due to a frequency change of $\pm 0.2\%$ (i.e., corresponding to a frequency variation of $\pm 0.1$ Hz in 50-Hz systems), and consequently the rejection ability is significantly degraded. A conclusion drawn from Figs. 4 and 5 is that the frequency sensitivity of the periodic current controllers (i.e., the PR, RES, and RC controllers) is poor, and thus enhancing the frequency adaptability is necessary in order to produce high-quality currents.

### III. Enhancing the Frequency Adaptability

As discussed in the last paragraph, in order to achieve a good current control in terms of a zero-error elimination of the harmonics even under a variable grid frequency (or a PLL tracking error), the current controllers have to be frequency adaptive. It means that the control gain should be infinite when $s = j\omega_{PLL}$. Thus, feeding back the frequency estimated by an advanced PLL system [10] or frequency estimator (e.g., using Kalman filter) [13], [14] to the current controllers is a possibility to decrease the frequency sensitivity. This is much convenient for the resonant controllers [30], [31], [35], which is given as

$$G_{RES}^h(s) = \frac{k_{rc}^h s}{s^2 + (\omega_{PLL})^2} = \frac{k_{rc}^h s}{s^2 + [h(\omega_0 + \Delta \omega)]^2}$$

(11)

Fig. 6(a) shows the implementation of a frequency adaptive resonant controller. It can be observed in Fig. 6(a) and (11) that, by feeding in the PLL estimated frequency, the resonant frequencies of the harmonic controllers $G_{RES}^h(s)$ will automatically be adjusted to the instantaneously estimated grid frequency. As a result, infinite gains of the resonant controllers are almost attained in the case of a varying grid frequency.

However, in respect to the RC controller, enhancing the frequency adaptability cannot be reached by simply feeding back the PLL estimated frequency, since the RC controller is normally implemented in a digital signal processor of a fixed sampling rate. In that case, the RC controller shown in (3) can be given as

$$G_{RC}(z) = \frac{k_{rc} z^{-(N+F)}}{1 - z^{-(N+F)}}$$

(12)

in which $N = \lfloor f_s / f \rfloor$ is an integer, $F = f_s / f - N$ is the order of a fractional delay (i.e., $z^{-F}$) with $f = \omega_{PLL} / (2\pi)$, \begin{align*}
\end{align*}
and $f_s$ is the sampling frequency. Therefore, to enhance the frequency adaptability of the RC controller, one possibility is that the fractional delay $z^{-F}$ induced by the frequency variations, which is neglected in practice, should be appropriately approximated. A cost-effective approach to approximate the fractional delay is using Finite-Impulse-Response (FIR) filters as discussed in [20], [37], [42], [50]. It should be noted that, the frequency adaptability of the RC harmonic compensator can be enhanced alternatively by varying the sampling frequency [21], [34], which in return is able to ensure an integer of $f_s/f$ (i.e., $F = 0$) in practical applications, but it will increase the cost and the overall complexity. Such enhancement of the frequency adaptability by varying the sampling frequency is impossible (or difficult) to implement in the case of a control system with a fixed sampling rate (e.g., a dSPACE DS1103 system). Furthermore, the adaptive scheme requires major modifications (reprogramming) when the control algorithm is transferred to another system, i.e., poor portability, in contrast to the frequency adaptive solutions based on digital filters.

The most popular but simple and effective solution to the FIR fractional delay $z^{-F}$ is based on the Lagrange interpolating polynomial, which can be expressed as

$$z^{-F} \approx \sum_{i=0}^{L} \left( z^{-l} H_l \right) \text{ with } H_l = \prod_{i \neq l}^{L} \frac{F-i}{F-l}$$

(13)

where $H_l$ is the Lagrange interpolating polynomial coefficient, $l, i = 0, 1, 2, \ldots, L$, and $L$ is the length of the Lagrange interpolation based fractional delay filter. For convenience, the coefficients of the Lagrange based fractional delay filter $z^{-F}$ are given in Table I. If $L = 1$, Eq. (13) corresponds to a linear interpolation between two samples, i.e., $z^{-F} \approx H_0 + H_1 z^{-1}$.

While in the case of $L = 3$, a cubic interpolating polynomial is formulated, i.e., $z^{-F} \approx H_0 + H_1 z^{-1} + H_2 z^{-2} + H_3 z^{-3}$, which has been proved in [20], [42], [47], [50] as a relatively good and accurate approximation of the fractional delay $z^{-F}$ in terms of the bandwidth and also the resultant phase delay. As a consequence, the Lagrange based FIR filter with $L = 3$ can be employed to enhance the frequency-variation-immunity of the RC compensator. Following, the general block diagram of a frequency adaptive RC harmonic compensator can be constructed as shown in Fig. 6(b).

Although the Lagrange-interpolation-polynomial based fractional delay filter has several advantages like easy formulas for the coefficients and good response at low frequencies [50], it may still consume certain memory space if not efficiently implemented in the digital control systems. Moreover, when comparing the frequency adaptive schemes for the RES and RC controllers in Fig. 6, the frequency delay order $F$ has an indirect mapping relationship with the frequency variations $\Delta \omega_c$ requiring an online calculation of the Lagrange coefficients according to the PLL estimated angular frequency $\omega_{PLL}$ and the system sampling frequency $f_s$, which is a fixed value in this case.

Fig. 7 gives two possibilities to implement digitally the fractional delay filter of (13) in low-cost digital signal processors. It can be observed that the Farrow structure [50] has less delay units and thus consumes less memory space compared to the direct structure that has been employed in [20]. Thus, the Farrow structure is a more efficient implementation of the fractional delay filter. Table II further summaries the computational burden (complexity) of the two fractional delay filter structures. It can be seen that, in terms of implementation, the frequency adaptive scheme for the RC harmonic compensator is more complicated than that for the RES controller. However, when compensating high-order harmonics is required, the memory consumed by the RES controller is increased, which is not the case for the RC compensator [24]. Nevertheless, the above discussions have revealed that an advanced PLL system in terms of accuracy and dynamics is crucial for the enhancement of the controller frequency adaptability, especially for single-phase grid-interfaced converters, as discussed at the beginning of § III.

IV. EXPERIMENTAL VERIFICATIONS

A. Test-Rig Description

In order to verify the above analysis and also to test the effectiveness of the enhanced frequency adaptability of the current controllers, experiments have been carried out on a single-phase grid-connected inverter system, where an AC
programmable power source has been used in order to change the frequency. Fig. 8 shows the experimental set-up, where a commercial inverter is adopted and an LC filter is used. The voltage of the capacitor $C_L$ is measured for synchronization. The other system parameters are listed in Table III. For comparison, a DB controller and the PR controller are adopted as the fundamental-frequency current controller, and the RES and RC controllers are used to compensate the harmonics. As for the synchronization, a Second Order Generalized Integrator based PLL (SOGI-PLL) algorithm [7], [8] has been adopted due to its robust immunity to background distortions and fast dynamics. Fig. 9 shows the structure of the SOGI-PLL system, in which two Third Order Integrators (TOI) [8] have been employed to realize the SOGI system, with $k$ being the control gain for the SOGI in-quadrature structure, and $k^p$, $k^i$ being the proportional and integral control gains for the PI controller (i.e., the PLL loop filter), respectively. In this paper, $k = 1.4$, $k^p = 0.283$, and $k^i = 5.663$, which will result in an optimal performance of the SOGI-PLL system in terms of overshoot and settling time [8], [16].

### B. Discrete Current Controllers

Since the control systems were done in a dSPACE DS 1103 digital control system, the resonant controller can easily be implemented in a discrete form using one Forward Euler method and one Backward Euler method [8], [35]. Then, the frequency adaptive RES harmonic compensator can be obtained in its discrete form as

$$G_{HRES}(z) = \frac{k^h(z^{-1} - z^{-2})T_s}{1 + (2\omega_p^2T_s^2 - 2)z^{-1} + z^{-2}} \tag{14}$$

with $T_s = 1/f_s$ being the sampling period. Notably, other discretization methods like the Tustin with pre-warping, the impulse invariant, and the Trapezoidal method can be employed to discretize the resonant controller of (4) at the cost of increased complexity [8]. While for the DB controller [16], it can be expressed as

$$G_{DB}(z) = \frac{z^{-1}}{(1 - z^{-1})G_f(z)} \tag{15}$$

where $G_f(z)$ is the filter model. In practice, a low pass filter is incorporated into the RC controller in order to improve the controller robustness [47]. Then, the RC harmonic compensator of (3) is modified as given by

$$G_{RC}(z) = \frac{k_R z^{-(N+F)}Q(z)}{1 - z^{-(N+F)}Q(z)} \cdot C(z) \tag{16}$$

in which $Q(z) = \alpha_0 + \alpha_1 + \alpha_2 z^{-1}$ is the pass filter with $\alpha_0 + 2\alpha_1 = 1$ and $\alpha_0, \alpha_1 > 0$, and $C(z) = z^m$ is a phase-lead compensator. The phase-lead number $m$ of $C(z)$ is practically determined in the experimental tests. All the parameters of these controllers are shown in Table IV, where it can be seen that only the 3rd, 5th, and 7th RES controllers were incorporated with the fundamental-frequency controller (i.e., the PR controller) for harmonic compensation.

### C. Experimental Results - Constant Loading

The frequency adaptability of the discussed current controllers in the case of a varying grid frequency has firstly

### Table II

**Complexity Comparison of the Fractional Delay Filter Implementations (Fig. 7).**

<table>
<thead>
<tr>
<th>Parallel structure</th>
<th>Farrow structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of summations</td>
<td>$L$</td>
</tr>
<tr>
<td>No. of multiplications</td>
<td>$L + 1$</td>
</tr>
<tr>
<td>No. of delays</td>
<td>$L(L + 1)/2$</td>
</tr>
<tr>
<td>Structure type</td>
<td>In-parallel</td>
</tr>
</tbody>
</table>

### Table III

**Parameters of the Single-Phase Grid-Connected System Shown in Fig. 8.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal grid voltage amplitude</td>
<td>$v_{gn}$</td>
<td>311 V</td>
</tr>
<tr>
<td>Nominal grid frequency</td>
<td>$\omega_0$</td>
<td>$2\pi \times 50$ rad/s</td>
</tr>
<tr>
<td>Rated power</td>
<td>$P^c$</td>
<td>1 kW</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$v_{dc}$</td>
<td>400 V</td>
</tr>
<tr>
<td>DC-link capacitor</td>
<td>$C_{dc}$</td>
<td>1100 $\mu$F</td>
</tr>
<tr>
<td>Grid impedance</td>
<td>$L_g$</td>
<td>2 mH</td>
</tr>
<tr>
<td>$R_g$</td>
<td>0.2 $\Omega$</td>
<td></td>
</tr>
<tr>
<td>LC filter</td>
<td>$L_1$</td>
<td>3.6 mH</td>
</tr>
<tr>
<td>$C_1$</td>
<td>2.35 $\mu$F</td>
<td></td>
</tr>
<tr>
<td>Switching and sampling frequencies</td>
<td>$f_{sw}, f_s$</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

![Fig. 8. Experimental set-up of a single-phase grid-connected inverter system: (a) test-rig photo and (b) implementation block diagrams.](image)

![Fig. 9. Digital implementation (structure) of the Second Order Generalized PLL (SOGI-PLL) system.](image)
been tested, when a constant loading condition (i.e., reference current amplitude $I_g^* = 5$ A) is considered. The test results are shown in Fig. 10, in which the grid frequency was programmed within a range of 49.5 Hz to 50.5 Hz (i.e., ±1%). It can be observed in Fig. 10 that the DB controller is almost immune to frequency deviations due to its model-dependent characteristic with a relatively low demand of the grid frequency information [46], [51]. In fact, the DB current controller behaves like a simple proportional controller according to (15), and hence its harmonic rejection capability is also poor. In contrast, the PR current controller is significantly affected by the grid frequency changes. Specifically, when the grid frequency increases, the performance of the PR controller is significantly degraded, thus resulting in a poor current THD that may exceed the limitation (e.g., THD < 5%) [5]. However, due to the inverter non-linearity and/or deadtime effect [52], which induces low-order harmonics, the harmonic compensators (e.g., RES and RC compensators) should be included in parallel with the two fundamental current controllers (i.e., DB and PR controllers) in order to improve the current quality. Nevertheless, those harmonic compensators may also be affected by the frequency variations. For instance, as it is shown in Fig. 10, both the RES and the RC periodic current controllers present poor frequency adaptability, since they are highly frequency-dependent controllers (i.e., their realizations rely on the grid frequency information provided by a PLL system or a frequency estimator). The test results are in close agreement with the analysis presented in § II.B (Fig. 4).

Moreover, the poor frequency adaptability is further verified by the steady-state performance of the RES and RC controllers under a severe abnormal grid frequency (i.e., $2\pi \times 50.5$ rad/s), as it is shown in Fig. 11. Although the control objective is to feed-in a high-quality current at unity power factor, it however can be observed in Fig. 11 that there is a phase shift between the grid voltage $v_g$ and the feed-in grid current $i_g$ due to the frequency deviation, and thus leading to a poor power factor. That is to say, the grid-connected inverter system is not operating at unity power factor mode, which may violate the integration demands. Nevertheless, the above experimental results have demonstrated the frequency-variation-immunity of the selected current controllers, i.e., the DB and PR fundamental frequency current controllers, and the RES and RC harmonic compensators.

According to the discussions in § III, the strategies to enhance the frequency adaptability of the periodic current controllers were applied and the single-phase grid-connected inverter system has been tested. Fig. 12 shows the steady-state performances of the enhanced periodic current controllers. It can be observed in Fig. 12(a) that, when the PLL estimated frequency $\omega_{PLL}$ is fed back to the resonant controller of (14), the tracking performance is improved. As a result, in the case of frequency variations induced by the grid disturbances, a unity power factor operation as well as an improved current quality is always achieved. Similarly, when applying the frequency adaptive scheme to the RC harmonic compensator, there is no phase shift between the grid voltage and the injected grid current as shown in Fig. 12(b), and i.e., the system is operating...
at a unity power factor to feed in high-quality currents. It should be pointed out that the parallel structure shown in Fig. 7(a) has been adopted for adapting the RC harmonic compensator to grid frequency changes without considering the implementation efficiency.

In addition, the dynamics of the frequency adaptive schemes were tested in the case of a grid-frequency step-up change (i.e., from 49.5 Hz to 50.5 Hz). The experimental results are presented in Fig. 13, which has verified the effectiveness of the proposed frequency adaptive schemes in terms of dynamics. Similar conclusions can be drawn; it is convenient to feed back the PLL estimated frequency according to Fig. 6(a) in such a way that the frequency adaptability of the RES controller is effectively improved; while by approximating the fractional order delay according to Fig. 7(a), the frequency adaptability of the RC harmonic controller is also enhanced. Both will contribute to an improved power factor as well as a lower THD of the feed-in currents.

Fig. 14 has further validated the effectiveness of the discussed adaptive schemes to enhance the frequency-variation-immunity of the selected current controllers under a wide range of grid frequency variations (i.e., 49.5 Hz - 50.5 Hz). When compared with the THD$g$ shown in Fig. 10, it can be observed in Fig. 14 that the periodic current controllers with the frequency adaptability schemes in § III can maintain an almost constant THD despite the variations of the grid frequency with a robust PLL system for the frequency estimation. It is also worth to point out that the RC harmonic controller consists of all the resonant controllers with the corresponding frequency below the Nyquist frequency. As a consequence, for the PR controller with a repetitive controller as the harmonic compensator, the grid current THD is lower than that in the case when the resonant controllers are paralleled as the harmonic compensator, where only a number of harmonics are selectively compensated.
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Fig. 15. Performance (experiments) of the proportional resonant controller without enhanced frequency adaptability under different power loading, where the resonant and the repetitive controllers are adopted for harmonic compensation and the grid frequency is: (a) 49.5 Hz, (b) 50 Hz, and (c) 50.5 Hz.

**D. Experimental Results - Various Loading**

Additionally, partial loading operations of the single-phase grid-connected inverter system have been conducted, where an extra inductor of 3.6 mH is connected between the LC filter and the isolation transformer. In this sense, a weak grid is to some extent simulated (i.e., the grid impedance is large, consisting of the leakage inductance of the transformer and the extra inductor), which may challenge the stability of the entire system [53]. Noted that the voltage across the filter capacitor \(C_f\) is still measured for synchronization like the previous case. All other parameters are listed in Table III. Fig. 15 shows the performance of the selected periodic current controllers under different grid frequencies. It has been observed that, in the case of an abnormal grid frequency, the feed-in current is getting worse with a lower THD \(i_g\), as illustrated previously. For instance, under the half-loading condition, the THD \(i_g\) has been drifted to around 3.9 % (49.5 Hz) and 4.5 % (50.5 Hz) from 3.2 % (50 Hz), when either the RES controller or the RC compensator is used for harmonic compensation. Those test results further illustrate the poor frequency adaptability of the selected periodic current harmonic controllers (i.e., the RES and RC controllers).

Hereafter, the frequency adaptive schemes are applied to these periodic current controllers, and the same single-phase grid-connected inverter system has been tested under various loading conditions. The THD \(i_g\) of the feed-in currents is plotted as shown in Fig. 16. The effectiveness of the frequency adaptive schemes can be identified in Fig. 16, where for example the THD \(i_g\) is reduced by around 0.5 % and 1 %, respectively, under abnormal grid conditions (i.e., the grid frequency is 49.5 Hz or 50.5 Hz) in contrast to the results presented in Fig. 15(a) and (c). Fig. 17 presents the steady-state performance of the frequency adaptive current controllers under a half-loading condition, in which it can be observed that the unity power factor operation is always achieved despite of the abnormal grid frequencies. Those experimental tests have demonstrated that, by feeding back the PLL estimated frequency to the RES controller and approximating the fractional delays for the RC compensator be means of...
Fig. 17. Steady-state performance (half-loading, i.e., output power is 500 W) of the frequency adaptive proportional resonant controller in the case of abnormal grid frequencies with frequency adaptability enhanced harmonic compensators (CH1 - grid current $i_g$ [5 A/div]; CH2 - grid voltage $v_g$ [100 V/div]): (a) resonant controllers and (b) the repetitive controller.

Lagrange polynomial filters, the frequency adaptability of the corresponding periodic controller can be enhanced.

Moreover, robustness is another important index to assess the current controllers. Hence, the selected current controllers with enhanced frequency adaptability have been further tested in the case of loading transients. The performance of these periodic current controllers is given in Fig. 18, where the output power experienced a step-up change from 100 W to 1 kW and a step-down change from 1 kW to 100 W, and the grid frequency was programmed as 49.5 Hz. It can be seen in Fig. 18 that the frequency adaptive schemes for the RES controllers and the RC compensator have negligible influence on the dynamics of the entire current controller. Specifically, both frequency adaptive periodic current controllers can quickly come into the steady-state without any compromise of stability. It should be noted that, when the repetitive current controller is adopted as the harmonic compensator, the entire controller can achieve almost zero-error tracking of the grid current within 3 cycles. Additionally, seen from the experimental results in Figs. 17(b), 18(c), and 18(d), the frequency adaptive RC harmonic compensation has superior performance over the frequency adaptive RES based harmonic compensator in terms of dynamics and harmonic mitigations, especially in light loading conditions.

Notably, the main idea behind the enhancement of the frequency adaptability in brief is to update the center frequency of the periodic current controllers according to the PLL estimated frequency. As a consequence, the response of the PLL system (i.e., the SOGI-PLL in this paper) to other abnormal grid conditions (e.g., grid voltage sags) may also affect the performance of the frequency adaptive current controllers. In order to validate the robustness of the selected periodic current controllers in the case of grid voltage sags, more experimental tests have been conducted on the single-phase system, where the grid frequency is 49.5 Hz and the grid current amplitude has been controlled as constant (i.e., $I_g^* = 6.43$ A, corresponding to the current at the rated power level) to avoid inverter shutdown due to over-current protection. The experimental results are presented in Fig. 19, where the grid voltage has sagged to 165 V in RMS (i.e., voltage sag level: 0.25 p.u.) during operation or reversely. It can be observed that the frequency adaptive current controllers by means of online updating the center frequency is robust even under grid voltage sags. In addition, the repetitive harmonic compensator has a faster response than the RES controller does, which is indicated by the current tracking errors ($e_i$) in Fig. 19, and however the dynamics are affected by the designed PLL systems. In this paper, the SOGI-PLL has relatively fast responses to abnormal grid conditions [8], and thus it also contributes to the frequency adaptability of the selected periodic current controllers to some extent. Nevertheless, all the above experiments have demonstrated the frequency adaptability of the selected current controllers, and also the effectiveness of the frequency adaptive schemes for these periodic current controllers even under various grid disturbances (e.g., frequency changes and voltage sags).

V. CONCLUSION

The sensitivity to frequency variations of selected current controllers for grid-connected power converters has been
Fig. 18. Performance (loading transients) of the frequency adaptive proportional resonant controller with frequency adaptability enhanced harmonic compensators (CH1 - grid current $i_g$ [5 A/div]; CH2 - grid voltage $v_g$ [250 V/div]; CH3 - current error $e_i = i^* - i_g$ [5 A/div]): (a) step-up load change from 100 W to 1 kW using RES harmonic compensator, (b) step-down load change from 1 kW to 100 W using RES harmonic compensator, (c) step-up load change from 100 W to 1 kW using RC harmonic compensator, and (d) step-down load change from 1 kW to 100 W using RC harmonic compensator, where the grid frequency is 49.5 Hz.

Fig. 19. Performance (voltage sag) of the frequency adaptive proportional resonant controller with frequency adaptability enhanced harmonic compensators (CH1 - current error $e_i = i^* - i_g$ [5 A/div]; CH2 - grid voltage amplitude (estimated by the PLL system) $v_{gm}$ [250 V/div]; CH3 - grid current $i_g$ [5 A/div]; CH4 - grid voltage $v_g$ [250 V/div]): (a) in the case of a voltage drop by 0.25 p.u. using RES compensator, (b) during the grid voltage recovery using RES compensator, (c) in the case of a voltage drop by 0.25 p.u. using RC compensator, and (d) during the grid voltage recovery using RC compensator, where the grid frequency is 49.5 Hz.
explored in this paper. The investigation has revealed that the dead-beat current controller is almost immune to the frequency deviations, since it is a model-based predictive controller. In contrast, the proportional resonant controller, the resonant (RES) harmonic compensator and the repetitive controllers (RC) are very sensitive to the frequency variations induced by the PLL synchronization errors and/or the grid disturbances. This is because those periodic current controllers (harmonic compensators) are strongly dependent on the center frequencies, and infinite control gains at the frequencies of interest (e.g., the fundamental frequency) cannot be achieved due to the frequency deviations. In addition, this paper has also introduced means to enhance the frequency adaptability of the discussed current controllers – simply feeding back the PLL estimated frequency to the RES controller or properly approximating the fractional delay for the RC harmonic controller. Experiments performed on a single-phase grid-connected inverter have verified the discussions. It is worth pointing out that an advanced frequency estimator (e.g., a PLL synchronization system) in terms of relatively high accuracy in the estimated grid frequency and fast responses to grid disturbances is important for the periodic current controllers to ensure high-quality currents into the grid.

REFERENCES

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