Dynamic Boiler Performance
- modelling, simulating and optimizing boilers for dynamic operation

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Ph.D. Thesis
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## Nomenclature

### Capital Letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$C$</td>
<td>Constant</td>
<td>—</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
<td>$N$</td>
</tr>
<tr>
<td>$F$</td>
<td>Cost function</td>
<td>DKK/EUR/USD</td>
</tr>
<tr>
<td>$H_u$</td>
<td>Heating Value</td>
<td>$kJ/kg$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>$m$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
<td>$kg$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number (see subscript)</td>
<td>DKK/EUR/USD</td>
</tr>
<tr>
<td>$NPV$</td>
<td>Net Present Value</td>
<td>DKK/EUR/USD</td>
</tr>
<tr>
<td>$NW$</td>
<td>Normal Water Level</td>
<td>—</td>
</tr>
<tr>
<td>$P$</td>
<td>Power/load</td>
<td>$W$</td>
</tr>
<tr>
<td>$R$</td>
<td>Thermal resistance</td>
<td>$K \cdot m^2/W$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>°C or K</td>
</tr>
<tr>
<td>$U$</td>
<td>Energy content</td>
<td>$J$</td>
</tr>
<tr>
<td>$U_{ht}$</td>
<td>Overall coefficient of heat transfer</td>
<td>$W/K \cdot m^2$</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight</td>
<td>$kg$</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt Number</td>
<td>—</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl Number</td>
<td>—</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds Number</td>
<td>—</td>
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### Lower Case

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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$c_p$</td>
<td>Specific heat capacity at constant pressure</td>
<td>$J/kg \cdot K$</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>$h$</td>
<td>Enthalpy</td>
<td>$J/kg$</td>
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## Lower Case cont’d

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<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>( m )</td>
<td>Mass flow</td>
<td>( kg/s )</td>
</tr>
<tr>
<td>( l )</td>
<td>Length</td>
<td>( m )</td>
</tr>
<tr>
<td>( f )</td>
<td>Factor</td>
<td></td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration of gravity (=9.81)</td>
<td>( m/s^2 )</td>
</tr>
<tr>
<td>( k )</td>
<td>Constant</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>Element</td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>Pressure</td>
<td>( \text{bar} )</td>
</tr>
<tr>
<td>( \dot{q} )</td>
<td>Energy flow</td>
<td>( J/s )</td>
</tr>
<tr>
<td>( r )</td>
<td>Radius</td>
<td>( m )</td>
</tr>
<tr>
<td>( s )</td>
<td>Thickness</td>
<td>( m )</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
<td>( s )</td>
</tr>
<tr>
<td>( u )</td>
<td>Specific energy content</td>
<td>( J/kg )</td>
</tr>
<tr>
<td>( v )</td>
<td>Velocity</td>
<td>( m/s )</td>
</tr>
<tr>
<td>( x )</td>
<td>Quality/dryness</td>
<td>( kg_{\text{steam}}/kg_{\text{mixture}} )</td>
</tr>
<tr>
<td>( x )</td>
<td>Mass fraction</td>
<td>( kg/kg )</td>
</tr>
<tr>
<td>( z )</td>
<td>Length/height</td>
<td>( m )</td>
</tr>
</tbody>
</table>

### Greek Letters

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Coefficient of heat transfer</td>
<td>( W/m^2 \cdot K )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Parameter</td>
<td>( kg/s )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Parameter</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_s )</td>
<td>Sand roughness</td>
<td>( mm )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Factor</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>Efficiency</td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Coefficient of friction</td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Thermal conductivity</td>
<td>( W/m \cdot K )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Conductivity</td>
<td>( \mu S/cm )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity</td>
<td>( kg/m \cdot s )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
<td>( m^2/s )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Specific volume</td>
<td>( m^3/kg )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
<td>( kg/m^3 )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Stress</td>
<td>( N/mm^2 )</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Angle of inclination (evaporator)</td>
<td>( rad )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Function</td>
<td></td>
</tr>
</tbody>
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1The duplication of symbols has been noted, but context will indicate correct meaning.
### Nomenclature

#### Subscripts

<table>
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<tr>
<td><code>act</code></td>
<td>Actual</td>
</tr>
<tr>
<td><code>all</code></td>
<td>Allowable</td>
</tr>
<tr>
<td><code>b</code></td>
<td>Bubble</td>
</tr>
<tr>
<td><code>boi</code></td>
<td>Boiler</td>
</tr>
<tr>
<td><code>circ</code></td>
<td>Circulation</td>
</tr>
<tr>
<td><code>comb</code></td>
<td>Combustion</td>
</tr>
<tr>
<td><code>con</code></td>
<td>Consumption</td>
</tr>
<tr>
<td><code>conv</code></td>
<td>Convection</td>
</tr>
<tr>
<td><code>cz</code></td>
<td>Convection zone</td>
</tr>
<tr>
<td><code>dc</code></td>
<td>Downcomer</td>
</tr>
<tr>
<td><code>dr</code></td>
<td>Drum</td>
</tr>
<tr>
<td><code>evap</code></td>
<td>Evaporator</td>
</tr>
<tr>
<td><code>ext</code></td>
<td>External</td>
</tr>
<tr>
<td><code>fg</code></td>
<td>Flue Gas</td>
</tr>
<tr>
<td><code>fric</code></td>
<td>Friction</td>
</tr>
<tr>
<td><code>fur</code></td>
<td>Furnace</td>
</tr>
<tr>
<td><code>fw</code></td>
<td>Feedwater</td>
</tr>
<tr>
<td><code>gt</code></td>
<td>Gas Turbine</td>
</tr>
<tr>
<td><code>int</code></td>
<td>Internal</td>
</tr>
<tr>
<td><code>mat</code></td>
<td>Material</td>
</tr>
<tr>
<td><code>m</code></td>
<td>Mean</td>
</tr>
<tr>
<td><code>op</code></td>
<td>Operation</td>
</tr>
<tr>
<td><code>p</code></td>
<td>Pipe</td>
</tr>
<tr>
<td><code>ref</code></td>
<td>Reference</td>
</tr>
<tr>
<td><code>rs</code></td>
<td>Riser</td>
</tr>
<tr>
<td><code>rel</code></td>
<td>Relative</td>
</tr>
<tr>
<td><code>sur</code></td>
<td>Surface</td>
</tr>
<tr>
<td><code>s</code></td>
<td>Steam</td>
</tr>
<tr>
<td><code>sat</code></td>
<td>Saturation</td>
</tr>
<tr>
<td><code>set</code></td>
<td>Set point</td>
</tr>
<tr>
<td><code>v</code></td>
<td>Volume</td>
</tr>
<tr>
<td><code>w</code></td>
<td>Water</td>
</tr>
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</table>

#### Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>T</code></td>
<td>Transposed</td>
</tr>
<tr>
<td><code>'</code></td>
<td>Fluid (water)</td>
</tr>
<tr>
<td><code>n</code></td>
<td>Gas (steam)</td>
</tr>
<tr>
<td><code>·</code></td>
<td>Rate (per sec)</td>
</tr>
</tbody>
</table>
Abstract

Traditionally, boilers have been designed mainly focussing on the static operation of the plant. The dynamic capability has been given lower priority and the analysis has typically been limited to assuring that the plant was not over-stressed due to large temperature gradients.

New possibilities for buying and selling energy has increased the focus on the dynamic operation capability, efficiency, emissions etc. For optimizing the design of boilers for dynamic operation a quantification of the dynamic capability is needed.

A framework for optimizing design of boilers for dynamic operation has been developed. Analyzing boilers for dynamic operation gives rise to a number of opposing aims: shrinking and swelling, steam quality, stress levels, control system/philosophy, pressurization etc. Common for these opposing aims is that an optimum can be found for selected operation conditions.

The framework has been developed as open, i.e. more dimensions and their corresponding quantification can be added. In the present study the dynamic capability has been quantified and the feasible set for the optimization has been limited by means of a number of constraints. The constraints are: (i) simple constraints related to the geometry and min/max gradients and/or load changes, and (ii) constraints derived from dynamic models related to shrinking and swelling and steam space load. Two boiler types were selected and an experimental verification of the dynamic model for one of these has been carried out.

As a result of the analysis for selected operating conditions the optimum design for dynamic operation of the plants has been assessed.
Synopsis

Traditionelt er kedler blevet designet under hensyntagen til deres statiske performance. Dynamisk performance er blevet tillagt en sekundær betydning og analyser mht anlæggenes dynamiske performance har typisk været begrænset til at sikre, at der ikke som følge af termiske spændinger forekommer overbelastning af anlæggene.

Nye muligheder mht køb og salg af energi har affødt stigende krav til dynamisk performance, effektivitet, emissioner mm. For at kunne optimere design af anlæg for dynamisk drift er der behov for at kunne kvantificere anlæggenes dynamiske performance.

I det aktuelle arbejde er der udviklet en model for optimering af kedler for dynamisk drift. Analyse af kedler med henblik på optimering af dynamiske drift giver anledning til en række modsatrettede tendenser: shrinking and swelling, damp kvalitet, spændingsniveau, kontrol system/filosofi, tryksætning osv. Fælles for disse modsatrettede tendenser er, at der for givne driftskonditioner kan findes et optimum.

Modellen er udviklet åben, dvs. yderligere dimensioner og deres tilhørende kvantificering kan løbende tilføjes. I det aktuelle arbejde er kedlers dynamiske performance blevet kvantificeret og løsningsmængden for optimeringen defineret af en række afgrænsninger. Disse er: (i) simple afgrænsninger relateret til kedlernes geometri og min/max lastgradienter, og (ii) afgrænsninger afdelt af dynamiske kedel modeller relateret til shrinking og swelling og damprumsbelastning. Der er udvalgt to kedeltyper, og for en af disse er der foretaget en eksperimental verifikation af den dynamiske model.

Som resultat af analyserne er der for udvalgte driftsbetingelser afdækket optimale designs og driftsbetingelser.
Preface

This thesis has been submitted as a partial fulfilment of the requirements for the Danish Ph.D. degree. It covers research work carried out at Institute of Energy Technology, Aalborg University and Aalborg Industries A/S during the period from August 2001 till July 2004. The work has been carried out under supervision of Associate Professor, Ph.D. Thomas Condra, Institute of Energy Technology, Aalborg University, Associate Professor, Ph.D. Niels Houbak, MEK - Energy Engineering Section, Technical University of Denmark and Head of Department, Ph.D. Jørgen A. Nielsen, Aalborg Industries A/S. I would like to express my deep gratitude to all of them for their patient and encouraging supervision during the project. The work has been carried out within the framework of The Industrial PhD Fellowship Programme (Danish Academy of Technical Sciences) funded by (Erhvervsfremme styrelsen) and Aalborg Industries A/S. The work has been co-funded by the Danish Academy of Technical Sciences under the grant of BOILERDYNAMICS, EF 923.

Based on a pre-print of the thesis a public defence took place at Aalborg University on the 13th September 2004 with Prof. Dr. techn. Reinhard Leithner, Institut für Wärme- und Brennstofftechnik, Technische Universität Braunschweig; General Manager Peter Overgaard, Elsam Engineering A/S and Associate professor, PhD, Lasse Rosendahl, The Institute of Energy Technology, Aalborg University as opponents. Incorporated into this version are clarifications suggested by the opponents.

Part of the work has been carried out under residence periods at Institut für Verfahrens Technik und Dampfkesselwesen, IVD at the University of Stuttgart. I owe a debt of gratitude to PD. Dr.-Ing. Uwe Schnell, Dipl. Ing. Christoph Sauer and the rest of the research group at the section Institute of Process and Power Plant Technology. Their generous hospitality and helpfulness during my stay at the institute has been very fruitful.

During the project period, a number of people have been very helpful with the project. I would especially like to thank: President of Aalborg Industries A/S, Mr. Freddy Frandsen for his very visionary contribution to the project. Associate Professor Brian Elmegaard, MEK - Energy Engineering Section, Technical University of Denmark for his encouraging contribution to the project. Associate Professor, Ph.D. Lisbeth Fajstrup and Associate Professor, Dr.rer.nat. (Ph.D.), Martin Raussen, Department of Mathematical Science, Aalborg University for their patient help with the mathematical aspects of optimization.
I would also like to express my deep gratitude to all my colleagues at the Institute of Energy Technology, Aalborg University and at Aalborg Industries A/S for their contributions and encouragement during the project.

Especially I would like to thank M.Sc. Claus M. S. Karstensen from Aalborg Industries’ R&D department for help with carrying out tests and implementing models for simulation.

Finally I would like to express my deep gratitude to my wife, Lotte and our two sons Anders and Christian for having always established the perfect environment and conditions for the carrying out the Ph.D. study.

Kim Sørensen
Aalborg, 2004
Structure of the Ph.D. Thesis

The main objective of the Ph.D. project has been to develop a framework for analyzing and optimizing boiler design and operation with respect to dynamic performance.

The scientific method applied in the present study has been an analytical approach supported by experimental verification of selected parts of the dynamic models developed. The experimental part of the project has been based on tests and experiments on a full-scale boiler installation.

The scientific aspects of the project focus on the development of a framework/methodology for optimizing the costs of boilers designed for dynamic operation. Furthermore, the project contributes scientifically with a development and verification of dynamic boiler models.

From an overall point of view the study consists of a General part that is independent of boiler type. This is followed by a part which is boiler type specific and is divided in two: (i) water tube boilers and (ii) fire tube boilers. The main report ends with a conclusion and perspectives.

A part of the study has been divided to cover the two, in principle, different boiler types: (i) the water tube boiler, where the evaporation takes place in a heating surface located external to the drum and (ii) the fire tube boiler where the evaporation takes place in a heat exchanger submerged in the water/steam drum.

The Ph.D. Thesis has been prepared with the following more detailed structure - refer also to Figure 1:

General

1 Introduction
In this chapter the background for the Ph.D. study, including the historical background for increased interest and research within dynamic boiler performance, is given. Furthermore, the objective of the study is detailed. The chapter concludes with a specification of the challenges and scope of the work and thereby the project limitations.

2 Design of Boilers
This chapter further details the background of the study and emphasizes the difference between different philosophies for the design of boilers: (i) static operational point of view and (ii) dy-
Figure 1: Structure of the Ph.D. Thesis consisting of a General part, a Boiler Specific part and a jointly Conclusion and Perspectives.

namical operation point of view. For the static point of view the most important aspects, included in the design of boilers, are discussed; this includes design philosophy, performance calculations, temperature profiles etc. This discussion is extended, for the dynamic point of view, to include fur-
ther aspects for designing boilers for dynamic operation including allowable temperature gradients, shrinking and swelling etc.

3 Objects of Design Optimization
In this chapter the different opposing aims that arise, when on the one hand aiming to cost optimize the boilers, whilst on the other hand aiming at the best possible dynamic performance, are specified and discussed.

4 Boiler Optimization
In this chapter the optimization challenges, presented in Chapter 3, are given in more detail. An introduction to the optimization method/theory applied is given.

5 Modelling and Simulation - Methodology
This chapter describes the modelling procedure developing from: developing a physical model to develop a mathematical model to which a numerical method is applied; to the implementation of the numerical model. Furthermore, the different types of equation systems (algebraic equation system, ordinary differential equation systems and differential-algebraic equation systems), arising from the modelling process and their characteristics, are described. This chapter includes a detailed description of characteristics and solution of DAE’s.

At this stage the analysis and experiments carried out as part of the study, are divided into two:

Water tube Boiler

6 Modelling and Simulation of the Water tube Boiler
In this chapter a physical and mathematical model for the water tube boiler is developed (Aalborg Industries boiler type: MISSION™ WHR-GT - see [162]). The model consists of three sub-models: heat exchanger model, where the energy flow from the flue gas to the water/steam side is calculated; evaporator circuit model, where the circulation in the evaporator is calculated; and drum model, where the water level in the drum is calculated. Furthermore, this chapter includes an example presenting results from the models developed.

7 Optimization of the Water tube Boiler
In this chapter the optimization of the water tube boiler is described. Based on the optimization procedure, introduced in Chapter 4, and the selected operation conditions, the feasible set for the optimization is constrained. The optimum design is assessed and results discussed.

Fire tube Boiler

8 Modelling and Simulation of the Fire tube Boiler
In this chapter a physical and mathematical model for the fire tube boiler is developed (Aalborg Industries boiler type: MISSION™ OB - see [162]). The model consists of three sub-models:
furnace model, which deals with the energy flow from the flue gas to the water/steam side; convective heating surface model, dealing with the energy flow from the flue gas to the water/steam side; and water/steam section, where the water level in the boiler is calculated. This chapter includes a combined simulation and verification example, and includes discussion of the results.

9 Optimization of the Fire tube Boiler
In this chapter the optimization of the fire tube boiler is described in a similar fashion to that of the water tube boiler - see Chapter 7.

10 Conclusion and Perspectives
The main results and conclusions for the present study are summarized. Furthermore, perspectives for further research work, including refinement, and more optimization dimensions are discussed.

Appendices:

A Terms and Definitions
In this appendix a more detailed explanation of selected terms and definitions is given.

B Basic Theory
In this appendix the theories for modelling applied in Chapter 6 and 8 are developed and explained in further detail.

C Water tube Boiler - tests
This appendix describes in detail the tests carried out on the MISSION™ WHR-GT boiler manufactured in Aalborg, Denmark, and installed at the Cruise Liner Coral Princess. Test results are included.

D Fire tube Boiler - tests
This appendix describes the tests carried out on the MISSION™ OB boiler installed at Aalborg Industries’ R&D test center in Aalborg. Test results are included.

E Modelling of Fire tube Boiler
In this appendix the fire tube boiler model - including reformulation of the equation system - is developed in details. The fire tube boiler model is given in Chapter 8.

F Modelling, Simulation and Optimization of Boilers - State of the Art
This appendix contains the results of the literature study carried out as a part of the present study.
As a part of the Ph.D. study the following papers have been written by the author:

- **Modelling of boiler heating surfaces and evaporator circuits** Sørensen, Kim; Condra, Thomas & Houbak, Niels; Presented at SIMS-Scandinavian Simulation Society, 43rd SIMS Conference (SIMS 2002), University of Oulu, Finland, September 26-27, 2002, [185].

- **Modelling, Simulating and Optimizing Boilers** Sørensen, Kim; Houbak, Niels & Condra, Thomas; Presented at The 16th International Conference on Efficiency, Costs, Optimization, Simulation and Environmental Impact of Energy Systems (ECOS 2003), Technical University of Denmark, June 30 - July 2, 2003, [167].

- **Modelling, simulating and optimizing boiler heating surfaces and evaporator circuits** Sørensen, Kim; Houbak, Niels & Condra, Thomas; Presented at SIMS-Scandinavian Simulation Society, 44th SIMS Conference (SIMS 2003), Västerås, Sweden, September 18 - 19, 2003, [186].

- **Modelling and simulating fire tube boiler performance** Sørensen, Kim; Karstensen, Claus M. S.; Houbak, Niels & Condra, Thomas; Presented at SIMS-Scandinavian Simulation Society, 44th SIMS Conference (SIMS 2003), Västerås, Sweden, September 18 - 19, 2003, [186].

- **Developing Boilers as Integrated Units** Sørensen, Kim; Houbak, Niels & Condra, Thomas; published in VGB PowerTech Volume 84/2004, ISSN 1435-3199, page 71-75, [190].

- **Solving Differential-Algebraic-Equation Systems by means of Index Reduction Methodology** Sørensen, Kim; Houbak, Niels & Condra, Thomas; Submitted for publication in SIMPRA, Simulation Practice and Theory, [184].

- **Optimizing design and operation of boilers with respect to dynamic performance** Sørensen, Kim; Houbak, Niels & Condra, Thomas. To be presented at the 17th International Conference on Efficiency, Costs, Optimization, Simulation and Environmental Impact of Energy Systems (ECOS 2004), Guanajuato, Mexico, July 7 - July 9, 2004, [168].


The major content of these papers have been incorporated in the thesis.

As a part of the Ph.D. study a literature study assessing *State of the Art for Modelling, Simulation & Optimization of Boiler* has been carried out - the results from this study are given in Appendix F.

A detailed explanation of selected terms and definitions from the thesis can be found in Appendix A.

In the thesis bibliographic references are given as [xx] - see the bibliographic list, page 95.

The thesis has been typeset in LATEX.
Chapter 1

Introduction

1.1 Background

Since the first boilers, for transforming fossil energy (solid fuels, oil or gas) to the energy carrying medium (water, steam, thermal fluid etc.), were developed at the start of the industrial era\(^1\), the advanced heat exchangers, as boilers actually are, have been the subject of continuous development aiming at: higher efficiency, lower emission levels, higher availability\(^2\), better operational performance etc. During this period the development focus has been strongly influenced by such factors as, higher energy costs demanding higher efficiency, environmental attention demanding lower emissions, higher salary levels demanding simpler operation (higher degree of automation) etc.

The characteristic of all these is that development has been kind of asymptotic, i.e. boiler efficiency $\rightarrow 100\%$ (state of the art: 94 - 95\%), emissions $\rightarrow 0$, availability $\rightarrow 100\%$ etc.

The continuous development towards more efficient plants with more advanced steam data has resulted in boilers that today are amongst the largest manmade steel structures on earth.

Over the years boilers have been subject of many different studies analyzing their performance and dynamic behavior\(^3\).

So, with this background: Why is it still interesting to analyze boilers? First of all boilers are still being applied to new purposes, related to change of fuels, increased requirements with respect to emissions, new inventions within areas like combustion technology, water-chemistry, materials, etc. Secondly, as market competition increases, boilers are being designed closer and closer to the limits of material strength, operation capability etc., i.e. better knowledge about dynamical behavior is required. As a

\(^1\)For more details on the historical background of boilers - see [126].

\(^2\)A plants availability is defined as the time the plant is available for operation divided by the total operational time.

\(^3\)Several authors have, over the years published different models. Amongst the more well known, which have been the basis for several Ph.D. studies, are [32]. Other often cited works are the boiler models developed by Åström [151] & Tyssö [5]. For more details - see Appendix F
result of the liberalization of the energy markets, where new opportunities for selling and/or buying energy arise, more focus is being put on the dynamic performance of the boilers\(^4\).

For maritime applications larger focus has been put on increasing efficiency and flexibility. This is mainly achieved by integration and optimization of the complete energy system\(^5\) which means strengthened requirements with respect to dynamic performance of maritime boilers - see [170].

Increased requirements, with respect to dynamic performance, have a number of built-in opposing aims, e.g.:

- **Drum size - shrinking and swelling.** Both natural circulation and once-through boilers need a reservoir for absorbing shrinking and swelling during the dynamic operation of the boiler (for example start-up or a sudden load change). Depending on the design of the drum, i.e. drum internals etc. the allowable water level fluctuations will be limited, e.g. from 10\% below Normal Water level to 10\% above Normal Water level.

- **Drum size - steam quality.** Depending on the end-use of the steam production, different steam quality (i.e. dryness) requirements are defined. A better steam quality required corresponds to a smaller accepted carry-over\(^6\). The steam quality performance is closely related to the size of the steam space (i.e. boiler drum). Depending upon the boiler plants operation philosophy the requirements with respect to steam quality will define the size of the drum. A quick start-up (or load change) on the boiler will therefore require a relatively large drum, though this limits the allowable gradients on the plant.

- **Drum size - stress level.** To be able to absorb the fluctuations within the boiler a large reservoir is required/desirable. The material thickness in a pressurized vessel (the reservoir) is approximately proportional to the diameter, i.e. the higher pressure, the larger material thickness. However, the allowable temperature gradients for the pressurized vessel decrease as the wall thickness increases. The stresses introduced in the thick-walled boiler parts related to temperature gradients are approximately proportional to the square of the material thickness.

- **Control system.** Depending on the complexity of the boiler control system the water level fluctuations can be controlled (i.e. limited), meaning that the required dynamic performance can be obtained with a smaller (i.e. cheaper) boiler. On the other hand a more complex control system is a larger investment (a typical control system is approx. 20\% of the boiler plant cost).

- **Drum size - pressure gradient.** For most boiler plants the dynamic performance is closely related to the allowable gradients in the thick-walled boiler parts. For boilers producing saturated steam the temperature gradients are closely related to the pressure gradient, \(dT_{\text{sat}}/dp = f(p_{\text{sat}})\), meaning that the pressure gradients define wall thickness requirements and hereby the boiler volume.

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\(^4\)The installation of wind mills in larger numbers in certain regions (for example Denmark) has especially increased the challenges related to the dynamic operation of boilers.

\(^5\)Main engine, boiler, auxiliaries etc.

\(^6\)See Appendix A.
Introduction

- Dynamic vs. static operation. Depending on the application of the boiler plant the importance of the boiler dynamic capability can vary significantly. For some applications (e.g. stand-by boilers foreseen to start producing steam very rapidly in special situations) a unique dynamic performance is required and therefore possesses a high value. For other applications the dynamic performance is of minor importance (for example base load plants foreseen to be operating at stationary load most of the time).

- Boiler construction and choice of materials. Depending on the boiler plant requirements, with respect to weight, height, foot print etc. opposing aims in the design process will press towards the cheapest boiler. For example boiler design based on few long tubes will be cheaper than boiler constructions based on many short tubes (fewer weldings etc.). Furthermore, the optimization could be extended to include exploitation of more advanced materials, i.e. alloyed materials with better material properties, e.g. higher allowable stresses. If this dimension is included in the optimization, the different manufacturing technologies applied for the different materials should also be included.

1.2 Challenges and scope of the present study

A number of opposing aims between a good dynamic performance and a cost-effective boiler design exist.

The objective of the present study is to:

- Establish a framework where dynamic performance is included in the optimization.

Furthermore to:

- Develop dynamic boiler models to be used, defining selected constraints for the optimization method. And for selected boiler types experimentally verify the models developed.

The optimization study will be based on a model consisting of, an Objective Function to be minimized, chosen Design Variables to be varied to minimize the objective function, and a number of Constraints defining the feasible set where the optimization is carried out.

The feasible set is constrained by:

- the size of the boiler - minimum and maximum
- the boiler load gradients/changes (min/max) on the heat input to the boiler (boiler load gradients)

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7Actually this dimension would require deeper in-sight into the different competence levels at different manufacturing locations, e.g. the Far East versus Western Europe.
the required steam drum volume to absorb shrinking and swelling related to dynamic operation

- the required steam drum volume to ensure the steam quality.

For specifying the constraints, dynamic models for analysis of boiler performance have been developed for water tube and fire tube boilers.

The framework for the optimization must be prepared openly, i.e. it should, at a later stage, be possible to include more Design Variables (for example pressure gradient) and a corresponding quantification as extra dimensions. In this manner the model could continuously be refined and extended to take more aspects into consideration.

1.3 Limitations

The main objective of the present study is to establish a framework for optimization, this means that a number of the above mentioned opposing aims will not be analyzed, i.e. those related to:

- Stress level. The proportionality between wall thickness and drum diameter means that increased requirements with respect to volume, i.e. diameter, yields a larger wall thickness. This limits the allowable gradients and hereby the dynamic performance, and this analysis has not been included.

- Control of boiler plants. Within this area intense research and development has been on-going related to hardware (computer based control systems) and software (advanced control algorithms). A better dynamic performance can, without doubt, be obtained by improving the control system. This has not been part of the present study.

- Pressurization of boiler plants. As the focus on dynamic boiler operation is expected to increase, the allowable gradients will naturally be challenged, but it has not been part of the present study - see Figure 2.3 and [136].

- Optimizing the complete boiler system - including all heating surfaces (i.e. distribution of heating surfaces, configuration etc). For these analysis, where for example exergy-destruction could be the object function to minimize, Pinch-technology could be applied. This has not been part of the present study.

- Optimization of material qualities to include material costs, manufacturing techniques and costs, etc. In this area intense research is ongoing, related to both material and manufacturing technology. This has not been included in the present study.

Furthermore, the present study has been limited to optimization of sub-critical steam boilers - see Appendix A.
**Introduction**

The main objective of the developed models for simulating the boilers dynamic performance is to define constraints for the optimization tasks. This means that a number of simplifications and presumptions have been made when developing the dynamic models and these are specified in the relevant sections.

A number of *inputs* to the study will be taken from the boiler lecture/theory (both *openly available* and company proprietary) and the validity of this will not be challenged in the study. This counts for:

- the allowable gradients in thick-walled constructions - see Figure 2.3
- the requirements with respect to steam space load - see Figure 3.1.

These phenomena and their consequences will be discussed in the thesis.
Chapter 2

Design of Boilers

2.1 Introduction

Historically the development within design of boilers has been closely related to the development of digital computers. Before computers the design of boilers was very tedious, and the solutions were not normally optimized for a complex pattern of operation. At that time the design of boilers had to be carried out as hand-calculations, i.e. either calculation by means of pen and paper or by means of different sets of curves or other graphical methods. Naturally, this approach limited the amount of calculations, and only relatively few attempts to optimize the design was carried out. Often boilers were designed more or less according to experience gained from previously designed plants. The different constitutive relations and thermodynamic state equations at the same time also had to be dealt with in a rather simple manner. The boiler business is relatively small and most boiler manufacturers have developed their own designs, which normally have not been standardized to a very high degree.

Designing boilers a distinction has to be made between static operation point of view and dynamic operation point of view.

Design for Static Operation, the traditional approach to boiler design, typically deals with performance of a given design exploiting the heating surfaces as efficiently as possible.

Design for Dynamic Operation of boilers has traditionally focussed on avoiding over stressing of the materials and limiting the temperature gradients. Furthermore, the shrinking and swelling of the water level in the drum has been analyzed and used to determine the required boiler drum size.

\(^1\)The reader should realize that this is only approximately 20 years ago.

\(^2\)It has been the tradition to design plants using standard components (e.g. engines, gas- and steam turbines) and afterwards design the boilers to meet the requirements of these specific components. Though for larger utility units there is a stronger coupling between design of the prime mover and the waste heat recovery unit.
More interested readers are referred to [6] and [7].

2.2 Design of boilers for static operation

Boiler design is, in principle, a matter of:

- Specifying an overall heat balance for the boiler to establish a *First Law of Thermodynamics* relationship for the complete boiler.
- Calculation of performance for a number of components, e.g. combustion chambers, economizers, superheaters and air pre-heaters.
- Using a number of constitutive relations, normally empirical relations for energy transfer, pressure loss, chemical reactions etc.
- Using thermodynamic state equations defining the relationship between the different thermodynamic properties (e.g. pressure and enthalpy). The most important thermodynamic relations are:
  - the water/steam properties
  - the flue gas properties
  - the combustion relations.
- Defining relations describing the coupling of the boiler components, i.e. how does the flue gas and/or the water/steam flow - see Figure 2.1.

The principle approach for design of boilers is different for:

- fired boilers (oil, gas, solid fuel etc.)
- waste heat recovery boilers (engine, gas turbine, process etc.)

and for the hybrid:

- waste heat recovery boilers with supplementary firing (engine, gas turbine, process etc.).

For fired boilers an overall heat balance is needed to integrate all the components, the purpose of the heat balance is to determine the heat input to the boiler, i.e. the energy fired into the boiler\(^3\). After having determined the heat input into the boiler, the energy fired into the furnace (i.e. combustion chamber) is known, and the furnace calculation can be carried out. The outputs from the furnace calculation are:

\(^3\)For waste heat recovery boilers the heat input to the boiler is typically externally given and cannot normally be controlled by the boiler.
Figure 2.1: Example of schematic flow of air, flue gas and water/steam in a fired boiler (multi stage heating surfaces are numbered in the water/steam flow direction) - see Appendix A.

- the flue gas composition, \( x_{O_2}, x_{N_2}, x_{CO_2} \ldots \)
- the furnace outlet temperature, \( T_{fur, out} \)
- the energy transferred from the flue gas to the water/steam cycle, \( \dot{q}_{fg\rightarrow w/s} \)

and for the more advanced furnace calculation routines:

- the flue gas velocity distribution profile
- the flue gas temperature distribution profile
• the furnace outlet gas composition (distribution across the outlet area)

• the furnace outlet emissions.

After finalizing the furnace calculation, the performance of the boiler components can be calculated and hereby the overall boiler performance - including the boiler efficiency (for fired boilers). The boiler performance calculation can include several energy flows: flue gas → high pressure steam, flue gas → low pressure steam, flue gas → combustion air, losses to the surroundings, etc. In general all these energy flows are outputs from the boiler performance calculations of the different components - see Figure 2.1.

Traditionally boiler performance calculations have been based on this component oriented approach and the philosophy has been to calculate the performance for each component successively and iterate until overall convergence is obtained - see [155] and [74]. Newer programs are based on solvers solving the entire systems of equations simultaneously - see [175].

The design of the different boiler components (i.e. furnace, superheaters, evaporators, economizers, air preheaters etc.) is typically based on openly available theory, as a basis, supplemented to some extent, by manufacturer methods of calculations. Apart from a few standardized boiler types, most manufacturers have their own boiler design and a design basis related to this. In general, this does not mean that one boiler manufacturer has a design which is more correct than another; it is a result of a process where the different manufacturers carry out performance tests on their boilers and assess to what extent the different theoretical formulae have to be corrected to fit their specific design. Examples of programs for boiler design can be found in [175], [74], [155] and [188].

The most critical component to design is the furnace. This area has, in recent years, been the subject of several research projects, e.g. [77], [16] and [17]. The results from these research activities are, today, integrated in several boiler manufacturers design. In general it has been realized that gaining deeper insight into especially the emission forming mechanisms and control requires detailed knowledge about the processes going on in the combustion chamber. Over the years many different approaches have been applied for furnace design:

• fully empirical models (e.g. \( T_{fur, out} = 1.200 \, ^\circ C \))

• semi empirical models (e.g. \( T_{fur, out} = C \cdot \frac{\dot{q}}{A} \, ^\circ C \))

• models with (partly) theoretical basis

\[
T_{fur, out} = 52.4 \cdot \left( \frac{\dot{q}}{A} \right)^{0.25} \, ^\circ C \text{ and } \dot{q}_{fur\rightarrow w/s} = C \cdot A \cdot (T_{comb}^{4} - T_{wall}^{4}).
\]

The dramatic increase in computer capacity the latest years combined with intensive development of especially Computational Fluid Dynamics have initiated, and been a catalyst for, the development of especially Computational Fluid Dynamics have initiated, and been a catalyst for, the development of

\footnote{Except for very simple boilers; different boiler manufacturers will typically calculate (a slightly) different performance. Though this does not include flue gas recirculation.}
far more advanced furnace models, and some of these have reached a level, where they are applicable for practical use in the boiler industry. The more advanced furnace models (i.e. models having, for example, temperature and velocity distribution profiles as output) yield new opportunities for optimizing boiler design, for example more compact boiler designs taking advantage of an unevenly distributed flow out of the furnace.

The design of combustion chambers is closely linked to the design of burners as the optimum performance requires that combustion chamber and burner (and control system) are designed and optimized as an integrated unit - see [128].

Using the furnace outlet conditions as input for the rest of the boiler calculation, the performance of the heating surfaces can be determined. The heating surfaces consist of:

- tube bundle heat exchangers (economizers, evaporators, superheaters etc.)
- water/steam cooled cavities (including water/steam cooled heat exchanger enclosures)
- support tubes
- air preheaters (tube heat exchanger, Ljungström types etc.).

For a more detailed description of the different components - see [6] and [7].

The most efficient use of heating surfaces is obtained if these, from an overall point of view, are configured as a counter-current flow heat exchanger - see Figure 2.2; i.e. the coldest flue gas heat exchanges with the coldest water/steam, the medium temperature flue gas heat exchanges with the medium temperature water/steam etc. Boiler design is therefore the art of approaching the counter-current flow principle to the greatest possible extent without compromising items such as, for example, the material temperature. The latter especially often causes deviations from the counter-current flow principle as the highest flue gas temperatures are achieved in the furnace and to avoid material failure due to over-heating, an effective cooling on the water/steam side is required, i.e. evaporation - see Appendix B.

In Figure 2.2 the flow in the evaporator is shown as counter-current flow with the flue gas flow. As the temperature on the water/steam side in the evaporator is constant this does not affect the energy transfer. For the water tube boiler analyzed in the present study, the water/steam and the flue gas are in co-current flow.

Depending on fuel, ambient temperatures etc. the counter-current flow principle often cannot be utilized in the coldest end of the boiler, as this for some operation conditions can cause low temperature dew-point deposits, corrosion etc. and thereby clogging or material failure\(^5\).

\(^{5}\)For some special applications requirements with respect to temperature levels can be defined at different locations in the boiler, for example a boiler equipped with a DeNOx-catalyst or a municipal waste boiler.
For the non-fired boilers (i.e. waste heat recovery boilers), the above described complexity with respect to furnace calculations will normally not be present, and the boiler design will typically be correspondingly simpler.

For fired boilers the highest flue gas temperature (in the furnace) will typically be much higher than for waste heat recovery boilers, this means that the overall counter-current flow principle cannot be exploited all the way through the boiler. Typically the combustion chamber will be enclosed by water/steam cooled walls, where evaporation takes place. It is necessary to take advantage of the high coefficient of heat transfer from evaporation for cooling the furnace walls and keeping the material temperature at an acceptable level.

A fired boiler with water/steam cooled walls will normally be equipped with several evaporation circuits. This means that part of the evaporation takes place with several hundreds (maybe more than a thousand) degrees K temperature difference. For this reason the term pinch point (see Figure 2.2) does not have the same importance for fired boilers as for waste heat recovery boilers.

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6This counts for a natural circulating (sub-critical) boiler.
The output from the boiler performance calculation typically consists of the performance of a pre-defined geometry, i.e.:

- the furnace heat load ([W/m²] and [W/m³])
- the water/steam and flue gas temperatures before and after the different components
- the material temperatures in the different boiler components
- the heat transfer to/from the different media
- the water/steam and flue gas velocities
- the pressure losses on the flue gas - and water/steam-side
- the velocities on the flue gas - and water/steam-side
- the absolute pressure at different locations in the boiler, e.g. furnace

and for drum boilers:

- the circulation number in the evaporator circuits
- the steam space load.

To the author’s knowledge design programs for optimizing boiler configurations have only been developed for very simple and fully predefined boiler geometries (for example waste heat recovery boilers and 3-pass boilers - [154]). For more complex geometries (for example power plant boilers), the manufacturer will typically design the boiler as described above and carry out the overall design procedure as an iteration, where the above mentioned output from the performance calculation (based on the predefined geometry) is evaluated after each iteration and the geometry corrected afterwards. Evaluation criteria for these, inter iterative, outputs will not be given here, references can be made to [153], [6] and [7].

One of the costly components in a boiler system is the steam drum. When boiler drums are designed a number of parameters have to be taken into consideration:

- the steam space load
- shrinking and swelling
- the manufacturing
- internals (separators, feed water preheaters, atemporators, stand-by heating etc.)
- the thermal stresses
- limits in the use of tube expansions.

For more details on design of steam drums reference should be made to [120].
2.3 Design of boilers for dynamic operation

As mentioned in Chapter 1 boilers have traditionally been designed for static operation without paying much attention to dynamic operation of the plant. This is partly a result of the fact that the requirements with respect to dynamic operation have traditionally been very limited, and partly because of the lack of expertise within calculation of dynamic boiler performance. Traditionally design of boilers for dynamic operation has been limited to:

- avoid over-stressing of the thick walled components (drum, superheater headers etc.)
- avoid problems with shrinking and swelling in the boiler reservoir (the drum).

2.3.1 Temperature gradients in thick walled pressurized components

The problems related to temperature gradients for thick walled pressurized components are detailed described in [32].

The calculation of the allowable gradients according to, for example [136], is rather comprehensive, as many parameters have to be included (for example specific design details of the pressure vessel as T-pieces, nozzles etc.). The outputs from the calculation are:

- the allowable temperature gradient with respect to time, i.e. \( \frac{dT}{dt} \) [K/sec]
- the allowable spatial temperature difference, \( \Delta T \) [K/m] resp. [K/mm], i.e. \( \frac{dT}{dz} \)

and both have to be considered in the design of the boiler. Output from a typical calculation is shown in the Figure 2.3.

In Figure 2.3 (left diagram) the curve with index 1 represents the start-up procedure with increasing temperature. The curve with index 2 represents the shut-down procedure with decreasing temperature \( (\Delta T = T_m - T_{int} > 0) \), i.e. the mean material temperature is higher than the water/steam temperature). In Figure 2.3 (right diagram) the curves define the allowable temperature gradients during start-up (i.e. positive temperature gradients, index 1) and during shut-down (i.e. negative temperature gradients, index 2).

A plant will be designed for a given number of cycles (i.e. start-up/shut-down procedures) during its lifetime. A larger number of cycles correspond to lower allowable temperature gradients and vice versa.

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7To the author’s knowledge [136] is the only design norm specifying requirements with respect to temperature gradients.

8A start-up of a plant is in principle similar to a load increase on the plant. This is similar for shut-down versus load decrease. Depending upon the application of the plant the start-up could also include a pressurization that would affect the gradients on the plant.

9The allowable gradients are rather sensitive to the number of cycles, i.e. the actual operation conditions have to be analyzed and planned carefully.
Figure 2.3: Allowable spatial temperature gradients and allowable temperature gradients with respect to time (feasible area between the curves) - see [136] and Figure 3.3.

Figure 2.4: Saturation temperature and temperature gradient versus pressure for water/steam.

If the plant, for example, is started up with lower gradients than the design values, this surplus\textsuperscript{10} can be saved for later use, i.e. more cycles or cycles with larger gradients\textsuperscript{11}. The two gradient curves on

\textsuperscript{10}Examples where the plant is designed for fewer start-ups/shut-downs in the entire lifetime are seen - see [159] and Appendix C.

\textsuperscript{11}By monitoring the actual operation conditions (number of cycles and gradients) the lifetime consumption can be tracked continuously.
Each diagram are linked together\textsuperscript{12}, i.e. if a smaller negative gradient is required, the curves can be displaced parallel upwards to allow for larger positive gradients and vice versa. In practice most boilers will remain pressurized after shut down, i.e. the only negative temperature gradients the boiler will experience are due to the temperature loss because of heat loss to the surroundings (cooling down), which is almost zero. This means that the curves can be displaced parallel upwards to zero negative gradient. For boilers producing saturated steam the temperature gradient with respect to time, $dT/\text{dt}$, determines how fast the pressure can be raised on (or lowered). For the evaporator, where the two-phase zone is present and where the boiler is equipped with the most thick walled components (typically the drum), the temperature gradient with respect to time is similar to the change in saturation temperature with respect to time, i.e.:

$$\frac{dT_{\text{boi}}}{\text{dt}} = \frac{dT_{\text{sat}}}{\text{dt}} = \frac{dT_{\text{sat}}}{dp} \cdot \frac{dp}{\text{dt}},$$

where $dT_{\text{sat}}/dp$ is a physical water/steam property and $dp/\text{dt}$ is determined by the operation of the plant. As $T_{\text{sat}} = f(p_{\text{sat}})$, the temperature gradient in the evaporator is a function of the pressure gradient. The gradient of the saturation temperature with respect to pressure, $dT_{\text{sat}}/dp$, is much larger at low pressure than at higher pressure (see Figure 2.4), i.e. the allowable pressure gradient for the boiler during low pressure operation will be lower than during operation at higher pressure levels.

When the boiler is operated during, for example, start-up or load change it is normally not the temperature that is controlled, but the pressure (or the steam production). Different philosophies are used for the pressure control of boilers, depending on the actual application, the boilers are either operated with fixed pressure or with sliding pressure (see Figure 2.5).

If the boiler is operated with sliding pressure, this will typically not be active in the complete operation range, as an example the boiler could operate with fixed pressure until 50% load (steam production)

\textsuperscript{12}The stress phenomena experienced during the cyclic load of the thick walled components is a fatigue stress phenomena where the linking together of the curves is well known.
is reached and then with sliding pressure to full load. One of the advantages of this operation philosophy is that the volumetric flow of steam is (almost) constant in the sliding pressure range, which is beneficial for example for a steam turbine that can operate with fully open control valves (limit losses) - an example of this can be seen in [159]. The pressure control during the initial phase of the start-up (shown dotted in Figure 2.5) will typically depend on the actual plant configuration\textsuperscript{13}. For some plants cooling of the superheater(s) is not required during the start-up in the initial phase, and it can be carried out without any steam production\textsuperscript{14}. For other plant configurations (i.e. plants with superheater(s) where the main heat input is due to radiation), the cooling of the superheater(s) during the initial phase of the start-up is critical and a certain steam production has to take place - according to the manufacturers specification.

If the firing rate controller loads a certain pressure gradient, \( \frac{dp_{\text{boi}}}{dt} \), on the plant, the allowable pressure gradient will depend significantly on the actual pressure on the boiler, \( (\frac{dp_{\text{boi}}}{dt})_{\text{all}} = f(p_{\text{boi}}) \). In Figure 2.6 the allowable pressure gradients, corresponding to a given allowable temperature gradient, are shown.

### 2.3.2 Shrinking and swelling in the boiler reservoir

Another important parameter to take into consideration designing boiler plants for dynamic operation is the shrinking and swelling of the water in the reservoir (the boiler steam drum).

\textsuperscript{13}The pressure during start-up will always slide from the start pressure to either the Sliding Pressure curve or the Fixed Pressure curve.

\textsuperscript{14}For plants where the start-up is carried out with no steam production in the initial phase, depending upon the design of the plant (i.e. the overall heat balance), problems such as steam production in the economizer can be seen.
The shrinking and swelling phenomena are closely related to changes of load and pressure on the boiler. During steady state operation the evaporator and the riser pipes will contain a certain amount and mixture of water and steam and the *feed water controller* will maintain the water level in the drum at the *normal* \(^{15}\) water level, NW.

At lower pressures the steam in the evaporator can *take up* an especially large volume. As the shrinking and swelling are closely related to the volume of water and steam in the evaporator the total volume of the evaporator (including connecting piping) is an important parameter. For boilers designed with a relatively small pinch-point (see Figure 2.2) the evaporators size (i.e. volume) will increase dramatically. This means that the volume to be *ejected* from or fed into the evaporator from the steam drum will increase correspondingly.

If the firing rate, \(^{16}\) for example, is increased the steam production in the evaporator will increase and presuming that the *pressure controller* maintains the pressure on the boiler, the volume of the steam in the evaporator will increase (see Figure 2.7). The only place the evaporator can *deliver* this extra volume is in the steam drum which means that the water *ejected* from the evaporator is sent to the drum where the water level increases - *swelling*.

The opposite situation occurs if the firing rate is decreased causing the steam production in the evaporator to decrease and hereby the steam in the evaporator to take up a relatively smaller volume. In this situation the volume in the evaporator is filled up from the drum which means that the water level in the drum is lowered - *shrinking*.

\(^{15}\)Typically at the drum center line - see Appendix C.

\(^{16}\)Change of load on the gas turbine for a waste heat recovery boiler corresponds to change of firing rate on the fired boiler.
Shrinking and swelling can be experienced in the same manner if the pressure on the boiler is decreased or increased. Increasing the pressure will cause the steam fraction in the evaporator to take up a smaller volume, i.e. lower the water level in the drum as the filling up of the evaporator is fed from the drum and vice versa for decreasing pressure. Normally the fluctuations in the water level in the drum due to changes in pressure are not as significant as the variations due to load change. This is due to the fact that typically the buffer capacity in the boiler limits the gradient for changing the pressure and the pressure changes will typically occur at normal operation where the relative change in volume of the steam in the drum due to the higher pressure is smaller\textsuperscript{17}.

The shrinking and swelling phenomena are experienced in the same manner for water tube and fire tube boilers as the same physical phenomena are taking place. The pinch-point defines the size of the evaporator (see page 25) and hereby the degree of shrinking and swelling.

A simple feed water controller will actually have an inverse response. If, for example, the water level is falling in the drum, the feed water controller will start to fill more water on the boiler. Since the water is normally sub-cooled (the degree of sub-cooling depends upon the economizer configuration) this will condense part of the steam in the boiler steam drum (and evaporator) causing the water level to lower even further. The water level will start to rise as soon as the steam production from the boiler increases - this will happen as the condensation of steam lowers the pressure on the boiler causing the firing rate controller to increase the heat input to the boiler and hereby increase the steam production.

In the following chapters the challenges related to design of boilers for dynamic operation including the described physical phenomena will be analyzed and described.

\textsuperscript{17} An exception is pressure changes during start-up of a cold boiler, i.e., a boiler started up from effectively zero pressure - see page 14.
Chapter 3

Objects of Design Optimization

3.1 Introduction

One of the major challenges designing boilers is, naturally, to minimize the mass of the boiler, i.e. the cost of the boiler\(^1\). This minimization should be carried out taking all other design parameters into consideration. Among the important design parameters, which can affect the optimum design of the boilers are:

- the number of operating hours per annum (boilers lifetime)
- the number of starts and stops in the plants lifetime
- the price of working hours in the manufacturing (production costs) versus material prices
- requirements with respect to dynamic performance
- requirements with respect to space occupied by the boiler (e.g. foot print)
- requirements with respect to the weight of the boiler (e.g. operational weight)
- the (mix of) fuels to be burnt in the boiler
- the local price level of manpower, fuel, etc.
- the required availability of the plant
- the required efficiency of the plant.

\(^1\)Given the same type of material.
In general the actual application of the boiler affects the optimum design of it to a large extent. If, for example, the boiler is foreseen only to be operated for a very few hours per year it may not be beneficial to optimize the boiler in a traditional manner, i.e. increase boiler efficiency. In other cases minimizing the mass and/or footprint of the boiler could be very important (for example oil platform or marine application), which again means that it would not necessarily be beneficial to optimize the boiler efficiency, i.e. the feasible set for optimizing the boiler design will be limited by the size of the boiler.

3.2 Optimizing design and operation of boilers

3.2.1 General

Optimizing design and operation of boilers from a dynamic point of view is a challenge where a number of design variables and related opposing aims have to be taken into consideration:

- Drum size - shrinking and swelling
- Drum size - steam quality
- Drum size - stress level
- Control system
- Drum size - pressure gradient
- Dynamic vs. static operation
- Boiler construction and choice of materials.

For a more detailed description of the opposing aims - see page 2.

3.2.2 Temperature gradients

Dynamic operation of the boiler means to be able to deal with the gradients, which the boiler will unavoidably experience. The allowable gradients are, for boiler manufacturers and the approving authorities\(^2\), given according to the norms [136] and manufacturing standards - see Figure 2.3.

The actual stress in the boiler material, \(\sigma_{\text{act}}\), which always has to be below the allowable stress level, \(\sigma_{\text{all}}\), can be seen as a sum of the stresses related to the internal pressure, \(p_{\text{boi}}\), in the boiler and the stresses related to the temperature gradients of the boiler components, i.e.:

\[
\sigma_{\text{act}} = \sigma_{p, \text{boi}} + \sigma_{dT/dt} \leq \sigma_{\text{all}}. \tag{3.1}
\]

\(^2\)For example DNV (see [165]), Lloyds (see [178]), TÜV (see [189]) and ABS (see [161]).
For thick walled components (i.e. pressure vessels) the stresses introduced because of the internal pressure, can be calculated according to the well known boiler-formula\(^3\) (in German: Kesselformel) [29] - see Equation 4.3 and 4.4.

\[
s_{boi,min} = \frac{p_{boi} \cdot d_{int}}{2 \cdot \sigma_{all}}
\]  

(3.2)

According to [29] the stresses introduced due to temperature gradients are proportional to the square of the wall thickness of the components and the temperature gradient, i.e.:

\[
\frac{\sigma_{st}}{\sigma_t} \propto s^2 \cdot \frac{dT}{dt}
\]  

(3.3)

A detailed derivation of the relationship between introduced stress and wall thickness can be found in [117].

This means that increasing the boiler volume for improving the dynamic performance (shrinking and swelling and steam space load) could actually, as a result of the physically required increase in wall thickness, result in restrictions on the allowable temperature gradients and hereby introduce heavy restrictions on the allowable pressure gradients.

\[m_s^3/m_{dr}^3 \cdot h\]

Steam Space Load

![Figure 3.1: Steam space load - see [153].](image-url)

\(^3\)The boiler-formula is applicable for cylindrical drums.
3.2.3 Shrinking and Swelling

As described in section 2.3.2 the water level in the drum will fluctuate due to changes in the pressure and the heat input to the evaporator (see Figure 2.7). For lower pressure levels the volume of the steam fraction in the evaporator will be relatively large causing larger fluctuations as a result of the changes in heat input or pressure level. For higher pressure levels the variations will be correspondingly smaller.

As the feed water controllers main purpose is to control the water level in the boiler steam drum, the design of the feed water controller may (strongly) affect the amount of shrinking and swelling.

Shrinking and swelling is closely related to the firing rate of the boiler (see Figure 2.7), i.e. increased firing rate gradient will compress the time interval where the shrinking and swelling affects the water level causing larger fluctuations in water level, and these fluctuations are a strong component in the specification of drum volume and thereby the wall thickness - see Chapter 3.2.2.

3.2.4 Steam Space Load

A boiler is normally designed with a certain water/steam volume to be able to absorb the fluctuations in the water level during dynamic operation of the plant. Furthermore, the steam space load has to be taken into consideration. As a rough rule of thumb for a certain quality (i.e. dryness) of the steam leaving the boiler, requirements can be put on the boiler drums steam volume in relation to the total steam production, i.e. \[ \frac{m_s}{m_{dr} \cdot h} = 1/\text{h} \]. This figure is normally given as a band between 2 curves (see Figure 3.1) specifying the required steam space. The shape of the curve(s), i.e. higher pressure correspond to lower specific steam space load, is related to the fact that the higher pressure, the smaller difference between the specific mass of water and steam. At this stage no relationship between the steam production and the water space/volume can be given.

\[ \text{Figure 3.2: Steam space load including the conductivity in the boiler drum water - redrawn from [45].} \]

4The band illustrates the uncertainty on the required steam space load, where for example salt content and efficiency of water/steam separation in the drum has to be taken into consideration - see Figure 3.2.

5For the water volume no requirements are specified (in principle once-through boilers do not have a water reservoir), but in general the water accumulated in the boiler drum is the greatest thermal buffer in the system, i.e. the stabilizer of the
Different types of *Steam space load* curves have been prepared over the years - see Figure 3.2.

Another philosophy for determining the required steam space in the boiler drum combined with the efficiency of the separation equipment in the drum is to specify the maximum *carry-over*, e.g. 0.1 % - [24].

### 3.2.5 Control of Boilers

Control of boiler is an area where intense research has been carried out during the last years. The control of boilers has traditionally been based on two controllers:

- *feed water controller*
- *firing rate controller*.

Traditionally these controllers have operated independently of each other, and not taking advantage of, for instance, *feed forward* signals from one controller to the other.

As the control of boilers has become fully computer based more advanced control algorithms can be easily implemented - see [64].

Furthermore, the more advanced controllers have begun to exploit more measurements (inputs) from the plants for improving the control, e.g. flame temperature.

For the present study the *feed water controllers* will be *single point feed water controllers* and the *firing rate controllers* will be generic for the two boiler types analyzed - see Chapter 6 and 8.

### 3.2.6 Pressurization

The requirements with respect to pressurization are very different and depend upon the use of the steam produced. The pressurization limitations are also closely related to the allowable temperature gradients - see section 2.3.1.

The relationship between allowable temperature gradients and wall thickness can be seen in Figure 3.3. The surfaces in the figure specifies the allowable gradients for different drum diameters. The required wall thicknesses are calculated for a given internal pressure - the surfaces are *centered* around 0 K/min, but could be displaced parallel up- or downwards depending upon the actual requirements - see page 14.

---

*system pressure. Requirements with respect to water volume are related to safety aspects, e.g. the boiler must not *dry out* due to lack of water.*

*For plants equipped with multi-stage superheaters a *superheating temperature controller* also has to be implemented. Furthermore relatively advanced controllers have been developed for plants configured for automatic start-up and shut-down - see [159] & [158].*
Allowable temperature gradients

![Temperature Gradient Graph]

**Figure 3.3:** Allowable temperature gradients (between the surfaces) for different drum diameters. The required wall thicknesses are calculated for a given internal pressure - calculations are carried out according to [136] by means of [105]. The figure is based on data for the water tube boiler test plant - see Appendix C.

From Figure 3.3 it can be seen that increasing the drum diameter, which corresponds to increasing the required wall thickness, decreases the allowable gradients - *narrow the surfaces*.

For plants where the steam is used, for example for heating purposes, the pressurization requirements of the plant will typically be rather *limited*. This is very different for plants where the steam is foreseen to be used in, for example, a steam turbine. In these situations a certain minimum pressure must be reached before the steam turbine can be put into operation\(^7\), i.e. steam production before this pressure is reached is (in principle) *lost*.

For the present study where the optimization has been limited not to include the pressurization of the plant, it has been chosen to control the pressure on the boiler according to Figure 7.1 for the water tube boiler and as described on page 84 for the fire tube boiler.

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\(^7\)Furthermore, for steam turbines it is typically required that a certain superheat is reached, e.g. 50 K above saturation temperature.
3.2.7 Optimizing complete boiler design

For the present study the optimization has been limited to the boiler steam drum, (water tube boiler) and the integrated drum/evaporator (fire tube boiler). The design of the evaporator (and/or the complete boiler including all heating surfaces) could potentially be included for further optimization. Designing evaporators for smaller pinch-points (i.e. higher boiler efficiency), see Figure 2.2, requires larger evaporator heating surfaces. The relationship between decrease in pinch-point and increase in evaporator size is exponential. This means that for evaporators designed with a relatively large pinch-point (for example 30-50 K) there is almost a linear relationship between increase in steam production (i.e. pinch-point decrease) and increase in evaporator size - see Figure 3.4. For plants designed with smaller pinch-points (for example 5-10 K) the exponential slope is more pronounced. For example, decreasing the pinch-point from 10 to 5 K (which could correspond to a 3 % increase in steam production) could require approx. 30 % extra heating surface.

A very important parameter, which at some stage should be included in an overall optimization of the boiler concept, is the design of the evaporator circuit including the circulation pumps, piping etc. As the quality, $x_{ev}$, in the evaporator circuit is the inverse of the circulation number, $N_{circ}$, an increased circulation number lowers the quality. This means that the amount of water ejected from the evaporator heating surface decreases (see page 17) which limits the shrinking and swelling in the steam drum.

A completely different approach for optimizing boilers for dynamic operation could be taken if focus were put on materials used and the manufacturing of the boiler. The first step could be to use higher alloyed materials, but even further steps could be taken in the direction of new materials, e.g. fiber- or polymeric based materials. This approach would entail the inclusion of manufacturing techniques. These have not been in concern in the present study.

3.2.8 Dynamic operation capability

One of the large challenges in the present study has been: how to quantify the plants capability with respect to dynamic operation?

Firstly, quantifying a boiler concept/configuration in favor of another requires an objective set of criteria for evaluating the concepts against each other. In the optimization an evaluation of the different concepts/configurations is a matter of minimizing an objective function taking the different design variables into consideration - see Chapter 4.

It is uncommon within the boiler business, to consider the quantification of the plant’s capability with respect to dynamic operation as a nice and smooth function. In the present study it has been analyzed to what extent data for this quantification is available from actual projects.

---

8These figures are from the plant FHKW Linz Süd (Austria) - see [159].

9For carrying out the optimization (depending on the optimization procedure applied) even further requirements with respect to differentiability of the function must be defined, i.e. as minimum a $C^1$ – function.
In general it has not been possible to find an expression for the quantification. From actual projects the following statements have been extracted:

- **the plant must be able to start-up from cold conditions in 20 minutes**
- **the plant must be able to change load by 20 % per minute**\(^{10}\)
- **the pressure on the plant must not be below 10 bar, 12 hours after shut-down.**

As mentioned, optimization requires an analytical function for the quantification - a detailed description of this function is given in section 4.4.2 - page 34.

### 3.2.9 Opposing aims - Optimization Challenges

As can be seen from the descriptions in this chapter a number of optimization challenges are present.

The main objective for the present study is to establish a framework for optimizing the design of boilers for dynamic operation.

Having defined this framework for the optimization each of these challenges could be included in the optimization model as extra dimensions.

For the present study the following dimensions will be included/analyzed:

\(^{10}\)Running plant, e.g. at 5-10 % load.
• Requirements with respect to boiler volume - maximum/minimum.

• The maximum and minimum allowable boiler load changes/gradients.

• The required steam drum volume to absorb the water level fluctuation due to shrinking and swelling operating the boiler dynamically.

• The steam space load defining requirements with respect to maximum allowable specific steam utilization, i.e. \( \frac{m^3_s}{m^3_{dr} \cdot h} = \frac{1}{h} \).

In the following chapters the optimization framework will be defined and the feasible set for the optimization will be limited by the above mentioned dimensions. The developed dynamic models will be applied to define these constraints.
Chapter 4

Boiler Optimization

4.1 Introduction

In this chapter a description of the applied optimization procedure is given, followed by a discussion of the optimization of boilers based on dynamic performance.

The optimization consists of a general part, described in detail in this chapter, and a boiler specific part described in Chapters 7 and 9.

4.2 Theory

In general an optimization task is about minimizing/maximizing an objective function\(^1\). The objective function quantifies the value of the chosen design and by means of the objective function it is possible to evaluate whether one design is better (i.e. closer to optimum) than another design\(^2\).

The optimization should not be limited to economical/technical optimization, it could also be a matter of maximizing/minimizing:

- the environmental impact
- the space requirement
- number of operation hours

\(^1\)The objective function is often named a cost function - a designation that has its origins in the economic optimization, which is an area, where optimization theory has been developed and applied intensively.

\(^2\)The numerical value of the objective function is not necessarily of use as a constant can be added to the function, or the function can be multiplied by a (positive) constant, without changing the location of the optimum.
In this context, a typical optimization task is to minimize, for example, the price of a plant, where the elements are capitalized, in a way, that makes it possible to carry out an overall optimization (for example put a value on emissions, space required etc). For some of the parameters, it is necessary to calculate a NPV (Net Present Value), which is a backwards discounting of the costs during the expected plants lifetime taking the planned operation of the plant into consideration.

This means that multi-variable optimization can be handled in the same way, but the involved objectives are to be included in one objective function.

The first challenge in the optimization is to define a set of design variables. These are the variables to be assigned numerical values as a result of the optimization and the variables to be varied to find the optimum.

When the design variables are defined, a number of constraints have to be defined. The constraints are normally very different in nature, they limit the feasible set (which is a multi-dimensional manifold) where the optimization task is carried out, e.g. material thickness > 0, resource limits, maximum footprint etc. The goal of the optimization is to find a feasible design, i.e. a design on the feasible manifold defined by the constraints. The constraints, which must depend upon the chosen design variables, are different in nature:

- implicit/explicit
- linear/non-linear
- equality/inequality.

Finally, a criteria to be able to judge whether one design is better than an other design is needed. This criteria is formulated in the above described objective function, which quantifies the different designs.

The objective function as well as the constraints may be linear or nonlinear functions.

---

The question about feasible or unfeasible design is very relevant for several of the numerical procedures for optimization, as some of these end up with designs, which are almost feasible (i.e. just outside the feasible set) and have to be brought back into the feasible set.
To summarize: An optimization problem is characterized by having an objective function to be minimized:

$$\text{minimize} \quad F(X) \quad \text{objective function}$$

which is subject to the constrains:

$$h_i(X) = 0, \quad i = 1, 2, \ldots, I \quad \text{equality constraints}$$

$$g_j(X) \geq 0, \quad j = 1, 2, \ldots, J \quad \text{inequality constraints}$$

where

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

is the design variable.

This optimization procedure will be applied in the actual optimization.

### 4.3 Optimizing Boilers with respect to Dynamic Operation

Optimizing the design of boilers, as described in Chapter 3, from a dynamic operational point of view, is a matter of, on the one hand, minimizing the volume of the boiler, \((X_2)\), though meeting the shrinking and swelling demands and meeting the steam space load requirements and at the same time maximizing the load gradient/change on the boiler, \((X_1)\).

This optimization will be carried out specifying an \textit{objective function} as a function of the two design variables, \(X^T = (X_1, X_2)\), and by means of dynamic models for the boilers defining the constraints. After having limited the \textit{feasible set} by means of the constraints, the optimum can be found.

### 4.4 Objective Function

The objective function will include:

- the investment costs
- the operational costs, where the value/quantification of a given dynamic performance is included

---

\(^4\)Without loss of generality we always operate with an objective function to be minimized. If an objective function \(F(X)\) has to be maximized, \(-F(X)\) has to be minimized.
the consumptions related to the dynamic operation of the boiler plant.

The following objective function has been applied for the analysis:

$$F(X) = F_{\text{mass}} + F_{\text{dyn \, op}} + F_{\text{cons}}.$$  \hfill (4.1)

The value of the investment:

$$F_{\text{mass}} = \text{M}_{\text{boi}} \cdot \text{specific material costs}.$$  

It should be noted that the specific material costs is not limited to the purchase price for the boiler material. This may actually make up a smaller part of the specific material costs, which also includes:

- the transportation (to include quantification of, for example, delivery time)
- the handling (could be a discontinuous function, e.g. jump in crane-sizes to be used)
- the manufacturing (welding, control etc.)
- the erection (site work should be limited, e.g. using package boilers)

and in special cases:

- penalty price for weight\(^5\).

The value of the dynamic operation:

$$F_{\text{dyn \, op}} = \text{quantified value of dynamic performance}$$

contains the quantification of, for example, the capability of starting the plant up within x minutes or allowing a certain gradient on the boiler load, \(d\text{P}_{\text{boi}}/dt\). This figure will normally be project specific and can be rather difficult to quantify - see Section 3.2.8.

The consumables during dynamic operation:

$$F_{\text{cons}} = \text{quantified value the consumables during dynamic operation of the plant}$$

quantifies the value of the consumptions related to the dynamic operation of the plant. As mentioned the consumables will have to be backward discounted to calculate a NPV to apply in the objective function. A number of terms could be relevant to include in \(F_{\text{cons}}\), e.g.:

\(^5\)For example for oil platforms designed for a certain load, extra weight due to heavier boilers will normally be penalized to compensate for the extra costs of the platform (prices as high as 8-10 times the material costs have been seen - see [123]).
• the fuel
• the feed water (chemically treated)
• electrical power (for pumps etc.)
• compressed air
• maintenance costs
• operating costs.

These components are rather different in nature, and the quantification of them will be, to some extent, proportional to the operating hours, some of them will consist of an initial investment plus a part proportional to the amount of operation. Finally, there will be some terms just consisting of the initial investment.

For the present study it is presumed that the efficiency of the boiler, \( \eta_{boi} \), is independent of the design chosen as a result of the optimization. As the present study has been limited to analyzing the boiler steam drum and not the entire boiler (including heating surfaces), this is a fair presumption. If this presumption does not hold, the NPV of the differences in boiler efficiency will have to be calculated and included in the objective function \( F_{cons} \).

4.4.1 \( F_{mass} \)

To simplify the model\(^6\) it is presumed that the boiler steam drum is spherical and with the internal pressure and allowable stresses in the boiler material given as input to the model in Figure 4.1. The volume of the boiler can be calculated as\(^7\):

\[
V_{boi} = \frac{4}{3} \cdot \pi \cdot r_{int}^3 \quad \Rightarrow \quad r_{int} = \sqrt[3]{\frac{3 \cdot V_{boi}}{4 \cdot \pi}} \quad (4.2)
\]

and the stresses introduced in the material due to

\[\sigma_{act} \]

[Drawing of a spherical drum with pressure and stresses labeled]

**Figure 4.1:** Model of spherical boiler.

---

\(^6\)The analysis in the present study is limited to the boiler steam drum.

\(^7\)It is here presumed that \( s_{boi} \ll r_{int} \).

\(^8\)This model for stress level in pressure parts is relatively simple, more detailed models can be found in [136], [137], [55], [56], [87] and [27].

\(^9\)\( \sigma_{spherical\ drum} = 1/2 \cdot \sigma_{cylindrical\ drum} \).
\[ p_{\text{int}} \cdot \pi \cdot r_{\text{int}}^2 = \sigma_{\text{act}} \cdot \pi \cdot r_{\text{int}} \cdot 2 \cdot s_{\text{boi}} \Rightarrow \]

\[ s_{\text{boi}} = \frac{p_{\text{int}} \cdot r_{\text{int}}}{2 \cdot \sigma_{\text{act}}} \quad (4.3) \]

\( \sigma_{\text{act}} \) always has to be below the allowable stress level, \( \sigma_{\text{all}} \). Allowable stress levels for different boiler materials can be found in norms and standards, e.g. [26] and [44].

As the thickness of the boiler material should be minimized the maximum allowable stress, \( \sigma_{\text{all}} \), will be applied, i.e.:

\[ s_{\text{boi}, \text{min}} = \frac{p_{\text{int}} \cdot r_{\text{int}}}{2 \cdot \sigma_{\text{all}}} \quad (4.4) \]

Combining Equation 4.2, 4.3 and 4.4, the following relationship between the internal pressure and the required wall thickness can be obtained:

\[ s_{\text{boi}, \text{min}} = \frac{p_{\text{int}} \cdot r_{\text{int}}}{2 \cdot \sigma_{\text{all}}} \cdot \sqrt[3]{\frac{3 \cdot V_{\text{boi}}}{4 \cdot \pi}} \quad (4.5) \]

The mass of the boiler can be calculated as:

\[ M_{\text{boi}} = \rho_{\text{boi}} \cdot s_{\text{boi}, \text{min}} \cdot \sqrt[3]{\frac{3 \cdot V_{\text{boi}}}{4 \cdot \pi}} \quad (4.6) \]

which after inserting Equation 4.2 and 4.5 yields:

\[ M_{\text{boi}} = \frac{2 \cdot \rho_{\text{boi}} \cdot \pi \cdot p_{\text{boi}} \cdot r_{\text{int}}^3}{\sigma_{\text{all}}} = \frac{3 \cdot V_{\text{boi}} \cdot p_{\text{int}} \cdot \rho_{\text{boi}}}{2 \cdot \sigma_{\text{all}}} \quad (4.7) \]

and when the mass of the boiler is known, the mass’ contribution to the cost function \( F_{\text{mass}} \) can be calculated as\(^\text{10}\):

\[ F_{\text{mass}} = M_{\text{boi}} \cdot \text{specific material costs} \quad (4.8) \]

\(^\text{10}\)It is presumed that the cost of the boiler including all the described items is proportional to the mass - see section 4.4.
As most boiler drums are designed and priced individually, data for verifying the price/weight ratio is not generally available.

Presuming that this ratio is the same for waste heat recovery boilers as for steam drums\(^{11}\), the course of \(F_{\text{mass}}\) has been assessed by means of \([154]\), the results can be seen in Figure 4.2.

As can be seen from Figure 4.2 the presumption \(F_{\text{mass}} \propto V_{\text{boi}}\) is almost fulfilled in practice.

### 4.4.2 \(F_{\text{dyn, op}}\)

The value of the boilers dynamic performance capability is difficult to quantify as many application based parameters will affect this.

In general, for very low allowable boiler load gradients, \(dP_{\text{boi}}/dt\), (firing rate on the boiler), it does not make sense to quantify the dynamic performance, as this means that the plant will have to accept very long start-up times. For higher values of \(dP_{\text{boi}}/dt\) the value of increasing boiler load gradient will increase until a certain level has been reached (asymptotic behavior)\(^{12}\).

The challenge is therefore to quantify the value of the dynamic operation in the range between the two extremes: \(F_{\text{dyn, op,max}}\) and \(F_{\text{dyn, op,min}}\). For the present study it is presumed, that the curves have a shape as shown in Figure 4.3. Mathematically this curve can be described as:

\(^{11}\)This presumption is most likely fair as many of the same phenomena/tendencies with respect to: purchase of materials, handling, welding, quality control, shipment etc. are present for both components.

\(^{12}\)The applied objective function is symmetrical in the \(dP_{\text{boi}}/dt\) direction. More detailed studies of the functions shape could assess non-symmetric behavior.
Quantification of $F_{dyn\ op}$

![Graph showing the quantification of dynamic plant operation.]

**Figure 4.3:** Quantification of dynamic plant operation.

\[
F_{dyn\ op} = F_{dyn\ op,max} - (F_{dyn\ op,max} - F_{dyn\ op,min}) \cdot \frac{2}{\pi} \cdot \tan^{-1} \left[ k \cdot \left( \frac{dP_{boi}}{dt} - \frac{dP_{boi,min}}{dt} \right) \right] \tag{4.9}
\]

where $k$ is a factor on the load gradient $dP_{boi}/dt$.

It is advantageous to apply this curve as it possesses the right properties in the extremes, i.e.

\[
\frac{dP_{boi}}{dt} \to -\infty \Rightarrow F_{dyn\ op} \to F_{dyn\ op,max} \\
\frac{dP_{boi}}{dt} \to \infty \Rightarrow F_{dyn\ op} \to F_{dyn\ op,min}.
\]

As can be seen from Figure 4.3 higher values of $k$ will cause the $F_{dyn\ op}$ to asymptotically approach a situation with a discontinuity, i.e. a situation where the plant cannot be utilized for the actual application, e.g. if the boiler is not able to start-up within a certain time, i.e. very high $F_{dyn\ op,max}$. Low values of $k$ causes $F_{dyn\ op}$ to asymptotically approach a situation where the plants dynamic capability has no/limited importance for the customer.

If, for example, a certain minimum boiler load gradient, $dP_{boi,min}/dt$, is required this can either be obtained by defining a constraint ($dP_{boi}/dt \geq dP_{boi,min}/dt$) or by using a high value of $k$ and a high $F_{dyn\ op,max}$ in Equation 4.9. By using the latter method an optimum based on, for example, installing an extra low-capacity boiler (for covering extreme operation conditions) would also be a feasible solution.
For the present study it is presumed that:

\[
F_{\text{dyn op;min}} \propto F_{\text{mass}} \\
F_{\text{dyn op;max}} \propto F_{\text{mass}}
\]  
(4.10)

the following figures are presumed\(^{13}\):

\[
F_{\text{dyn op;min}} = 0,05 \cdot F_{\text{mass}} \\
F_{\text{dyn op;max}} = 0,5 \cdot F_{\text{mass}}
\]  
(4.11)

which, inserted into Equation 4.9 yields:

\[
F_{\text{dyn op}} = 0,5 \cdot F_{\text{mass}} - (0,5 \cdot F_{\text{mass}} - 0,05 \cdot F_{\text{mass}}) \cdot \frac{2}{\pi} \cdot \tan^{-1} \left[ k \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi; min}}}{dt} \right) \right]
\]

i.e.,

\[
F_{\text{dyn op}} = F_{\text{mass}} \cdot \left( 0,5 - 0,45 \cdot \frac{2}{\pi} \cdot \tan^{-1} \left[ k \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi; min}}}{dt} \right) \right] \right)
\]  
(4.12)

### 4.4.3 \(F_{\text{cons}}\)

It is presumed that the \(F_{\text{cons}}\) is constant, i.e., \(dF_{\text{cons}}/dX = 0\).

\(F_{\text{cons}}\) being constant means that the consumption of fuel, water, electricity, chemicals etc. are proportional to the actual load on the plant and independent of whether the plant is operated dynamically or stationary.

As described on page 31 this is a fair presumption and the error introduced will be limited, i.e. the course of \(F_{\text{total}}\) will almost be independent of whether \(F_{\text{cons}}\) is added or not (the change in location of the optimum will be marginal).

---

\(^{13}\)In general the value of \(F_{\text{dyn op;min}}\) must be larger than 0, and the value of \(F_{\text{dyn op;max}}\) must be smaller than the purchase value of the plant, i.e. \(F_{\text{mass}}\). This presumption is fair as it fulfills these requirements and at the same time has a reasonable distance between \(F_{\text{dyn op;min}}\) and \(F_{\text{dyn op;max}}\). Consequences of decreasing \(F_{\text{dyn op;min}}\) or increasing \(F_{\text{dyn op;max}}\) are discussed in Chapter 7 and 9.
4.4.4 \( F_{\text{total}} \)

Summarizing the results from the sections 4.4.1, 4.4.2 and 4.4.3 the following objective function is obtained:

\[
F_{\text{total}} = F_{\text{mass}} + F_{\text{dyn op}} + F_{\text{cons}}
\]  

which after inserting Equation 4.12 yields:

\[
F_{\text{total}} = F_{\text{mass}} \cdot \left( 1,5 - 0,45 \cdot \frac{2}{\pi} \cdot Tan^{-1}\left[ k \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi}, \text{min}}}{dt} \right) \right] \right) + F_{\text{cons}}
\]  

applying Equation 4.8:

\[
F_{\text{total}} = M_{\text{boi}} \cdot \text{specific material costs} \cdot \left( 1,5 - 0,45 \cdot \frac{2}{\pi} \cdot Tan^{-1}\left[ k \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi}, \text{min}}}{dt} \right) \right] \right) + F_{\text{cons}}
\]  

as the numerical value of \( F_{\text{total}} \) is not relevant (see page 28) the specific material costs can be set to unity, which means that Equation 4.15 after inserting Equation 4.7 is reduced to:

\[
F_{\text{total}} = \frac{3 \cdot V_{\text{boi}} \cdot p_{\text{int}} \cdot \rho_{\text{boi}}}{2 \cdot \sigma} \cdot \left( 1,5 - 0,45 \cdot \frac{2}{\pi} \cdot Tan^{-1}\left[ k \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi}, \text{min}}}{dt} \right) \right] \right) + F_{\text{cons}}
\]  

which can be written as:

\[
F_{\text{total}} = C_1 \cdot V_{\text{boi}} \cdot \left( C_2 + C_3 \cdot Tan^{-1}\left[ C_4 \cdot \left( \frac{dP_{\text{boi}}}{dt} - \frac{dP_{\text{boi}, \text{min}}}{dt} \right) \right] \right) + C_5
\]  

and hereby:

\[
F_{\text{total}} = \int \left( \frac{dP_{\text{boi}}}{dt} \cdot V_{\text{boi}} \right)
\]  

As can be seen from Figure 4.4 the shape of the quantification of the dynamic operation of the boiler (see Figure 4.3) can be recognized in the Relative boiler load gradient direction and the linearity of \( F_{\text{mass}} \) can be recognized in the Boiler Volume direction.

4.5 Design Variables

Since the optimization task is to minimize the price\(^{14}\) of the boiler, the following design variables have been chosen for the optimization:

\(^{14}\)Price is seen as a broader term including quantification of other important parameters, see page 31.
For the present optimization task the relative boiler load gradient, \((dP_{boi}/dt)_{rel}\), will be given as input to the dynamic model. As a result of the boiler simulation, the required boiler volume, \(V_{boi}\), for a given relative boiler load gradient, \((dP_{boi}/dt)_{rel}\), can be calculated.

The function shown in Figure 4.4 is a nice and smooth function. Depending on the shape of, for example, \(F_{mass}\) (see Figure 4.2) the function could have a not so nice and smooth course. In the extreme cases discontinuities could be present, but presuming that the overall shape is as shown in Figure 4.4 the location of the optimum design will not be affected, i.e. monotone surface.
4.6 Constraints

A feasible set has to be defined/limited by means of a number of constraints, which for the present optimization task are inequality constraints. For the design variable $V_{boi}$, the following (simple) constraint can be defined:

$$V_{boi} \geq 0.$$  \hspace{1cm} (4.19)
Depending on the application of the plant constraints with respect to maximum allowable boiler volume/size could be defined, e.g. replacement of existing boilers, limited access possibilities etc.

For the design variable \((dP_{boi}/dt)_\text{rel}\), the following (simple) constraints can be defined:

\[
0, 1 \leq \left(\frac{dP_{boi}}{dt}\right)_\text{rel} \leq 1, 0 \tag{4.20}
\]

For the water tube boiler fired by the gas turbine outlet gas flow, the mass flow from the gas turbine during ignition and loading can be seen in Figure 7.4 and the operating temperature course in Figure 7.3. \((dP_{boi}/dt)_\text{max}\) correspond to maximum gas turbine load gradient.

For the re tube boiler analyzed in the present study the ring rate is limited by the fresh air fan load gradient and the burner used in general - for more details on the re tube boiler, see [128], Chapter 8 and Appendix D. \((dP_{boi}/dt)\) is defined as the fraction of Maximum Continuous Rate (MCR) to be loaded on the plant per minute, e.g. 10 %. In the actual optimization task the dynamic operation of the boiler is simulated as a step in the steam production.

Furthermore, the volume of the steam drum has to be checked against the steam space load (see Figure 3.1 and 3.2) and the requirements with respect to shrinking and swelling related to dynamic operation. For the selected operating conditions this is only relevant for the water tube boiler, because the steam production has been controlled in the re tube boiler simulations - see Chapter 9.
Chapter 5

Modelling and Simulation - Methodology

5.1 Introduction

In this chapter procedures for modelling and simulation of dynamic systems are described. Furthermore, the general characteristics of the developed equation systems are described. The equation systems consist of algebraic and/or differential equations depending on the model developed.

A number of tools (i.e. software packages) are available for solving these equation systems. Some of these have been analyzed and for the present study MATLAB/SIMULINK has been chosen for integration of the equation systems - see [46], [65] and [60]. For the present study Modelica has also been considered - see [181] and [132].

A more detailed description of the different tools can be found in [52].

5.2 The Modelling Process

Modelling and simulation of, for example, energy systems is a process, which for clarity can be split in the following steps [51] and [50]:

- Developing a physical model, that is the process where the actual plant/process is *drawn*, i.e. specifying boundaries, presumptions, what to neglect etc.

---

1From the analysis it has not been possible to point out one of the tools as *the best*. In general the tools investigated are good and robust tools which could have been applied for the present study. *Minor differences are naturally present* - MATLAB/SIMULINK was chosen because it is a widespread tool with a large group of users, i.e. support is widely available.
Develop a mathematical model applying the physical model as a basis, that is the process where the physical model by means of the fundamental physical laws (First Law of Thermodynamics, ideal gas equation etc.) is transformed into a mathematical model.

Apply a numerical method to solve the mathematical model. The choice of numerical method depends on the nature of the equation system developed (algebraic equations for static simulations, ordinary differential equation systems or differential/algebraic equations systems for simulating dynamic systems). The choice of numerical method also depends on the type of the equation system to solve (stiff/non-stiff equation system, size of the system, discontinuities etc.).

Implement the numerical model in a program. This process is related to the program (computer system) to be applied, i.e. numerical representation of real numbers, handling of errors etc.

Verification of the model developed. Ideally this is based on results from measurements on plants/installations operating in the full load range. The verification includes a discussion of the results and deviations between measurements and simulations.

In the present study main focus is on the two first steps. For the last steps already developed models/tools will be applied. A good overview of the last steps including references is given in [51].

In the present study the models have been developed starting from simple models, which step by step have been refined.

5.3 Equation systems

Equation systems describing the dynamic boiler performance consist of a number of dynamic equations (differential equations) and a number of static equations (algebraic equations/constraints). An equation systems with this characteristic is a Differential Algebraic Equation system (DAE). DAE’s are to be seen as an extension of Ordinary Differential Equation systems (ODE). To some extent the same theories for integrating the systems can be applied.

5.3.1 Algebraic Equation (AE) Systems

Static (non-dynamic) design of boilers is characterized by algebraic equation systems - see section 2.2. In general an AE consist of a number of linear and/or non-linear algebraic equations:

\[ f(y) = 0; \ f : \mathbb{R}^n \to \mathbb{R}^n. \] (5.1)

The equation systems can be solved either by direct methods, for example Gauss-elimination (linear systems), or indirect/iterative methods, for example Newton-Raphson iteration (non-linear systems).

More methods for solving Algebraic Equation systems are described in [150].
Numerous numerical tools are available for solving algebraic equation systems - [169] is recommended.

### 5.3.2 Ordinary Differential Equation (ODE) Systems

Modelling the dynamic behavior of physical systems results in systems of differential equations. In the simplest form the system variables are given by a vector \(\mathbf{y}(t) : \mathbb{R} \rightarrow \mathbb{R}^n\) (the system state). The system model relates the system variables to its derivatives. The first order system\(^2\) has the following explicit form:

\[
y'(t) = f(y,t); \quad f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n. \tag{5.2}
\]

This means that an ODE model (given a set of initial value conditions) defines how each quantity changes as a function of time. Solving ODE’s are often referred to as solving or integrating initial value problems - see [62].

A detailed description of the historical approach to solution of ODE’s can be found in [61]. Today the theory is well understood and robust software applications for solving ODE’s are available - see [81].

### 5.3.3 Differential Algebraic Equation (DAE) Systems

Differential Algebraic Equation systems describe the behavior of a physical system in the same way as ODE’s. During the modelling phase often a number of algebraic constraints are required to give a full description of the system being modelled\(^3\). In the semi-explicit form a DAE can be written as:

\[
y'(t) = f(y,z,t); \quad f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n
\]

\[0 = g(y,z,t); \quad g : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^m. \tag{5.3}\]

Where \(y(t)\) contains the differential variables and \(z(t)\) the algebraic variables - see [103].

A DAE only makes sense if the initial values, \(z(t_0) = z_0\), are consistent, i.e. the values are on the feasible manifold defined by the algebraic equations.

It is presumed that \(g_z(y,z,t)\) (i.e. \(\partial g/\partial z\)) has a bounded inverse in the neighborhood of the solution, i.e. \(\partial g/\partial z\) is non-singular, then it follows from the inverse function theorem (see [1]) that \(z\) can be written as a function of \(y\), i.e. \(z = \phi(y,t)\). From this presumption the local existence and uniqueness and regularity of the solution follows - see [104].

\(^2\)Higher order systems can always by means of new variables be reduced to a first order system - see [69].

\(^3\)The algebraic constraints could be related to physical properties of the media included in the systems (e.g. water/steam properties). Another typical algebraic constraint is a definition of the manifold to operate on (e.g. the length of a pendulum or movements of a robot arm).
Index and Solvability for DAE’s

Historically, the solution of DAE’s has been closely related to the definition of index and solvability. A precise definition of solvability can be found in [10]. Different definitions of index have been given in the literature, a precise definition is given in [10]:

**Theorem 1** (Index of DAE’s) The minimum number of times that all or parts of Equation 5.3 must be differentiated with respect to \( t \) in order to determine \( y'(t) \) as a continuous function of \( y \) and \( t \), is the index of the DAE in Equation 5.3.

The index is also referred as the differentiation index - see [104]. Generally the index of a DAE can be thought of as a measure of the DAE’s complexity - the higher the index, the more complex, i.e. the higher the index, the more the DAE deviates from an ODE.

As DAE’s of index 0 (i.e. ODE’s) and index 1 (\( \partial g/\partial z \) is non-singular) are best described and easiest to analyze, many references name DAE’s of index 2 (\( \partial g/\partial z \) is singular) and higher as higher index DAE’s. There are no general methods for direct solution of higher index DAE’s, but several special DAE forms have been analyzed in details - some of these are described in [3] and [10], especially the Hessenberg forms.

Another way to look at DAE’s is as constrained optimization problems where the constraints force the solution to lie on the feasible manifold defined by the algebraic equations - see [3].

Reducing the index of the DAE’s by means of differentiation is called index reduction, and in principle the method can be applied until an ODE (index 0) has been reached. This ODE is named the underlying ODE. The index reduction methodology has several implications that can cause drift-off and instability - a more detailed discussion of these phenomena can be found in [104].

A severe problem for index reduction is that a solution to the reduced system is not necessarily a solution to the original DAE. This is due to the fact that any solution on the manifold defined by the system of algebraic equations is a solution to the reduced system, but only the solution that satisfies the consistent initial values is also the solution to the original DAE - see [63].

Numerical Methods for solving DAE’s

For solving DAE’s in principle two methods can be applied:

- the direct discretization methods
- the methods based on reformulation of the problem (i.e. index reduction).

---

4 Actually the index should be named the index along a solution \( y(t) \) as the index depends on the solution and not only on the form of the DAE - see [3].

5 Another index related to the smoothness of the solution is the perturbation index - see [3] and [9].
As mentioned previously DAE’s of higher index can only be solved numerically for some special forms of the DAE. It is therefore recommended to carry out the index reduction for higher index DAE’s.

The first numerical methods to be applied for solving DAE’s were the linear multi-step methods (e.g. Backward Differentiation Formulas - BDF). Later, one-step methods like Runge-Kutta methods have been applied successfully - see [10].

DAE’s are closely related to stiff ODE’s. And it is therefore natural to consider numerical methods suited for these for solving DAE’s. Different methods have been applied successfully:

- Backward Euler
- BDF and General Multistep Methods
- Runge-Kutta Methods (explicit, implicit and semi-implicit).

More detailed discussion of pro’s and con’s for the different methods can be found in [3], [10] and [104]. It should be mentioned that the popular DASSL code, which is based on BDF methods, has proved to be very successful - a more detailed description of DASSL can be found in [10].

5.3.4 Simulation of Equation Systems

In the present study the models have been developed as relatively small sub-models which subsequently have been merged together to larger complete boiler system models, where the sub-models exchange data - see Figure 6.1 and 8.1.

This component-oriented approach has several advantages:

- simple and clear models
- easy to develop (debug)
- easy to document and maintain
- sub-models developed can be used for other models.

The models developed in Chapter 6 and 8 (see Figure 6.1 and 8.1) are based on this approach which in this present study has proven to be successful.
Chapter 6

Modelling and Simulation of the Water tube Boiler

6.1 Introduction

In this chapter a model for the water tube boiler has been developed. The model is developed specifically for the Aalborg Industries type MISSION™ WHR-GT [162]. A more detailed description of the test plant can be found in Appendix C.

The overall model has been split into three sections, i.e. sub-models for heat exchanger, evaporator circuit and drum are developed - see Figure 6.1. The objective has been to develop a relatively simple model for simulating the shrinking and swelling of water in the boiler steam drum and the steam production during dynamic operation of the plant and hereby define the constraints related to these dimensions. More detailed dynamic models are described in [144], [109], [91] and [30].

Furthermore, tests have been carried out on a full scale boiler plant for verifying the model developed.

6.2 Overall Modelling

Mainly to simplify the modelling process, the model has been split in three sections:

- the heating surface (see section 6.3.1)
- the evaporator circuit (see section 6.3.2)
- the drum (see section 6.3.3)
models have been developed for each of the above mentioned components. The split in these sub-
models minimizes the flow of data between the models. The overall flow of data during a simulation

The first step in the simulation is the calculation of the heating surface performance, where the input
parameters are:

- the mass flow of the flue gas, $m_{fg}$
- the temperature of the flue gas, $T_{fg}$
- the drum pressure, $p_{dr}$.

Furthermore, the heating surface calculation also requires the flow of water/steam. For single phase
heat exchangers (for example economizers and superheaters) the flow of water/steam is required to
determine the internal coefficient of heat transfer - see Appendix B.5. For evaporators the flow of
water/steam is required to verify that the water/steam mixture at the outlet is in the two-phase region. As a result of the heating surface simulation, the energy transferred from the flue gas to the water/steam \( \dot{q}_{fg\rightarrow w/s}(t) \) is calculated, this figure is used as an input to the evaporator simulation. Furthermore, the following values are calculated as output from the heating surface simulation:

- the energy transferred from the flue gas to the pipe, \( \dot{q}_{fg\rightarrow p} \)
- the energy transferred from the pipe to the water/steam, \( \dot{q}_{p\rightarrow w/s} \)
- the gas outlet temperature, \( T_{fg,\text{out}} \)
- the pipe temperature, \( T_p \)
- the water/steam outlet enthalpy, \( h_{\text{ev, out}} \)
- the water/steam outlet quality, \( x_{\text{out}} \).

The second step in the boiler simulation is the evaporator circuit simulation, where the performance of the evaporator circuit including downcomer is simulated (with \( \dot{q}_{fg\rightarrow w/s}(t) \) from the heating surface simulation as input). As a result of the evaporator simulation, the following figures are calculated:

- the mass flow in the downcomer, \( \dot{m}_{dc} \)
- the mass flow in the evaporator circuit, \( \dot{m}_{w/s} \)
- the mass in the evaporator circuit, \( M_{\text{ev}} \)
- the circulation number in the evaporator, \( N_{\text{circ}} \).

The third and last step in the simulation is the drum simulation, where the above mentioned figures are given as inputs. As a result of the drum simulation the actual water filling of the drum is calculated. As the mass flow of feedwater, \( \dot{m}_{fw}(t) \), and the pressure development, \( p_{dr}(t) \), are externally given inputs to the model, the mass flow of steam, \( \dot{m}_s(t) \), is calculated after each integration step to obtain the specified pressure in the drum with the calculated internal energy.

When a complete integration of the three sub-models has been carried out, the overall model has to be iterated to convergence. For the next iteration the output from the evaporator and drum simulations are used as input to the heating surface simulation. As a convergence criterion for the overall iteration \( \| T_{fg,\text{iteration } n} - T_{fg,\text{iteration } n-1} \|_2 < \epsilon \) is applied. The overall iteration convergence of the model is very good - typically convergence is obtained after two to three iterations.
6.3 Physical and Mathematical Modelling

6.3.1 Heating Surface

To be able to connect the flue gas side (i.e. the energy source) and the water/steam side (i.e. the energy absorber) it is important to calculate $q_{fg\rightarrow w/s}(z,t)$, and at the same time $T_p(z,t)$ to assess any temperature peaks that could over-stress the pipe material. The model in Figure 6.2 (left diagram) has been applied for modelling the heating surfaces.

For the more detailed analysis of the pipes in the heat exchanger, the model in Figure 6.2 (right diagram) has been applied.

The model is based on the First Law of Thermodynamics (Energy Conservation) - see Chapter B.2.3.

Dynamic (differential) equations

Dynamic Energy balance for the water/steam\(^1\):

\[ m_{w/s} \cdot (h_{w/s,\text{out},n} - h_{w/s,\text{in},n}) - q_{p\rightarrow w/s,n} + M_{w/s,n} \cdot c_{p,w/s,n} \cdot \frac{dT_{w/s,n}}{dt} = 0 \]  
(6.1)

Energy balance for the heating surface pipe:

\[ q_{p\rightarrow w/s,n} - q_{fg\rightarrow p,n} + M_{p,n} \cdot c_{p,p,n} \cdot \frac{dT_{p,n}}{dt} = 0 \]  
(6.2)

\(^1n\): number of sections in heating surface.
Dynamic Boiler Performance

Energy balance for the flue gas:

\[
\dot{q}_{fg-p,n} + \dot{m}_{fg} \cdot c_{p,fg,n} \cdot (T_{fg,\text{out},n} - T_{fg,\text{in},n}) + M_{fg,n} \cdot c_{p,fg,n} \cdot \frac{dT_{fg,n}}{dt} = 0 \quad (6.3)
\]

Non-dynamic (algebraic) equations

Heat transfer:

\[
\dot{q}_{fg-p,n} - \alpha_{ext,n} \cdot A_{p,s,ext,n} \cdot (T_{fg,n} - T_{p,n}) = 0 \quad (6.4)
\]

\[
\dot{q}_{p-w/s,n} - \alpha_{int,n} \cdot A_{p,s,int,n} \cdot (T_{p,n} - T_{w/s,n}) = 0 \quad (6.5)
\]

Mean Temperatures:

\[
\overline{T}_{w/s,n} = \frac{T_{w/s,\text{in},n} + T_{w/s,\text{out},n}}{2} \quad (6.6)
\]

\[
\overline{T}_{fg,n} = \frac{T_{fg,\text{in},n} + T_{fg,\text{out},n}}{2} \quad (6.7)
\]

The equations are formulated as a finite volume model (see [101]) with discretization in the spatial variable. Depending on the required degree of accuracy more or less elements are applied (typically each row of tubes in the heating surface will define an element).

The above developed set of equations are for heating surfaces applied for either preheating of water (economizers) or superheating of steam (superheaters) i.e. heating surfaces, where the single-phase media experiences a temperature change. For the evaporator section the equations have been prepared with the enthalpy as independent variable, and the energy content in the evaporator has to be added as an extra variable, these equations which have been used for the evaporator simulations, are, in principle, similar to the above given equations and will not be presented here. At the evaporator the inlet pressure will be slightly higher than the drum pressure (i.e. saturation pressure), this means that the water at the evaporator inlet is slightly sub-cooled and preheating to the saturation temperature has to take place before evaporation can start. After having preheated the water the evaporation takes place in the evaporator, where the media temperature does not change².

²For low-pressure boilers the temperature change due to pressure change in the evaporator is relatively larger - see Figure 2.4.
6.3.2 Evaporator Circuit

Evaporation is characterized by the very high internal coefficient of heat transfer, $\alpha_{int}$, i.e. good cooling of the pipe material - see Appendix B.5. For this reason the evaporator is typically physically located in the boiler where the highest flue gas temperatures are present - see page 11. This means that if the cooling fails (for example insufficient circulation in the evaporator) the consequences will normally be severe to the boiler and safety in general - see Figure B.7. For modelling the evaporator circuit the model in Figure 6.3 has been applied.

\[ \Delta p_{pump} = 0 \] corresponds to a natural circulating evaporator. The simulation of the two-phase flow in the evaporator circuit is based on a simple homogenous model - see Appendix B. The modelling is based on the following equations:

**Dynamic (differential) equations**

**Mass Balance for the evaporator-element:**

\[ \frac{dM_{ev}}{dt} + \dot{m}_{ev, out} - \dot{m}_{ev, in} = 0 \] (6.8)

**Momentum Balance for the evaporator-element:**

\[
\frac{d\tau_{ev}}{dt} + \tau_{ev} \cdot \frac{v_{ev, out} - v_{ev, in}}{L_{ev}} - \frac{p_{ev, in} - p_{ev, out}}{\tau_{ev} \cdot L_{ev}} + \frac{\lambda_{ev} \cdot \tau_{ev}^2}{2 \cdot d_{ev}} + g \cdot \sin \phi = 0 \] (6.9)

**Energy Balance for the evaporator-element:**

\[ \frac{dU_{ev}}{dt} + \dot{m}_{ev, out} \cdot h_{ev, out} - \dot{m}_{ev, in} \cdot h_{ev, in} - \dot{q}_{fg\rightarrow ev} = 0 \] (6.10)

and

\[ \dot{q}_{fg\rightarrow ev} = \sum_{j=1}^{N_{elements}} \dot{q}_{fg\rightarrow ev,j} \]
Momentum Balance for the downcomer:

\[
\frac{dV_{dc}}{dt} = \frac{P_{dc} - P_{dc,out}}{\rho_{dc} \cdot z_{dc}} + \frac{\lambda_{dc} \cdot \tau_{dc}^2}{2 \cdot d_{dc}} - g = 0
\]  
(6.11)

Non-dynamic (algebraic) equations

Qualities in the evaporator-element:

\[
x_{ev,out} = \frac{h_{ev,out} - h'}{h'' - h'}
\]  
(6.12)

\[
x_{ev} = \frac{x_{ev,out} + x_{ev,in}}{2}
\]  
(6.13)

Densities in the evaporator-element:

\[
\rho_{ev,out} = \left[ \frac{x_{ev,out}}{\rho''(p_{dr})} + \frac{1 - x_{ev,out}}{\rho'(p_{dr})} \right]^{-1}
\]  
(6.14)

\[
\overline{\rho}_{ev} = \left[ \frac{x_{ev}}{\rho''(p_{ev})} + \frac{1 - x_{ev}}{\rho'(p_{ev})} \right]^{-1}
\]  
(6.15)

Total mass in the evaporator-element:

\[
M_{ev} = \overline{\rho}_{ev} \cdot A_{ev,int} \cdot L_{ev}
\]  
(6.16)

Mean enthalpy in the evaporator-element:

\[
\overline{h}_{ev} = \frac{h_{ev,out} + h_{ev,in}}{2}
\]  
(6.17)
Energy content in the evaporator-element:

\[ U_{ev} = M_{ev} \cdot u_{ev} = M_{ev} \cdot \left( \bar{h}_{ev} - \frac{\bar{P}_{ev}}{\bar{p}_{ev}} \right) = M_{ev} \cdot (\bar{h}_{ev} - \bar{P}_{ev} \cdot \bar{p}_{ev}) \] (6.18)

Mean mass flux in the evaporator-element:

\[ \bar{\rho}_{ev} \cdot \bar{p}_{ev} = \frac{\nu_{ev,in} \cdot \rho_{ev,in} + \nu_{ev,out} \cdot \rho_{ev,out}}{2} \] (6.19)

Mass flow into the evaporator-element:

\[ \dot{m}_{ev,in} = \rho_{ev,in} \cdot v_{ev,in} \cdot A_{ev,int} \] (6.20)

Mass flow out of the evaporator-element:

\[ \dot{m}_{ev,out} = \rho_{ev,out} \cdot v_{ev,out} \cdot A_{ev,int} \] (6.21)

Pressure levels in the evaporator circuit:

\[ \bar{P}_{ev} = \frac{P_{ev,in} + P_{dr}}{2} \] (6.22)

\[ P_{ev,in} = P_{dc,out} + \Delta p_{pump} \] (6.23)

The equation system for the evaporator element has been formulated with the evaporator-element as one element absorbing the integrated energy flow from the pipe \( \int_0^L \dot{q}_{p-w/s}(z,t)dz \) - output from the heating surface simulation.

6.3.3 Drum

The boiler drum is, in principle, a vessel with inlet and outlet flows as shown in figure 6.4. The sub-model for the drum consists of the following equations:
Dynamic (differential) equations

Mass balance for the drum

\[ \frac{dM_{dr,t}}{dt} = \dot{m}_w + \dot{m}_{rs} - \dot{m}_s - \dot{m}_{dc} \]  
(6.24)

Energy balance for the drum

\[ \frac{dU_{dr,t}}{dt} = \dot{m}_w \cdot h_{fw} + \dot{m}_{rs} \cdot h_{rs} - \dot{m}_s \cdot h_s - \dot{m}_{dc} \cdot h_{dc} \]  
(6.25)

Non-dynamic (algebraic) equations

Total mass of water and steam in the drum:

\[ M_{dr,t} = M_{dr,w} + M_{dr,s} \]  
(6.26)

Total Energy content in the drum:

\[ U_{dr,t} = U_{dr,w} + U_{dr,s} = M_{dr,w} \cdot u_{dr,w} + M_{dr,s} \cdot u_{dr,s} \]
\[ = M_{dr,w} \cdot (h_{dr,w} - p_{dr,w} \cdot \nu_{dr,w}) + M_{dr,s} \cdot (h_{dr,s} - p_{dr,s} \cdot \nu_{dr,s}) \]  
(6.27)

Total volume of the drum:

\[ V_{dr,t} = V_{dr,w} + V_{dr,s} = M_{dr,w} \cdot \nu_{dr,w} + M_{dr,s} \cdot \nu_{dr,s} \]  
(6.28)

The DAE system defined by the Equations 6.24, 6.25, 6.26, 6.27 and 6.28 has the following dynamic variables:

\[ Y^T = [M_{dr,t}, U_{dr,t}] \]

and the following algebraic variables:

\[ Z^T = [M_{dr,w}, M_{dr,s}, \dot{m}_s] \]

Applying the notation from Equation 5.3, the drum model can be written as:
\[
y'(t) = \begin{bmatrix}
\dot{m}_w f_w + \dot{m}_{rs} - \dot{m}_{dc} \\
\dot{m}_w h_w + \dot{m}_{rs} h_{rs} - \dot{m}_{dc} h_{dc}
\end{bmatrix} + \begin{bmatrix}
0 & 0 & -1 \\
0 & 0 & -h_s
\end{bmatrix} \cdot z = f(y, z, t)
\]

and

\[
o = \begin{bmatrix}
0 \\
0 \\
-V_{dr, t}
\end{bmatrix} + \begin{bmatrix}
-1 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \cdot y + \begin{bmatrix}
h_w - p_{dr} \nu_{dr, w} & 1 \\
\nu_{dr, w} & h_s - p_{dr} \nu_{dr, s} & 0 \\
\nu_{dr, s} & 0 & 0
\end{bmatrix} \cdot z = g(y, z, t)
\]

i.e.

\[
\frac{\partial g}{\partial z} = \begin{bmatrix}
h_w - p_{dr} \nu_{dr, w} & 1 \\
\nu_{dr, w} & h_s - p_{dr} \nu_{dr, s} & 0 \\
\nu_{dr, s} & 0 & 0
\end{bmatrix}
\]

Since \(\frac{\partial g}{\partial z}\) is singular (the last column is 0) the index of this DAE system is greater than 1 (if \(\frac{\partial g}{\partial z}\) is non-singular the index of the DAE system is 1 - because one differentiation of \(g(y, z, t)\) in principle yields \(z'(y, t)\) - see [3]).

**Index reduction for Boiler Drum model**

Since the index of the boiler drum model is greater than 1, it is expected that the integration of this model will cause difficulties. An attempt was made to solve this DAE by means of MATLAB’s ode23t (Trapezoidal rule) and ode15s (BDF) solvers, but without results. These solvers are applicable for *Moderately stiff* and *stiff differential equations* and *index 1 DAE’s* respectively. *Stiff differential equations and index 1 DAE’s* - see [179].

The philosophy applied for resolving this problem is to rewrite the DAE into an ODE with 4 differential variables (independent of the 5\(^{th}\) algebraic variable - decoupling), i.e.:

\[
\tilde{y}'(t) = f(\tilde{y}, t) \\
0 = g(\tilde{y}, z, t)
\]

(6.29)
with the following dynamic variables:
\[ \tilde{y}^T = [M_{dr,t}, M_{dr,s}, M_{dr,w}, U_{dr,t}] \]
and the following algebraic variable:
\[ \tilde{z}^T = [\dot{m}_s] \]

The first step is to differentiate the mass balance for the drum (Equation 6.26):
\[
\frac{dM_{dr,t}}{dt} = \frac{dM_{dr,w}}{dt} + \frac{dM_{dr,s}}{dt} \tag{6.30}
\]
and in the same manner differentiate the total volume of the drum (Equation 6.28):
\[
0 = \frac{dV_{dr,t}}{dt} = \nu_{dr,s} \cdot \frac{dM_{dr,s}}{dt} + M_{dr,s} \cdot \frac{d\nu_{dr,s}}{dp_{dr}} \cdot \frac{dp_{dr}}{dt} + \nu_{dr,w} \cdot \frac{dM_{dr,w}}{dt} + M_{dr,w} \cdot \frac{d\nu_{dr,w}}{dp_{dr}} \cdot \frac{dp_{dr}}{dt} \tag{6.31}
\]

The derivatives \( d\nu_{dr,s}/dp_{dr} \) and \( d\nu_{dr,w}/dp_{dr} \) are water/steam properties and they will be calculated according to [141]. \( dp_{dr}/dt \) is given as a user input\(^3\), i.e. the operator decides how to build-up pressure. If the heat input to the boiler cannot be controlled by the boiler operator (for example a waste heat recovery boiler located after a gas turbine), then the mass flow of steam from the boiler, \( \dot{m}_s(t) \), will be controlled by the pressure controller, which is a part of the overall boiler control system [47].

**Differentiation and reduction of the total energy content in the drum (Equation 6.27):**
\[
\frac{dU_{dr,t}}{dt} = (h_{dr,w} - p_{dr} \cdot \nu_{dr,w}) \cdot \frac{dM_{dr,w}}{dt} + M_{dr,w} \cdot \frac{dp_{dr}}{dt} \cdot \left[ \frac{dh_{dr,w}}{dp_{dr}} - p_{dr} \cdot \frac{d\nu_{dr,w}}{dp_{dr}} - \nu_{dr,w} \right] +
(h_{dr,s} - p_{dr} \cdot \nu_{dr,s}) \cdot \frac{dM_{dr,s}}{dt} + M_{dr,s} \cdot \frac{dp_{dr}}{dt} \cdot \left[ \frac{dh_{dr,s}}{dp_{dr}} - p_{dr} \cdot \frac{d\nu_{dr,s}}{dp_{dr}} - \nu_{dr,s} \right] \tag{6.32}
\]

Rewriting Equation 6.24 and 6.25 yields:
\[
\dot{m}_s = \dot{m}_{fw} + \dot{m}_{rs} - \dot{m}_{de} - \frac{dM_{dr,t}}{dt} \tag{6.33}
\]

\(^3\)Normally it is the pressure at the boiler outlet.
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\[ \dot{m}_s = \frac{1}{h_{dr,s}} \left[ \dot{m}_{f,w} \cdot h_{f,w} + \dot{m}_{r,s} \cdot h_{r,s} - \dot{m}_{dc} \cdot h_{dc} - \frac{dU_{dc,t}}{dt} \right] \]  

(6.34)

These equations reduce to (by elimination of \( \dot{m}_s \)):

\[ \dot{m}_{f,w} \cdot \left( 1 - \frac{h_{f,w}}{h_{dr,s}} \right) + \dot{m}_{r,s} \cdot \left( 1 - \frac{h_{r,s}}{h_{dr,s}} \right) - \dot{m}_{dc} \cdot \left( 1 - \frac{h_{dc}}{h_{dr,s}} \right) = \frac{dM_{dr,t}}{dt} - \frac{1}{h_{dr,s}} \cdot \frac{dU_{dr,t}}{dt} \]  

(6.35)

Equation 6.30, 6.31, 6.32 and 6.35 forms an ODE (4 ordinary differential equations with 4 differential variables), which can be written in the form:

\[ A(\tilde{y}, t) \cdot \tilde{y}' = B(\tilde{y}, t) \cdot \tilde{y} + C, \quad \tilde{y} = \tilde{y}_0 \quad \text{for} \quad t = t_0 \]

\[ \tilde{y}^T = \begin{bmatrix} M_{dr,t}, M_{dr,s}, M_{dr,w}, U_{dr,t} \end{bmatrix} \quad \text{and} \quad \tilde{z}^T = [\dot{m}_s] \]

where:

\[ A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & \nu_{dr,s} & \nu_{dr,w} & 0 \\ 0 & -(h_s - p_{dr} \cdot \nu_{dr,s}) & -(h_{dr,w} - p_{dr} \cdot \nu_{dr,w}) & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ C^T = \begin{bmatrix} 0, 0, 0, \dot{m}_{f,w} \cdot \left( 1 - \frac{h_{f,w}}{h_{dr,s}} \right) + \dot{m}_{r,s} \cdot \left( 1 - \frac{h_{r,s}}{h_{dr,s}} \right) - \dot{m}_{dc} \cdot \left( 1 - \frac{h_{dc}}{h_{dr,s}} \right) \end{bmatrix} \]

this ODE can be solved by means of an integration procedure for ODE’s. The algebraic variable \( \dot{m}_s \) does not appear in the dynamic part of the system but it can be calculated by means of Equation 6.33 or 6.34. Depending on the numerical accuracy, drift-off can occur when \( \dot{m}_s \) is calculated - see page 44.

A long line of quantities in the model are here presumed as (time dependent) parameters but in the overall model their values come from the other sub-models.
6.4 Presumptions/Simplifications

The main purpose of the present study has been to optimize the design and operation of boilers with respect to dynamic performance using dynamic models as tools to define the constraints for the optimization, i.e. the dynamic model of the water tube boiler has been developed with a number of simplifications.

The main simplifications and presumptions are:

- The internal coefficient of heat transfer, $\alpha_{\text{int}}$, in the evaporator (two-phase flow) is decades larger than the external coefficient of heat transfer, $\alpha_{\text{ext}}$, therefore the thermal resistance at the inside of the tubes has been neglected - see Appendix B.

- In developing the model for the evaporator circulation, the evaporator has been modelled as a one element evaporator. The water/steam properties have been modelled as arithmetic mean properties\(^4\). Especially, the density of the mixture in the evaporator does not change linearly through the evaporator. For the present study the exact value of the quality out of the evaporator is not very important as there is no risk for dry out - see Appendix B. An error on the specific density affects calculation of the pressure loss through the heating surface and hereby the circulation number.

- For developing the evaporator model the homogenous model has been applied. Practical experience shows that the error on the circulation number, $N_{\text{circ}}$, applying the homogenous model is very limited - [157].

- In the heating surface model the temperature change due to sub-cooling of the water at the heating surface inlet related to the increase in pressure level (the static height from the drum to the heating surface inlet) with constant enthalpy has not been taken into consideration. Enthalpy wise the sub-cooling is included in the model as a negative steam quality\(^5\) - see Figure 6.7. The performance of the heating surface, where evaporation takes place, has been calculated with the same (saturation) temperature and the actual pressure in the complete heating surface.

\(^4\)Arithmetic average of in- and outlet conditions.
\(^5\)This corresponds to a two-phase state located in the single-phase water section of the water-/steam-diagram.
6.5 Simulations

6.5.1 General

The simulations in this chapter have been carried out with (see Figure 7.3 and 7.4):

\[
\left( \frac{dP_{\text{boi}}}{dt} \right)_{\text{rel}} = \frac{dP_{\text{boi}}}{dt} \frac{dP_{\text{boi, max}}}{dt} = 50 \%
\]

and the pressure on the boiler developing (building up) according to Figure 7.1 - see detailed description page 68.

The simulations are based on the geometry of the test plant - see Appendix C. This boiler consists of 12 rows of tubes, numbered from flue gas outlet, i.e. the top of the boiler - see Figure C.3. The volume of the drum is approx. 14 m³.

The simulations have been carried out with the on/off feed water controller, as can be seen from Chapter 3, the type of feed water controller affects the numerical value of the outputs from the simulations. But without loss of generality the controller can, as well as the single point controller, be applied for illustrating the results from the simulations - see page 67.

6.5.2 Integration of Heating Surface model

The equation system for the heating surface (see Chapter 6.3.1) is an index 1 DAE system and have successfully been solved by means of the MATLAB solvers ode23t and ode15s, described in [179] and [114].

The solvers have shown to be stable for large equation systems - simulations have successfully been carried out for more than 25 rows of tubes, i.e. \( n = 25 \) (se page 50).

In Figure 6.5, 6.6, 6.7 and 6.8 outputs from the heating surface simulations are shown. These simulations have been carried out for a boiler start-up. Since the boiler is a waste heat recovery boiler, the firing rate is controlled by the gas turbine (see Figure 7.3 and 7.4).

In the simulation shown in Figure 6.5 the flue gas and pipe material temperatures are initially lower than the water/steam temperature, i.e. the flue gas cools the pipe material resp. the water/steam until the flue gas temperature raises to a higher level according to Figure 7.3 and 7.4 - see detail in Figure 6.5.

As the developed model of the water tube boilers heating surface (see Chapter 6.3) is formulated as an element-model using the local temperature differences as driving potentials for energy flow, the change in direction of energy flow (initially with water/steam to flue gas and as the flue gas temperature raises
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Temperature profiles in Heating Surface

![Temperature profiles in Heating Surface](image)

**Figure 6.5:** Heating Surface simulation - temperature profiles for flue gas, heating surface material and water/steam. For detail shown on RHS of figure see page 60.

above the water/steam temperature from the flue gas to the water/steam does not cause the model any difficulties. Other models\(^6\) based on an overall heat balance for the heating surface (see [49]) and using a log-mean temperature difference based model can have problems with this change in direction of energy flow - see detail in Figure 6.5.

In Figure 6.6 the pressure profile on the water/steam side through the heating surface during the start-up of the gas turbine is given. The shape of the pressure curve (see Figure 7.1) can be recognized along the time axis. The inclination of this surface in the Tube Row direction shows the development in pressure through the heating surface.

\(^6\)Typically these models will be much faster and simulations can be carried out for larger and more complex plants - e.g. power plant boilers.
Figure 6.6: Heating Surface simulation - water/steam pressure profile through heating surface.

In Figure 6.7 the quality (dryness) profile through the heating surface during the start-up of the gas turbine is given. As can be seen the quality at the heating surface inlet is negative in the beginning of the simulation period, this corresponds to a sub-cooling of the water at the heating surface inlet (see page 50) due to increase in pressure (from the drum to the evaporator inlet). As can be seen from Figure 6.7 the quality increases as the load on the gas turbine is increased, and the pressure is still low at the boiler, though the quality decreases as the boiler pressure increases, following the profile shown in Figure 6.6.

As the pressure on the evaporator increases the density of the evaporator content increases causing the mass flow through the evaporator to increase as $\Delta p_{pump}$ is constant.

In Figure 6.8 the energy transferred from the pipe material to the water/steam side profile through the heating surface during the gas turbine start-up is shown. The negative value of the energy transfer initially corresponds to the cooling of the water/steam and pipe material from the flue gas - see Figure 6.5.
6.5.3 Integration of Evaporator Circuit model

In Figure 6.9 the circulation number in the evaporator circuit during the gas turbine start-up is shown. The simulations have been carried out with $\Delta p_{\text{pump}} = 1.5$ bar, i.e. a flat characteristic for the circulation pump. As can be seen from Figure C.8 this is a fair assumption.

In Figure 6.9 the change in pressure (see Figure 7.1) causing the density of the water/steam mixture in the evaporator to increase and hereby the total mass in the evaporator to increase, can be seen from 500 to 800 s.

As the heat input to the boiler starts, the evaporation increases causing the circulation number to decrease. During the gas turbine start-up the circulation number, $N_{\text{circ}}$, depends on the actual heat input to the water/steam side. As the pressure starts increasing in the boiler (from $t = 500$ sec) the pressure loss in the evaporator circuit decreases, causing the flow in the evaporator to increase.
Figure 6.8: Heating Surface simulation - profile of energy flow from pipe to water/steam through the heating surface.

6.5.4 Integration of Boiler Drum model

The index reduced equation system for the boiler drum developed in section 6.3.3 has been solved by means of MATLAB’s solver ode15s. This solver applies a BDF method and is suitable for Stiff differential equations and DAE’s - see [179] and Appendix B.

The equation system is reduced to an ODE which means that the solution is based on robust and well-known solvers - see Chapter 5. The solution of the equation system has shown to be stable without any problems.

As the equations are decoupled in a manner where the differential equations do not depend on the algebraic variable, the differential equations can be solved first. When the differential variables and their derivatives are known, the algebraic variable $\ddot{z}(t)$ can be calculated - see Chapter 6.3.3.

The overall philosophy for operating the boiler has been to control the boiler pressure and calculate the mass flow of steam, $\dot{m}_s$, corresponding to the specified pressure built-up. Simulations according
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Figure 6.9: Evaporator circuit simulation - circulation number, $N_{\text{circ}}$ and mass of water/steam mixture in evaporator circuit.

to this philosophy could, if for example a very steep curve for pressure built-up is specified result in a negative steam flow\(^7\).

Figure 6.10: Drum simulation - Mass flows, $\dot{m}_{fw}$ and $\dot{m}_{s}$ and Mass of water and steam in the drum.

\(^7\)From a purely mathematical/numerical point of view this result is correct. If, on the other hand, a philosophy based on a specified mass flow of steam was chosen the model could, by specifying a (too) low mass flow of steam, cause the steam pressure to rise to a very high level. Again from a purely mathematical/numerical point of view this result is correct.
In Figure 6.11 the filling of the drum during the start-up of the gas turbine is shown with the flow of feed water and steam as shown in Figure 6.10 - left diagram. In Figure 6.10 (right diagram) the mass of water and steam in the drum can be seen. Initially the mass of steam is negligible compared to the water, but as the boiler pressure starts increasing (from $t = 500$ sec), the mass of the steam increases slightly.$^8$

**Figure 6.11**: Drum simulation (% of full drum) - filling of drum, i.e. water level, $L_{\text{w}}$.

### 6.5.5 Verification

A limited number of tests for verifying the water tube boiler model have been carried out on a full-scale boiler plant.

The results from these measurements are given in Appendix C.

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$^8$In practical design/operation water level fluctuations can be dealt with, apart from designing the drum a priori to contain the water transferred, by blow down during start up or starting at low water level.
Chapter 7

Optimization of the Water tube Boiler

7.1 Introduction

In this chapter the optimization of the water tube boiler introduced in Chapter 4 is finalized. The constraints for the optimization will be derived from the dynamic simulations. Furthermore, the results and consequences of the optimization are discussed.

7.2 Optimization

As can be seen from the overall model of the water tube boiler - see Figure 6.1, a number of inputs are given to this model. Flow and temperature of flue gas from the gas turbine are externally given and can only be partially controlled during the operation of the boiler - see page 47.

The pressure at the boiler outlet, \( p_{dr} \), is for the present study also given as an input to the model - see Figure 7.1.

In the simulations used for the optimization the feed water flow, \( \dot{m}_{fw} \), has been controlled by means of the single point feed water controller - see page 67. The Gain in the feed water controller has been set to a typical value - see page 67.

Applying this control philosophy the fluctuations of the water level\(^1\) in the drum for different boiler load gradients, \( dP_{boil}/dT \), have been calculated. The results of these simulations can be seen in Figure 7.5.

For the present study of the water tube boiler (see Chapter 6), two feed water control philosophies:

\(^1\)The results from the simulations of the water level fluctuations are, in principle, similar to Figure 6.11
single point feed water controller - see Figure 7.2 (left diagram)

on/off feed water controller - see Figure 7.2 (right diagram)

have been applied.

The simulations with the on/off feed water controller have been carried out with the following criteria:

\[
\begin{align*}
\text{Water level in drum} & \leq 45 \% \Rightarrow \dot{m}_w &= \dot{m}_{w,\text{max}} \\
\text{Water level in drum} & \geq 55 \% \Rightarrow \dot{m}_w &= 0 \\
\end{align*}
\]

i.e. the feed water is shut off when *water level in drum* increases to a level above 55 %, and turned on when *water level in drum* decreases to a level below 45 %.

The single point feed water controller is implemented as:

\[
\begin{align*}
\text{Water level in drum} & \geq 50 \% \Rightarrow \dot{m}_w &= 0 \\
\text{Water level in drum} & < 50 \% \Rightarrow \dot{m}_w &= \frac{L_{w,\text{set}} - L_{w,\text{act}}}{100} \cdot \text{Gain} \cdot \dot{m}_{w,\text{max}} \\
\end{align*}
\]

and if \(\dot{m}_w \geq \dot{m}_{w,\text{max}}\):

\[
\dot{m}_w = \dot{m}_{w,\text{max}}.
\]

For the simulations different values of the *Gain* have been applied. As can be seen from Figure 7.5 increasing the *Gain* on the feed water controller limits the fluctuations of the water, but typically a number of limits will be present with respect to choice of *Gain*, e.g. feed water control valve.
Figure 7.2: Examples of simulation of Steam flow and Feed water flow for the two feed water control philosophies. Depending upon the Gain of the feed water controller the feed water flow follows the steam flow differently. In the figure the steam flow is controlled to build the pressure up on the boiler according to Figure 7.1.

The on/off feed water controller is a special case of the single point feed water controller corresponding to infinity Gain.

For the water tube boiler analyzed in the present study, which is a waste heat recovery boiler located after a gas turbine, the firing rate controller has to a large degree been defined by the gas turbine. After the start-up of the gas turbine it is operating at the Full Speed - No load point (see Figure 7.3 and 7.4). Thereafter the gas turbine can be loaded with a user specified gradient until the Full Speed - Full load point has been reached.

The surfaces shown in Figure 7.5 have been prepared for a given drum size. For a typical boiler operation only a certain fluctuation in water level during start-up and operation of the plant will be permitted.

By means of a surface as shown in Figure 7.5 (corresponding to a specific value of the Gain in the feed water controller) and by specifying an allowable fluctuation in drum water level the required drum size can be calculated. The results shown in Figure 7.6 are based on an allowable water level fluctuation - between 40 and 60 % of the drum volume, i.e. [NW - 10 %; NW + 10 %].

Specifying this limitation on the water level fluctuations in the drum means that for each of the boiler load gradients analyzed a certain drum size/volume will be required. Using the required drum size for a specific boiler load gradient as a basis/reference, the required drum size is shown in Figure 7.6.
Optimization of the Water tube Boiler

On the basis of the steam production the required steam space in the boiler drum has been calculated for the different Relative Boiler load Gradients. The allowable steam space load during start-up
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Figure 7.5: Drum filling as a function of \((dP_{boi}/dt)_{rel}\) during start-up for increasing Gain on the feed water controller.

has been defined as twice the allowable steam steam space load during steady state operation, i.e. \(1.800 \text{ [m}^3/\text{m}^3_{dr}/\text{h}]^2\). For calculating the required drum volume to meet the requirements with respect to steam space load it is presumed that the steam volume is 50 % of the drum volume.

A plot of the Objective Function as a function of the two design variables, \(X^T = [(dP_{boi}/dt)_{rel}, V_{boi}]\), is shown in Figure 7.7. In the figure the constraints related to the minimum drum volume (Equation 4.19), minimum and maximum boiler load gradients (Equation 4.20) define the feasible set (see also Figure 4.5).

For the simulations it has been presumed that a relative boiler load gradient lower than 10 % would not be relevant, i.e. \((dP_{boi}/dt)_{rel} \geq 10 \%\).

Furthermore the constraint related to shrinking and swelling and the constraint related to allowable steam space load are shown in Figure 7.7.

The results from the optimization can also be seen in the contour plot in Figure 7.8.

\(^2\) During the start-up of the plant, where the pressure builds up according to Figure 7.1, the operation data are almost out of range according to Figure 3.1. Extrapolating the data to the lower pressures indicates that even higher steam space load could be allowed especially for a shorter period.
7.3 Discussion of optimization results

As can be seen from Figure 7.7 and 7.8 the global minimum of the objective function can be found at the intersection between the maximum allowable relative boiler load gradient and the maximum allowable steam space load\(^3\). Furthermore, it can be seen from Figure 7.7 and 7.8 that a local minimum of the objective function is located at the intersection between the minimum allowable relative boiler load gradient and the maximum allowable steam space load.

With the chosen parameters, in the quantification of the boilers capability with respect to dynamic operation (see Figure 4.3), the surface of \( F_{\text{total}} \) in Figure 7.7 and 7.8 is rather flat in the Relative Boiler load Gradient direction, i.e. the optimum design could by minor changes of the equation parameters move to the local minimum - see Figure 7.8. This change of optimum in the direction of a cheaper boiler with lower dynamic capability is the obvious development if the quantification of dynamic performance is low. For many practical applications the requirements with respect to dynamic performance will be rather rigid, e.g. the plant must be able to change load by 20 % per minute. In these situations the constraint with respect to minimum allowable boiler load gradient, \( dP_{\text{boi,min}}/dt \), would be located at the specified value. Alternatively, the parameters: \( k \) and \( F_{\text{dyn,op, max}} \) in Equation 4.9 should be chosen to simulate a discontinuity at the minimum required boiler load gradient - see page 35 for a more detailed discussion.

\(^3\)Attention should be drawn to the fact that the quantification of the boilers capability with respect to dynamic operation is based on the shape/course of the Objective function as defined in Equation 4.9.
Optimizing dynamic design and operation of Water tube Boilers

![Graph showing the relationship between Boiler Volume, Relative Boiler load Gradient, and Objective Function with Feasible area.](Image)

**Figure 7.7:** Objective Function with *Feasible area* defined by means of constraints on *Boiler Volume* and *Relative Boiler load Gradient* (Equation 4.19 and 4.20) and constraints related to *Shrinking* and *Swelling* and *Steam Space Load*.

Depending on the shape of the quantification of the boiler’s dynamic capability (Equation 4.3), the global minimum can change/move. If, for example, a rather low relative boiler load gradient is acceptable for the actual application, this would correspond to a low $k$-value (see Figure 4.3), lowering the
Optimizing dynamic design and operation of Water tube Boilers

Figure 7.8: Contour plot of the Objective Function with Feasible area defined by means of constraints on Boiler Volume and Relative Boiler load Gradient (Equation 4.19 and 4.20) and constraints related to shrinking and swelling and steam space load.

- As can be seen from Figure 7.7 and 7.8 the requirements/constraints with respect to steam space load are more restrictive than the requirements/constraints with respect to shrinking and swelling, i.e. a larger drum volume. Decreasing the steam space load requirements would move this constraint to the left in Figure 7.8. At a certain stage the two constraints would cross each other, but still, with the selected operation conditions, the global minimum of the objective function would be found at the intersection between the maximum allowable boiler load gradient and the maximum allowable steam space load requirements or the requirements/constraints with respect to shrinking and swelling.
If the requirements/constraints with respect to water level fluctuations, i.e. shrinking and swelling were strengthened, allow smaller fluctuations, this would move this constraint to the right in Figure 7.7 and in principle the same phenomena would take place.

If requirements with respect to maximum boiler volume/size were defined this would correspond to a $V_{\text{boi, max}}$ surface parallel to the $V_{\text{boi, min}}$ surface (see Figure 7.7). This surface could, depending on the location, move the present global minimum towards the present local minimum. If the $V_{\text{boi, max}}$ surface were located to the left of the Steam Space Load constraint (see Figure 7.8), the feasible area would be empty, i.e. no solution.

With the analyzed operation conditions it can be seen that the optimum drum volume is smaller than the actual drum size, approx. 8.5 m$^3$ vs. approx. 14 m$^3$. The actual drum was designed taking a number of uncertainties into consideration:

- pressure built-up on the boiler, i.e. steam space load
- shrinking and swelling (extent of)
- actual feed water quality - see Figure 3.2.
Chapter 8

Modelling and Simulation of the Fire tube Boiler

8.1 Introduction

In this chapter a model for the fire tube boiler has been developed. The model is developed specifically for the MISSION™ OB boiler developed at Aalborg Industries A/S in 2002 - see [162] and [128]. A more detailed description of the test plant can be found in Appendix D.

The overall model has been split into three sections, i.e. sub-models for furnace, convection zone and water/steam zone are developed - see Figure 8.1. The objective has been to develop a relatively simple model for simulating the shrinking and swelling of water in the boiler steam drum during dynamic operation of the plant. More detailed dynamic models are described in [144], [109], [91] and [30].

Furthermore tests have been carried out on a full scale boiler plant for verifying the model developed.

8.2 Overall Modelling

The MISSION™ OB boiler is built up as a one-pass vertical boiler consisting of a water submerged furnace and a convection heating surface (partly water and partly steam submerged). The integrated burner is located in the bottom of the furnace - see Figure 8.1.

The model developed has been prepared for simulation in SIMULINK and for that reason the model developed consisting of a number of DAE’s, has been reduced to an ODE by means of index reduction methodology - see [129] and Chapter 5.

A detailed derivation of the model (physical and mathematical) is given in Appendix E.
Mainly to simplify the modelling process, the model has been split in three sections:

- the furnace (see section 8.2.1)
- the convection zone (see section 8.2.2)
- the water/steam section (see section 8.2.3).

The split in these sub-models minimizes the flow of data between the models. The overall flow of data during a simulation can be seen in Figure 8.1.

8.2.1 Furnace

The equation system modelling the furnace is composed of:

- the energy balance for the furnace (differential equation)
- the mass balance for the furnace (algebraic equation)
and constitutive equations for:

- \( U_{fur} \), \( \dot{q}_{fuel} \), \( \dot{q}_{air} \), \( \dot{q}_{fg,1} \), \( \dot{q}_{rad\rightarrow p} \), \( \dot{q}_{conv\rightarrow p} \), \( \overline{T}_{fur} \) and \( T_{comb} \).

The complete set of equations is specified in Appendix E.2.1.

Reformulating the equations, the differential equation for the furnace temperature, \( \overline{T}_{fur} \), can be written as:

\[
\frac{d\overline{T}_{fur}}{dt} = \frac{\dot{q}_{fuel} + \dot{q}_{air} - \dot{q}_{rad\rightarrow p} - \dot{q}_{conv\rightarrow p} - \dot{q}_{fg,1}}{V_{fur} \cdot c_{p,fu,fur} \cdot \rho_{fg,fur}}
\]  

(8.1)

### 8.2.2 Convection Zone

The equation system modelling the convection zone is composed of:

- the energy balance for the convection zone (differential equation)
- the mass balance for the convection zone (algebraic equation)

and constitutive equations for:

- \( U_{fg,cz} \), \( \dot{q}_{fg,2} \), \( \dot{m}_{fg,2} \), \( \dot{q}_{fg\rightarrow p,cz} \), \( \overline{T}_{fg,cz} \) and \( M_{fg,cz} \).

The complete set of equations is specified in Appendix E.2.2.

Reformulating these equations, the differential equation for the mean flue gas temperature in the convection zone, \( \overline{T}_{fg,cz} \), can be written as:

\[
\frac{d\overline{T}_{fg,cz}}{dt} = \frac{\dot{q}_{fg,1} - \dot{q}_{fg,2} - \dot{q}_{fg,cz}}{V_{fg,cz} \cdot c_{p,fg,cz} \cdot \rho_{fg,cz}}
\]  

(8.2)

### 8.2.3 Water/steam section

For modelling the dynamic behavior of the water/steam section, models for the complete water/steam section and for the system under the water level have been developed - see Figure E.3.

The complete set of equations is specified in Appendix E.2.3.

Combining the developed equation systems, the final set of equations, with the differential variables: \( p_s \), \( V_w \) and \( V_b \) can be written as:
\[ A \cdot y' = B, \quad \text{i.e.} \quad \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \cdot \begin{bmatrix} \frac{dp_s}{dt} \\ \frac{dV_w}{dt} \\ \frac{dV_b}{dt} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \]

Which is an ordinary set of differential equations (ODE)\(^1\) - see Chapter 5.

### 8.3 Presumptions/Simplifications

As for the water tube boiler the model developed for the fire tube boiler model has been based on a number of presumptions and simplifications - see also section 6.4.

The main simplifications and presumptions are:

- The coefficient of heat transfer, \(\alpha_{ext}\), in the evaporator (two-phase flow) is decades larger than the internal coefficient off heat transfer, \(\alpha_{int}\), therefore the thermal resistance at the outside of the tubes has been neglected - see Appendix B.

- The convection zone has been modelled as a one element evaporator. The flue gas properties have been modelled at mean temperatures.

- The combustion chamber has been modelled as a one element furnace, i.e. a medium temperature has been used for calculating the heat radiated to the furnace walls.

### 8.4 Simulations

The integration of the developed equation system has been carried out by means of SIMULINK - see Figure 8.2.

The simulations are based on the geometry of the test plant - see Appendix D.

The simulations have been carried out with a PI feed water controller.

### 8.5 Tests - experimental verification

To verify the model developed, a number of tests where step-input has been given to the test plant have been carried out, i.e. step-response tests.

\(^1\) \(A_{ij}\) and \(B_k\) - see Appendix E.
Figure 8.2: SIMULINK- Model of the bottom red boiler.

Results from a test with step-response on the steam flow, $\dot{m}_s$, can be seen in Figure 8.3. The data measured during the tests have subsequently been utilized for verifying the model.

From the curves in Figure 8.3 it can be seen that as the steam flow decreases (due to the closing of the valve), the boiler pressure starts to increase. The decreasing steam flow causes the water level in the boiler to increase which lowers the feed water flow.

In the model the following parameters are difficult to calculate/measure:

- the convective coefficient of heat transfer in the furnace, $\alpha_{\text{conv, fur}}$ - see Equation E.8
- the radiative coefficient of heat transfer in the furnace, $\alpha_{\text{rad, fur}}$ - see Equation E.6
- the convective coefficient of heat transfer in the convection zone, $\alpha_{\text{conv, cz}}$ - see Equation E.17
- the calorific value of the fuel, $H_u$ - see Equation E.3

and finally the empirical parameters:

---

2In practise the main steam valve was partly closed momentarily.
3As the boiler produces saturated steam the temperature on the boiler, $T_{\text{sat}}$, increases along the saturation curve.
4These parameters can only be calculated with some uncertainty.
Dynamic Boiler Performance

- β and γ - see Equation E.33.

For verifying the developed model, these parameters have been determined by means of a least-square algorithm - see [76].

Outputs from a test on the plant and the corresponding simulation results can be seen in Figure 8.4. The results of parameter estimation by means of [76] in Figure 8.4 are:

- the convective coefficient of heat transfer in the furnace, \( \alpha_{conv,fur} = 22.1 \text{ W/m}^2 \cdot \text{K} \)
- the radiative coefficient of heat transfer in the furnace, \( \alpha_{rad,fur} = 5.2 \cdot 10^{-9} \text{ W/m}^2 \cdot \text{K}^4 \)
- the convective coefficient of heat transfer in the convection zone, \( \alpha_{conv,cz} = 115.7 \text{ W/m}^2 \cdot \text{K} \)
- the calorific value of the fuel (HFO), \( H_u = 37.0 \text{ MJ/kg} \)
- the parameters \( \beta \) and \( \gamma \) in Equation E.33 have been estimated to: 2.9318 kg/s resp. 0.87.

During the tests the measured steam flow, \( \dot{m}_s \), was always approx. 100 kg/h lower than the feed water flow, \( \dot{m}_{fw} \). It is presumed that this deviation is due to water droplet carry over in the steam. Since the steam flow is an input to the model it is presumed that this causes the deviation in the steam pressure - see Figure 8.4. This will be analyzed further in the following tests to be carried out on the plant.

8.6 Simulations and Experimental Verification - Conclusion

From Figure 8.4 it is seen that the least-square algorithm applied for estimating the parameters in the developed equation system ensures a good agreement between the sampled data and the simulations.

The estimated parameters: \( \alpha_{conv,fur} \) and \( \alpha_{conv,cz} \) fit very well with calculated values - see [153] and [134]. For the fuel applied during the test no measurement of heating value has been carried out - \( H_u = 37.0 \text{ MJ/kg} \) is a relatively low value (normally approx. 40 MJ/kg).

As can be seen from Figure 8.4 the simulations of the Stack Temperature and the Furnace Temperature also seems to follow the measurements well. Especially the measurement of the Furnace Temperature which was carried out in co-operation with Risø National Laboratory [183] by means of a suction pyrometry, is subject to some uncertainty.

The estimated parameter for \( \alpha_{rad,fur} \) is relatively low compared to black body radiation\(^4\). The following reasons/assumptions could cause this deviation:

\[^4\]The estimated coefficient of heat transfer by means of radiation in the furnace, \( \alpha_{rad,fur} \), corresponds to an emissivity in the range 10-15 % which even for the actual plant configuration, (i.e. relatively short radiation length), is very low.

- the measured temperature out of the furnace is too high (the measurement is carried out near the flame)
- the calculated temperature in the furnace model \( T_{fur} = \left[ T_{comb} + T_{fg,1} \right]/2 \) is too high.

Furthermore, the estimated parameters are considered constant and their variation with, for example, temperature or load are not taken into consideration.

Despite the mentioned uncertainties on the measurement, this relatively simple model predicts the measured figures very well.
Test with step-response on steam flow

**Steam Flow**
![Steam Flow Graph](image)

**Steam Pressure**
![Steam Pressure Graph](image)

**Steam Temperature**
![Steam Temperature Graph](image)

**Water Level**
![Water Level Graph](image)

**Stack Temperature**
![Stack Temperature Graph](image)

**Furnace Temperature**
![Furnace Temperature Graph](image)

Figure 8.3: Results from test with a step response in Steam Flow given to the test plant.
Simulations with estimated parameters

Figure 8.4: Results from test with step responses on steam flow (see Figure 8.3) given to the test plant and the corresponding simulation results.
Chapter 9

Optimization of the Fire tube Boiler

9.1 Introduction

In this chapter the optimization of the fire tube boiler introduced in Chapter 4 is finalized. The constraints for the optimization will be derived from the dynamic simulations. Furthermore, the results and consequences of the optimization are discussed.

9.2 Optimization

For simulating the water level fluctuations in the fire tube boiler the load change on the boiler (boiler load gradient) has been simulated as a change in steam production, i.e. a step-input. This approach has been used from the viewpoint that this matches the operation conditions verified most closely.

The changes in steam flow have been carried out while increasing the firing on the boiler by approx. 3.75 % per minute (i.e. 50 % load change in 800 sec) and the nominal firing on the boiler has been controlled to assure that the pressure at the end of the simulation period is the same, as at the start of the simulation period (i.e. 8 bar) - see Figure 9.1.

The simulations have been carried out with constant feed water supply to the boiler, \( \dot{m}_{fw} = 0.65 \text{ kg/s} \).

The simulations of the fire tube boiler have been carried out controlling the steam production and not the steam pressure, the opposite to that used in the water tube boiler.

The increase in steam production starts at \( t = 200 \) sec (see Table 9.1 and Figure 9.2).

\footnote{The pressure drops during the simulation as the steam production increases and is regained as the firing increases.}
The control of the steam production instead of the steam pressure means that the optimization constraint related to the steam space load will be a constant value corresponding to full load in the optimization of the fire tube boiler.\(^2\)

\(^2\)Opposite to the water tube boiler (see Chapter 7) where the defined operation conditions result in fluctuations in the steam production.
For the fire tube boiler plant analyzed in the present study the following constraint has been defined:

\[ 10\% \leq \Delta \text{load} \leq 50\% \quad \text{(9.1)} \]

The main data for the fire tube boiler are given in Appendix D.

The simulations have been carried out with the load changes (steps) on the steam production as given in Table 9.1.\(^3\)

The results on the water level fluctuations from the simulations can be seen in Figure 9.2. In Table 9.1 the required extra steam space volume in the boiler for absorbing the shrinking and swelling with the same fluctuations are given for the different boiler load changes simulated these data are plotted in Figure 9.2 with the boilers present steam space volume \(V_s = 0,8477\, \text{m}^3\) and no load change as reference.

\(^3\)This approach has been chosen to simulate operation conditions similar to the conditions applied for the model verification.
Optimizing dynamic design and operation of Fire tube Boilers

Figure 9.4: Objective Function with Feasible area defined by means of constraints on Boiler Volume and Boiler load change (Equation 4.19 and 4.20) and constraint related to shrinking and swelling.

9.3 Discussion of optimization results

As can be seen from Figure 9.4 and 9.5 the global minimum of the objective function can be found at the intersection between the maximum allowable boiler load change and the required steam space.
Optimizing design and operation of Fire tube Boilers

\[
\Delta \text{load} [\%]
\]

Figure 9.5: Contour plot of the objective Function with Feasible area defined by means of constraints on Boiler Volume and Boiler load change (Equation 4.19 and 4.20) and constraints related to shrinking and swelling and steam space load.

For the selected operation conditions, the fire tube boiler optimization does not have a local minimum of the objective function at the intersection between the minimum allowable boiler load change and the required boiler steam volume for different boiler load changes, which is opposite to the optimization carried out for the water tube boiler - see Chapter 7. This is because the surface defining the required volume for the selected operation conditions for different boiler load changes has the tendency shown in Figure 9.5.

Furthermore the selected operation conditions causing relatively small water level fluctuations result in a Shrinking and Swelling constraint being almost parallel with the \( \Delta \text{load} \) direction. If the boiler was designed with a smaller water volume the fluctuations in the water level would be correspondingly larger causing larger requirements with respect to \( V_s \) - see Figure 9.5.

\[4\] Attention should be drawn to the fact that the quantification of the boilers capability with respect to dynamic operation is based on the shape of the objective function as defined in Equation 4.9.
In general for the selected operation conditions the constraint defined by the dynamic operation of the boiler result is a very flat constraint, i.e. the required volume is almost insensitive to the boiler load change.

This means that to develop a minimum of the objective function at the intersection between the the minimum allowable boiler load change and the required volume for different boiler load changes would require an extremely flat objective function, i.e. a boiler plant where the dynamic operation capability is without any importance at all - an unrealistic Objective Function.

This means that for the defined operation conditions the most feasible boiler design would always be the design with the maximum allowable load change. This is mainly because of the course of the constraint related to Shrinking and Swelling - see in Figure 9.5.

If requirements with respect to maximum boiler volume/size were defined this would, as for the water tube boiler, correspond to a \( V_{\text{boi,max}} - surface \) parallel to the \( V_{\text{boi,min}} - surface \) (see Figure 9.4). This surface could, depending on the location, move the present global minimum towards a feasible solution with a smaller boiler volume/size. If the \( V_{\text{boi,max}} - surface \) were located to the left of the Shrinking and Swelling constraint (see Figure 9.5), the feasible area would be empty, i.e. no solution.
Chapter 10

Conclusions and Perspectives

Few years ago the main focus designing boilers was on the boilers efficiency at static load conditions, normally full load. The plants were not designed with special attention paid to the dynamic performance. Designing boilers for dynamic operation has traditionally focussed on avoiding over stressing of the materials and limiting the temperature gradients. Furthermore, the shrinking and swelling of the water level in the boilers water/steam reservoir (drum) have been analyzed and used to determine its required size.

As a result of increasing requirements with respect to efficiency and emissions and the liberalization/deregulation of the energy markets, where new opportunities for selling or buying energy arise, increased flexibility is required and more focus is put on the dynamic performance of the boilers.

10.1 Conclusion

In the present study it has been the objective to (see Chapter 1):

- Establish a framework for optimizing boiler design and operation with respect to dynamic performance.
- Develop and experimentally verify dynamic boiler models to be used defining selected constraints for the optimization model.

Designing boilers for dynamic operation offers a number of opposing aims that all could form the basis of a detailed optimization.

Essential opposing aims are (see page 2):

- Drum size - shrinking and swelling
Conclusions and Perspectives

- Drum size - steam quality
- Drum size - stress level
- Control system
- Drum size - pressure gradient
- Dynamic vs. static operation
- Boiler construction and choice of materials.

The framework for the optimization has been developed as *open*, i.e. it is at a later stage possible to include more *Design Variables* (for example pressure gradient) and corresponding *quantifications* as extra dimensions. In this manner the model could continuously be refined to take more aspects (opposing aims) into consideration.

In the present study a framework for optimizing the dynamic design and operation of boilers has been developed. The methodology for the optimization is based on an *Objective Function* quantifying the different boiler designs against each others (see Chapter 4). The optimization has been carried out on a *feasible* set limited by a number of constraints:

- boiler volume
- minimum and maximum boiler load gradients/changes
- minimum boiler volume defined by the *shrinking* and *swelling* of the water/steam in the boiler drum
- minimum boiler volume defined by the requirements with respect to *steam space load*\(^1\).

The two first mentioned constraints are *simple constraints* defined by the physical and geometrical limitations. For defining the two last constraints, models for simulating the dynamic boiler performance have been developed for Water tube Boilers and Fire tube Boilers. These constraints have been defined for selected operation conditions for the plant, e.g. controllers, pressure development and feed water flow.

The dynamic model developed for the fire tube boiler has been experimentally verified.

On the basis of the presumptions\(^2\) with respect to operation of the plants and quantification of the plants dynamic capability, optimizations of Water tube Boilers and Fire tube Boilers have been carried out.

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\(^1\)For the analyzed water tube boiler.
\(^2\)Presumptions have been made with respect to firing rate, pressure development and feed water flow on the boiler for the analysis conducted.
The optimization of the Water tube Boiler shows that a global minimum (cheapest boiler) is present for a plant designed for operation with the highest possible load gradient. The feasible area has been constrained by the requirements with respect to *Steam Space Load* and the constraint defined by the *shrinking* and *swelling* is not active, i.e. outside the feasible set. If the requirements with respect to *Steam Space Load* are defined less restrictively, i.e. allowing a higher steam space load during start-up of the plant, the constraint defined by the *shrinking* and *swelling* gets active, but still the global minimum will be located at the point with the highest possible load gradient. If the quantification of the plants dynamic operation capability is changed in a direction lowering the value of *good* dynamic capability the local minimum turns into a global minimum, i.e. the cheapest boiler is the boiler only allowed to be operated with the minimum allowable boiler load gradient.

The optimization of the Fire tube Boiler shows that a global minimum (cheapest boiler) is present for a plant designed for operating with the highest possible load change (simulation of load gradient). The feasible area has been constrained by the requirements with respect to *Shrinking* and *Swelling*. For the selected operation conditions a constraint has not been defined by the requirements with respect to *Steam Space Load*. For the Fire tube Boiler the quantification of the plants dynamic operation capability should be changed dramatically to move the optimum design away from a boiler designed for maximum allowable load change.

For defining the constraints with respect to *Steam Space Load* and *Shrinking* and *Swelling* two dynamic models have been developed. Both models are developed *component oriented* for modelling the different boiler components and afterwards merged into overall boiler models. Simulations have been carried out for a qualitative analysis and discussion of the results. Furthermore, measurements have been carried out on full scale boiler plants for a quantitative verification of the models. For the Fire tube Boiler a very good verification of the developed model has been possible and it is by means of the model possible to simulate the observed performance. For the Water tube Boiler the measurements were very limited and it has not been possible to verify the simulations by means of the measurements.

The main methodologically/scientifically contributions from the present study are:

- development of a framework for optimizing the design and operation of boilers with respect to dynamic performance
- development of an Objective Function that is open for inclusion of further dimensions (i.e. optimization challenges) for the optimization method
- *component oriented* development of dynamic boiler models
- experimental model verification of the Fire tube Boiler model
- demonstration of solution DAE’s of higher order by means of index reduction methodology for the *Steam drum model*.

The solutions favoring a good dynamic capability would probably also be the result of *traditional design* of boilers. A quantification of the *shift* in optimum as seen for the water tube boiler would
presumably not be possible with traditional design, i.e. the true optimum would most likely not be found. Developing the model a number of presumptions have been made, e.g. shape of objective function for quantifying the dynamic operation capability. And operation conditions to be analyzed have been selected. On the basis of these for the water tube boiler minor changes in for example the quantification could change the optimum. To change the optimum found for the fire tube boiler larger changes should be made in the presumptions.

The major strength of the developed framework is the ability to merge the different Design Variables into one scalar function (the objective function). This opens for a quantitative comparison of different designs. The major weakness of the developed framework is the determination of the parameters in the objective functions different terms, e.g. \( F_{\text{mass}} \) and \( F_{\text{dyn op}} \).

### 10.2 Perspectives

In the present study only a few of the possible dimensions have been included in the optimizations. During the study it became clear that a number of opposing aims all affecting the optimum design were present - see page 90.

In general all of these opposing aims (the list is probably not even complete) could affect the location of the optimum design. For the present study only two dimensions have been included, i.e. it has been possible to analyze the results of the optimization graphically. If more dimensions are included, the feasible set will turn into a multi-dimensional manifold and the optimization has to be carried out on this manifold.

The new dimensions are of course different in nature, as some of them (for example the overall operation pattern of the plant) are economically oriented and would probably be a completely new study. Other dimensions (for example pressurization of the plant) are technically oriented and could, to some extent, be based on the work carried out in the present study.

For future studies within Dynamic Boiler Performance - modelling, simulating and optimizing boilers for dynamic operation the following topics are recommended:

- selection of materials for boilers to optimize dynamic behavior further - to include manufacturing optimization. This dimension could include dealing with discontinuities and non-monotone surfaces - see section 4.5.

- Validation of the Objective Function developed in the present study - to include optimizations with discontinuities in, for example, prices

- analysis including different operational conditions. This could include analysis of a number of predefined operation conditions - each valued by the end-user of the plant.

- sensitivity analysis of the model developed in the present study - to include the Objective Function and the dynamic models.
• improvement of the dynamic models developed. This should include analysis of the presumptions made in the present work.

• maturing the developed model, i.e. preparation for practical use. This should include analysis of the input needed for the model, i.e. transforming of end-user requirements into the model.

• include long term respectively short term optimization in the model. Especially differences in the plants technical and commercial life-time changes focus between long and short term perspective.

• stochastic models, e.g. life time analysis. This could include analysis of the consequences of changes in e.g. operation conditions.

• analysis including the opposing aims described, but not analyzed in the present study - see Chapter 1

• Introduction of soft constraints vs. hard constraints. This should include analysis of designs almost within the feasible area.
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[187] Homepage of "Institut für Technisch Wärmelehre der Technischen Universität Wien".
     http://www.itw.tuwien.ac.at/

[188] Homepage of ThermoFlow, Inc.
     http://www.thermoflow.com/

[189] Homepage of TÜV.
     http://www.de.tuv.com/germany/de/produkte/industrie_service/
     druckgeraete_und_werkstofftechnik/index.html

[190] Homepage of "VGB".
     http://www.vgb.org/index.html
Appendix A

Terms and Definitions

Carry over

Carry-over is the amount of water carried with the steam out of the boiler drum. The simpler water steam separation equipment (or higher steam space load) in the drum the larger carry-over.

Convex Set

Mathematically a convex set is characterized as:

Theorem 2 (Convex set) If $p, q$ are points of $V$, the line segment from $p$ to $q$ is the set $\{tp + (1 - t)q : 0 \leq t \leq 1\}$. A subset $B \subset V$ is said to be convex if for every two points $p, q \in B$, the line segment from $p$ to $q$ is contained in $B$ - see [84].

In optimization challenges this means if two points $p, q$ are in the feasible set, any points on the line segment between $p$ and $q$ will also be in the feasible set.

Economizer

The economizer is the heating surface, where the feed water fed into the boiler is preheated to a temperature typically few degrees (approach) below the evaporation point - see Figure 2.2. The economizer is typically located in the coldest end of the boiler (on the flue gas side). As the economizer is a relatively cold heating surface, extended heating surfaces (fins, pins etc.) can be used - see [120].
**Evaporator**

The evaporator is the heating surface, where the evaporation in the boiler takes place. The evaporator is characterized by the high internal coefficient of heat transfer. This means that the evaporator physically can be located where the highest flue gas temperatures are present, i.e. in the combustion chamber - see [120].

For fired boiler typically a number of evaporator circuits will be present, all operating independently of each others. For every circuit the requirements with respect to *critical heat flux* have to be fulfilled - see Appendix B.5.3.

**HFO**

Heavy Fuel Oil is oil with a high content of long-chained components, i.e. asphaltthenes etc. HFO is characterized by having a very high viscosity, i.e. HFO has to be pre-heated to temperatures in the range 120-130 °C for atomizing and 50 °C for pumping. More details on HFO can be found in [8].

HFO is applied for most maritime applications.

**Manifold**

In the present Thesis a manifolds can be seen as a (constrained) set. For more details on Smooth Manifolds and Topological Manifolds reference should be made to [83] and [84].

**Subcritical Boiler**

A subcritical boiler is a boiler operating below waters critical pressure, i.e. \( p = 22,064 \) MPa and \( T = 647,096 \) K. Typically subcritical boilers are characterized by having an evaporator circuit, where the water/steam circulates (naturally or forced) before it is fully evaporated. Once-through boilers (e.g. Benson) could also be designed for subcritical operation (during low load operation and start-up the Benson boilers are operated subcritically). For more details on boilers see [153], [6], [120] and [7].

**Superheater**

The superheater is the heating surface, where the saturated steam leaving the boiler steam drum is superheated to the final steam temperature. The superheater is characterized by the *single phase* steam flow, i.e. relatively low internal coefficient of heat transfer. This means that the superheater physically has to be located in the boiler, where the flue gas temperature on the one hand is high enough to ensure
the superheating of the steam and on the other hand not too high (damaging the superheater material). For more details on superheaters - see [120].

**Supercritical Boiler**

A supercritical boiler is a boiler operating above water’s critical pressure, i.e. \( p = 22,064 \) MPa and \( T = 647,096 \) K. Supercritical boilers are also named once-through as these (at higher load) are characterized by not having an evaporator circuit, where the water/steam circulates (naturally or forced) before it is fully evaporated (Benson)\(^1\). For more details on boilers see [6] and [7].

\(^{1}\)In general Benson boilers are once-through boilers, but not necessarily supercritical boilers.
Appendix B

Basic Theory

B.1 Introduction

In this section the basic theory as being applied during the project is described. It should be emphasized that this is not an attempt to write a theory book for *Fluid Flow and Heat Transfer*, that are the main physical areas being relevant for the project. The intention is shortly to describe the theory applied and especially how the theory is applied for the actual purpose. In general [12] and [25] has been used and reference to these should be made.

This section has primarily been limited to the applications in the actual projects, but for part of the theory considerations with respect to the projects perspectives (see Chapter 10) have been made.

B.2 Fundamental equations

For the modelling the following general equations have been applied:

- Mass Balance (see section B.2.1)
- Momentum Balance (see section B.2.2)
- Energy Balance (see section B.2.3).

B.2.1 Mass Balance

The control volume shown in Figure B.1 has been applied for formulating the mass balance\(^1\) equations.

\(^1\)Often the term *Continuity Equation* is applied.
\begin{equation}
\frac{dM(t)}{dt} + \dot{m}_{out}(t) - \dot{m}_{in}(t) = 0
\end{equation}

**Figure B.1**: Mass Balance for a control volume - see [12].

\begin{equation}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.
\end{equation}

(B.1)

The mass balances are being formulated on the basis of an average velocity through the control volumes boundaries, i.e.

\begin{equation}
\frac{dM(t)}{dt} + \dot{m}_{out}(t) - \dot{m}_{in}(t) = 0
\end{equation}

(B.2)

### B.2.2 Momentum Balance

The control volume shown in Figure B.2 has been applied for formulating the momentum balance equations.

In the general form the momentum balance can be written as (Newtons 2. Law):

\begin{equation}
\frac{d}{dt} \int_{V_{M(t)}} \rho \mathbf{v} \, dV = \mathbf{F}
\end{equation}

(B.3)

Applying the *Leibnitz rule* for differentiation of a volume integral - see [25]:

\begin{equation}
\frac{d}{dt} \int_{V_{M(t)}} \mathbf{u} \, dV = \int_{V_{M(t)}} \frac{\partial \mathbf{u}}{\partial t} \, dV + \int_{S(t)} (\mathbf{n} \cdot \mathbf{v}_S) \mathbf{u} \, dS
\end{equation}

(B.4)
Basic Theory

and the Divergence Theorem - see [25]:

\[
\int_{V(t)} \nabla \cdot (\mathbf{v} \mathbf{u}) \, dV = \int_{S(t)} \mathbf{n} \cdot (\mathbf{v} \mathbf{u}) \, dS \tag{B.5}
\]

which combined yields:

\[
\frac{d}{dt} \int_{V_{st}(t)} \mathbf{u} \, dV = \int_{V_{st}(t)} \left[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{u}) \right] \, dV \tag{B.6}
\]

This identity which is also called the Reynolds’ transport theorem applied to eqn. B.3 with \( \mathbf{u} \) replaced by \( \rho \mathbf{v} \) yields for the bracket:

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = \mathbf{v} \left[ \frac{\partial (\rho)}{\partial t} + \nabla \cdot \rho \mathbf{v} \right] + \rho \left[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right] \tag{B.7}
\]

**Figure B.2**: Momentum Balance for a control volume.
From the *continuity equation* (B.1) it is seen that the first bracket on the right hand side is equal to zero and using the Substantial Derivative\(^2\) yields:

\[
\frac{d}{dt} \int_{V_M(t)} \rho v \, dV = \int_{V_M(t)} \rho \frac{Dv}{Dt} \, dV
\]

(B.8)

i.e.,

\[
\int_{V_M(t)} \rho \frac{Dv}{Dt} \, dV = F
\]

(B.9)

The forces \(F\) acting on the control volume are normally split in forces acting on:

- the *surface*: \(F_{\text{sur}}\)
- the *volume*: \(F_V\)

of the control volume i.e.,:

\[
F = F_{\text{sur}} + F_V
\]

(B.10)

For the control volume in Figure B.2 the forces acting on the surface are the *pressure forces* and the *pressure loss*, i.e.,:

\[
F_{\text{sur}} = F_p + F_{\text{fric loss}}
= A \cdot (p_{\text{in}} - p_{\text{out}}) - A \cdot \Delta p_{\text{fric}}
\]

(B.11)

The - sign on the *friction loss* is due to the fact that this force is against the motion of the fluid. The force acting on the volume is the *gravity force*, i.e.:

\[
F_V = A \cdot \frac{d}{dt} \cdot \rho \cdot g \cdot \sin \varphi
\]

(B.12)

combining the equations B.9 (after having integrated the volume, i.e., \(\int_0^V dv = V\)), B.11 and B.12 yields:

\[
\frac{2}{Dt} \equiv \frac{\partial}{\partial t} + v \cdot \nabla - \text{the term Material Derivative is also applied.}\]
Basic Theory

\[ A \cdot dz \cdot \rho \frac{Dv}{Dt} = A \cdot (p_{in} - p_{out}) - A \cdot \Delta p_{fric} - A \cdot dz \cdot \rho \cdot g \cdot \sin \varphi \]

\[ A \cdot dz \cdot \rho \left[ \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial z} \right] = A \cdot (p_{in} - p_{out}) - A \cdot \Delta p_{fric} - A \cdot dz \cdot \rho \cdot g \cdot \sin \varphi \]

integrating the whole length (i.e. \( \int_{0}^{Z} dz = Z \)) and setting \( v \cdot \frac{\partial v}{\partial z} = \bar{v} \cdot \frac{v_{out} - v_{in}}{Z} \) yields:

\[ \frac{dv}{dt} + \bar{v} \cdot \frac{v_{out} - v_{in}}{Z} = \frac{p_{in} - p_{out} - \Delta p_{fric}}{Z \cdot \rho} - g \cdot \sin \varphi \] (B.13)

with \( \sin \varphi \equiv \frac{\text{Height of element}}{\text{Length of element}} = \frac{H}{Z} \)

For a pipe element the Friction Loss (\( \Delta p_{fric} \)) can be written as - see [12]:

\[ \Delta p_{fric} = \lambda \cdot \frac{Z}{d} \cdot \frac{1}{2} \cdot \rho \cdot v^2 \Rightarrow \frac{\Delta p_{fric}}{Z \cdot \rho} = \frac{\lambda}{2 \cdot d} \cdot v^2 \] (B.14)

inserting Equation B.14 in Equation B.13 yields:

\[ \frac{dv}{dt} + \bar{v} \cdot \frac{v_{out} - v_{in}}{Z} = \frac{p_{in} - p_{out} - \Delta p_{fric}}{Z \cdot \rho} - \frac{\lambda}{2 \cdot d} \cdot v^2 - g \cdot \frac{H}{Z} \] (B.15)

which is the Momentum Balance to be applied developing the dynamic models.

Developing the Momentum Balances it is presumed, that:

- viscous forces
- buoyancy terms

can be neglected - see [12].

**B.2.3 Energy Balance**

The control volume shown in Figure B.3 has been applied for formulating the energy balance equations.
Applying the First Law of Thermodynamics for the element shown in Figure B.3 the Energy Balance can be written as:

\[ \frac{dU(t)}{dt} + \dot{m}_{\text{out}}(t) \cdot h_{\text{out}}(t) - \dot{m}_{\text{in}}(t) \cdot h_{\text{in}}(t) + \dot{q}_{\text{out}}(t) - \dot{q}_{\text{in}}(t) = 0 \]

which is the Energy Balance to be applied developing the dynamic models.

The equations are formulated on the basis of the Internal Energy, \( U(t) \), and the Specific Internal Energy, \( u(t) \):

\[ U(t) = M(t) \cdot u(t) \]

as the specific internal energy, \( u(t) \), is not at state property, the enthalpy, \( h(t) \), is applied together with the following relationship:

\[ u(t) = h(t) - p(t) \cdot \nu(t) \]
\[ u(t) = h(t) - \frac{p(t)}{\rho(t)} \]

For the properties of the medium in the element it is presumed that these can be calculated at the mean of inlet and outlet conditions (e.g. \( T_{\text{element}} = (T_{\text{in}} + T_{\text{out}})/2 \)).
The terms $q_{\text{out}}(t)$ and $q_{\text{in}}(t)$ are calculated by means of the constitutive relations given in Section B.5. Developing the Energy Balances it is presumed, that:

- axial conduction in pipes (diffusion)
- radiation
- viscous stresses

can be neglected.

B.3 Water/steam Properties

In general water/steam properties have been calculated according to the industrial formulation IAPWS-IF97, that has been developed by The International Association for the Properties of Water and Steam (see [171]) and is detailed described in [141].

B.4 Flue gas Properties

The Flue Gas properties applied in the simulations are calculated according to [13] and [82]. In general the properties are calculated as weighted properties - mass fractions\(^3\). According to [22] the property values from [13] are the same (within very little margin) as the values calculated from [20].

B.5 Heat transfer

B.5.1 Fouling

Probably the most important single discipline within boiler design is heat transfer and numerous books within this area has been written - see for example [92] and [70]. Together with calculation of coefficients of heat transfer the fouling is very important. The overall coefficient of heat transfer from the gas side to the water/steam side (the typical heat flow direction in boilers) can be written with the basic relationship:

\[
q = U_{\text{ht}} \cdot A \cdot \Delta T
\]

\(^3\)This method is applied for Enthalpy, Density, Specific Heat capacity etc., but not for Entropy, which is increased due to this mixing of the different components.
where

\[ U_{ht} = \frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}} + \frac{1}{R_{\text{metal}}} + R_{\text{int}} + R_{\text{ext}} \]  

(B.16)

The temperatures at the different interfaces (see Figure B.4) can be calculated by means of a simple heat balance for each layer - knowing the total transferred amount of heat from the overall heat balance.

To include fouling in the heat transfer calculations is primarily relevant for applications with heavy fouling, e.g. HFO-, coal- or wood fired plant. For these applications it is important to calculate the temperature profile through the layers - see Figure B.4. Especially for applications where internal fouling \( R_{\text{int}} \) is possible, i.e. cooling of metal can be limited, which normally has severe consequences.

For the water tube boiler (see Chapter 6) no fouling has been included in the calculations, this is due to the facts that the boiler is operated with high-quality feedwater and on the flue gas side it is fired with light oil. This means that \( R_{\text{int}} \) and \( R_{\text{ext}} \) are both close to zero.

A detailed description of fouling can be found in [70] - especially the different mechanism controlling fouling.

\[ ^4 \text{B.16 is based on a flat plate with the same internal and external area. For extended heating surface - see [66].} \]

\[ ^5 \text{See [116], [138] and [140].} \]

\[ ^6 \text{For simple applications an overall fouling coefficient (calculated as a percentage to be multiplied on the overall coefficient of heat transfer - related to a clean heating surface) has often been applied. Especially for heavy fouled heating surfaces this approach will only yield the correct amount of energy transferred, but not the correct material temperatures} \]
B.5.2 Single Phase Flow

For boiler design the fluids:

- air
- flue gas
- water
- steam

are calculated as single phase flow. Except for a very few applications all heat transfer in boilers take place as *forced convection* and *radiation*. For the *forced convection* inside tubes the Dittus-Boelter
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Formula\(^7\) will typically be applied\(^8\):

\[ Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \] (B.17)

[12] includes the ratio between the wall and bulk viscosity:

\[ Nu = 0.026 \cdot Re^{0.8} \cdot Pr^{1/3} \cdot \left( \frac{\mu_{bulk}}{\mu_{wall}} \right)^{0.14} \] (B.18)

More references of Nu-number for forced convective flow can be found in [67] and [139] - the latter to include the Gnielinski formula.

For a water tube boiler the flue gas flows on the outside of the heating surface/tubes. Normally numerous tubes are located next to each other and typically in several layers - either inline or staggered configuration. For the flue gas side of the tubes, the coefficient of heat transfer is typically given as:

\[ Nu = f(Re, Pr, f_A) \] (B.19)

where \( f_A \) is a factor related to the geometry of the heating surface - see [14].

For heating surfaces with circular fins the heat transfer problem can be solved analytically - a detailed description can be found in [14].

**B.5.3 Two Phase Flow**

In general designing boilers the high coefficient of heat transfer in the two-phase flow region is taken advantage of for cooling the boiler materials in the regions expiring the highest flue gas temperatures - see Figure 2.2. Very comprehensive studies of heat transfer in two-phase flow regions have been carried out\(^9\). The studies have been intensively reported in e.g. [174] and in the classical books [21] and [142] furthermore reference should be made to [11] and [147], newer references can be found in [135], [102], [72], [73] and [67]. On an overall level the two-phase flow splits into pool boiling and boiling flow in tubes - the latter further split into flow in horizontal and vertical tubes, this is due to the effect from gravitation. Designing boilers one of the most critical topics to analyze in the critical heat flux (CHF), which is characterized by a sharp reduction of the local coefficient of heat transfer resulting from a replacement of liquid by steam adjacent to the heating surface causing the temperature to rise two or three orders of magnitude - see [21] and [67]. In some references the term dry out is applied for this point - see [146]. In general the prediction of CHF is very complex\(^10\) since many parameters are involved, [21] lists the following:

---

\(^7\)The Dittus-Boelter equation is very similar to the well-known Colburn equation \( Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{1/3} \) - [67].

\(^8\)In the full version, a factor compensating for inlet conditions \(1 + (d_{int}/L)^{2/3}\), is multiplied on the Nu-number - see [139].

\(^9\)The most intensive research was carried out in the sixties and seventies and were primarily related to cooling of the rods in nuclear powerplants - see [133].

\(^10\)According to [67] more than 400 different correlations for predicting CHF has been published since 1982.
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Figure B.5: Flow Patterns in Vertical and Horizontal flow - see [21].

- the tube diameter
- the tube Length
- the inlet flow rate
- the inlet temperature
- the system pressure

as the most important parameters for prediction of CHF and illustrates the importance of the different parameters. The most important correlations for predicting CHF are listed in [67].

Many attempts have been made to establish a systematic structure for the different flow patterns depending on the mass velocities of water and steam, the first Flow Pattern Map for horizontal flow was prepared by Baker in 1954 - see [21] and Figure B.6. Hewitt and Roberts prepared a similar map for vertical flow in 1969 - see [21].

More flow pattern maps have been developed for vertical and horizontal flow - see e.g. [21]. Many of these are based on the same idea as Baker’s diagram and typically the axis have been changed to include more phenomena - see for example Taitel, 1990, [21].

For modelling the two-phase flow a number of models have been developed, in general the models split in two types:
**Figure B.6:** Flow Pattern map for Horizontal flow (Baker 1954) - see [21].

- homogenous model(s)
- separated models.

In the *homogenous model* the two-phase flow is assumed to be a single-phase flow having pseudo-properties (suitable weighting of the properties of the individual phases), and there is no relative velocity between the phases [21] and [147].

In the *separated phase flow model* the two phases are treated independently of each other and relations for the interactions between the phases are included in the analysis - see [133].

As mentioned the coefficient of heat transfer for two-phase flow (i.e. boiling - forced convective boiling or pool boiling) is normally very large compared to coefficient of heat transfer for the heating surfaces flue gas side and the heat conduction in the boiler material\(^{11}\). Relatively simple correlations for coefficient of heat transfer taking *pressure* and *heat flux* into consideration can be found in [14] - the Jens/Lottes correlation:

\[
\alpha = 1,267 \cdot q^{0,75} \cdot e^{63} \text{ W/m}^2 \cdot \text{K} \quad [50 - 150 \text{ bar}]
\]  

(B.20)

can be applied for *nucleate boiling* (i.e. *sub-cooled boiling* - see [67]). More formulas for heat transfer at evaporation can be found in [14] and [21].

\(^{11}\)This does not count fully for the regions of the boiler with the highest specific heat input, e.g. furnace.
As can be seen from Figure B.7 the high coefficient of heat transfer related to two-phase flow (boiling) controls the wall temperature.

For practical applications [153] recommends that the value:

\[ \alpha_{\text{int}} = 10.000 \text{ W/m}^2 \cdot \text{K} \]  

(B.21)
is applied for the two-phase flow regions\(^{12}\).

For the actual project B.21 will be applied.

### B.6 Pressure Loss

For boiler design the fluids are calculated as Newtonian fluid and the pressure loss is calculated according to:

\[
\Delta p = \lambda \cdot \frac{L}{d} \cdot \frac{1}{2} \cdot \rho \cdot v^2
\]  

(B.22)

Where the coefficient of friction is given according to the *Moody chart* - giving the friction factor as a function of \(Re\) and relative roughness \(\frac{s}{D}\) for round pipes - see [95].

Typically the flow will be fully turbulent, i.e. \(Re \gg 10,000\), and it is a practical experience that the relative roughness for pipes being applied within the boiler industry correspond to a friction factor being:

\[
\lambda = 0.02
\]  

(B.23)

For the actual project B.23 will be applied.

For calculation of pressure drop in the two-phase region the homogenous model has been applied.

---

\(^{12}\)The coefficient of heat transfer at the flue gas side will normally be orders lower than the coefficient of heat transfer at the water/steam side at evaporation, i.e. the coefficient of heat transfer for evaporation has normally low/no importance - see [14].
Appendix C

Water tube boiler - tests

C.1 Test Plant

It was planned to apply a full-scale plant for the verification of the water tube boiler model. The plant chosen was a MISSION™ WHR-GT manufactured in Aalborg, Denmark and installed at the Cruise Liner Coral Princess, being built at the Chantiers de l’Atlantique Shipyard in France. Pictures from the manufacturing and assembling in the workshop can be seen in Figure C.3 and C.4. Boiler and drum drawings can be seen in Figure C.6. For more details on the project see [121] and [19]. The tests were carried out as a part of the commissioning and the performance test of the plant which means that it was not possible to adjust the load and the gradients on the plants as required/planned. Pictures from the performance test can be seen in Figure C.5.
Figure C.1: Coral Princess at the Chantiers de l’Atlantique Shipyard in France.

Figure C.2: Coral Princess at sea.
Figure C.3: Workshop pictures from manufacturing and assembling of the MISSION™ WHR-GT.
Figure C.4: Workshop pictures from manufacturing and assembling of the MISSION™ WHR-GT.
Figure C.5: Pictures from performance test on MISSION™ WHR-GT.
Figure C.6: Boiler and drum drawings.
Figure C.7: Boiler installment on ship.
C.2 Results from the performance tests at Coral Princess

The performance test on the plant were carried out in the period 08-09 November 2002. Except from the performance measurements being relevant for the dynamic modelling, Aalborg Industries also carried out measurements of the temperature gradients in casing plate, flue gas bypass (to reheat the cavity between the headers and thereby avoid thermal stress gradients).

As mentioned in Chapter 6 it was decided to apply a full scale plant for the test; on the one hand benefiting from the more precise results that hereby can be obtained, but on the other hand giving up part of the desires with respect to what load cases to operate the plants in.

Unfortunately the test period was postponed several times until at the end it was very close to the deadline for delivery of the ship to the owner. This caused some limitations with respect to what measurements could be carried out. The results from the measurements can be seen in the Figure C.11. An example showing the different measurements can be seen in Figure C.10. The location of the measurement points can be seen on the P & I diagram in Figure C.9.

During the test the following parameters were measured/sampled:

- Gas Turbine load [MW]
- the Gas Turbine compressor discharge temperature [°C]
- the pressure in boiler drum [bar]
- the drum level [mm]
- Pressure difference - circulation flow measurement [bar]
- the pressure at the boiler outlet [bar]
- the feedwater temperature i.e. temperature in hot well [°C]
- Boiler inlet header temperature [°C]
- temperature of Water/Steam in circulation circuit [°C]

During the tests it was not possible to sample the feed water flow and the steam flow, these were monitored manually, and the feedwater controller was not trimmed to the optimum.

---

1The Gas Turbine compressor discharge temperature is not applied in the model verification.
Figure C.8: Characteristic for evaporator circulation pump.
Figure C.9: Boiler P & I diagram with measurement points.
Figure C.10: Example of output from tests.
Figure C.11: Result from measurements on test plant - 8. Oct 2002, 02:00:00 - 02:30:00 & 03:00:00 - 03:30:00.
C.3 Tests - experimental verification

It was planned to carry out a number of dynamic tests (start-up, load changes and especially dry-running of the plant).

Due to circumstances which the present study could not control the full scale plant was heavily delayed which limited the extent of measurements dramatically\(^2\).

The lack of good measurement data (and possibilities for improving these) means that it has only to a very limited extent been possible to exploit the tests for verifying the water tube boiler model.

For the measurements it has not been possible to verify the Mass Balance and the First Law of Thermodynamics, which indicates that not all measurements are correct.

As an example of verification the measurements carried out 8. Oct 2002 (see Figure C.11) has been used.

Data for the gas turbine outlet mass flow, \(m_{fg}\), and flue gas temperature, \(T_{fg}\), have been taken from [156] and [182] based on the gas turbine load data from the test.

---

\(^2\)The MISSION™ WHR-GT boiler was supplied according to schedule, but due to other delays on the ship the access to the boiler and possibilities for carrying out further test ended up being very limited.
C.4 Simulations and Experimental Verification - Conclusion

As mentioned in Chapter 6.4 a number of presumptions have been made during the development of the models. In general these presumptions are made to simplify the models and will cause minor errors on the results from the simulations.

For the actual purpose of the dynamic models, it shows the correct/expected characteristics - see Chapter 6.5. Improving the developed model or substituting it with more advanced models will cause minor changes of the results and could marginally affect the location of the optimum design - see Chapter 7.

The experimental verification of the developed models has not been possible. In general the extent of measurements was very limited and a chance for correcting errors in measurements has unfortunately not been given. Furthermore the feed water controller was not trimmed, i.e. it has been very difficult to carry out simulations verifying the measurements.

The circulation flow in the evaporator circuit has been measured during load and pressure changes on the plant and can be seen from Figure C.12. With all the reservations\(^3\) with respect to the results already given, the curve seems to have the right tendency as the flow drops for increasing pressure and (with a delay) starts to increase as the pressure increases again.

The water level in the evaporator has been measured in the same time interval, again the curve seems to have the right tendency as the level lowers for increasing pressure (corresponding to a compression of the steam in the evaporator). The level starts increasing again as the pressure decreases.

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\(^3\)Including the presumptions/simplifications - see Chapter 6.4
Appendix D

Fire tube boiler - tests

D.1 Introduction

As the present study was initiated it was planned to include analysis of and measurements on the MISSION™ WHR-GT as a part of the study. Shortly after the Ph.D. study was initiated Aalborg Industries decided to start the development of the MISSION™ OB boiler and analysis of and measurements on this boiler were therefore naturally included in the project.

A detailed description of the MISSION™ OB development project can be found in [128].

D.2 Test Plant

The overall objective in the MISSION™ OB boiler development project was to develop a cost effective boiler based on an integrated unit design point of view. The main idea in the integrated unit design is to integrate and optimize the different components:

- the pressure part
- the burner
- the control system.

A very detailed test programme including:

- overall boiler performance
- burner performance
Dynamic Boiler Performance

Figure D.1: P&I diagram of the 3.5 t/h MISSION™ OB prototype plant - burner system.

- convective heat exchanger performance
- furnace temperature measurements

was initiated as a part of the MISSION™ OB development project.

And finally detailed measurements of the plants dynamic performance were carried out.

For assessing the dynamic performance the following parameters were measured:

- the fuel flow, $\dot{m}_{\text{fuel}}$
- the fuel temperature, $T_{\text{fuel}}$
- the air temperature, $T_{\text{air}}$
- the air flow, $\dot{m}_{\text{air}}$
- the flue gas1 temperature, $T_{f_{g,1}}$
- the flue gas2 temperature, $T_{f_{g,2}}$
- the feedwater temperature, $T_{f_{w}}$
Fire tube boiler - tests

Figure D.2: Drawing of the 3.5 t/h MISSION™ OB prototype plant.

- the feedwater flow, $m_{fw}$
- the water level, $L_w$
- the steam pressure, $p_s$
- the steam flow, $m_s$
- the steam temperature, $T_s$

The data sampling has been carried out by means of xPC & Realtime Workshop - see [179]
D.3 Results from the tests

In the following examples of test results obtained from measurements on the MISSION™ OB test plant are given.
Figure D.3: Result from measurements on MISSION™ OB test plant, to be applied for estimating parameters in the simulation model - date 04. April 2003.
Figure D.4: Result from measurements on MISSION™ OB test plant, operation at approx. 1000 kg/h - date 04. April 2003.
**Figure D.5:** Result from measurements on MISSION™ OB test plant, operation at approx. 1500 kg/h - date 04. April 2003.
Figure D.6: Result from measurements on MISSION™ OB test plant, operation at approx. 2000 kg/h - date 04. April 2003.
Appendix E

Modelling of Fire Tube Boiler

E.1 Introduction

In this appendix a derivation and detailed description of the model applied for modelling and simulating the fire tube boiler is given (see Chapter 8).

From an overall point of view boiler systems consist of three major components:

- pressure part
- burner
- control system.

Traditionally these systems have been analyzed and developed relatively independent of each other, i.e. not optimized for integrated operation. To be able to meet the futures requirements with respect to emission, efficiency and dynamic performance, these components must be optimized as an integrated system.

The modelling and simulation of the fire tube boiler is foreseen as:

- a continuation of the development of the MISSION\textsuperscript{TM} OB boiler - see [128]
- a predecessor of a continued development of advanced control systems - see [127], [131] and [98].

The overall objective of these activities which are central in Aalborg Industries’ R&D activities is to develop boilers as integrated units.
For carrying out the integrated optimization of boilers; modelling, simulating and optimizing boiler
designs with respect to dynamic performance have become significantly important. In the present
study a dynamic model for the boiler performance has been developed. The overall objective has been
to be able to control the boilers:

- water level, \( L_w \)

and

- steam pressure, \( p_s \)

more efficiently and hereby optimize the boiler design with respect to size and dynamic performance.

The model (see Figure E.2) consist of sub models for:

- the furnace (combustion chamber) - see section E.2.1
- the convection zone - see section E.2.2
- the water/steam section - see section E.2.3.

These models have been combined into an overall model (see Figure E.1) having the following *controllable input*\(^1\) parameters:

- the fuel flow, \( \dot{m}_{fuel} \)
- the air flow, \( \dot{m}_{air} \)
- the feed water flow, \( \dot{m}_{fw} \)

\(^{1}\textit{Controllable parameters} are parameters which can be adjusted by the operator during operation of the plant, e.g. increase feed water flow.\)
the following non-controllable input parameters:

- the fuel temperature, $T_{\text{fuel}}$
- the air temperature, $T_{\text{air}}$
- the feed water temperature, $T_{fw}$

and the following output parameters:

- the steam flow, $\dot{m}_s$
- the boiler pressure, $p_s$
- the water level in the boiler, $L_w$.

This approach has also been applied for water tube boilers - see [125] and [126].

For verifying the developed model tests and measurements have been carried out on a full scale boiler plant.

A sketch of the boiler and the models can be seen in Figure E.2.

In general the equation systems developed in this appendix are DAE-systems - see Chapter 5. For the future development of advanced control systems the equation systems will be reduced to ODE-systems.

### E.2 Modelling

As mentioned the overall model consists of a number of sub models that are exchanging data according to Figure E.2. In the modelling and simulation water/steam properties have been calculated according to [141], flue gas properties according to [82] and constitutive relations for e.g. combustion and heat transfer according to [153].

#### E.2.1 Furnace

The equation system modelling the furnace is composed of:

- the energy balance for the furnace (differential equation)
- the mass balance for the furnace (algebraic equation)
and constitutive equations for:

- $U_{fur}$, $\dot{q}_{fuel}$, $\dot{q}_{air}$, $\dot{q}_{fg,1}$, $\dot{q}_{rad}$, $\dot{q}_{conv}$, $T_{fur}$ and $T_{comb}$

**Energy balance for furnace**

$$\dot{q}_{fuel} + \dot{q}_{air} - \dot{q}_{fg,1} - \dot{q}_{rad} - \dot{q}_{conv} - \frac{dU_{fur}}{dt} = 0 \quad (E.1)$$

**Internal Energy in Furnace**

$$U_{fur} = M_{fg,fur} \cdot h_{fg,fur} = V_{fur} \cdot \rho_{fur} \cdot c_{p,fur} \cdot T_{fur} \quad (E.2)$$

---

$^2$For calculation of enthalpies, $T_{ref} = 0^\circ C$ is used.
Energy flow with fuel into furnace

\[ \dot{q}_{fuel} = \dot{m}_{fuel} \cdot (H_u + h_{fuel}) = \dot{m}_{fuel} \cdot (H_u + c_{p,fuel} \cdot T_{fuel}) \quad (E.3) \]

Energy flow with air into furnace

\[ \dot{q}_{air} = \dot{m}_{air} \cdot h_{air} = \dot{m}_{air} \cdot c_{p,air} \cdot T_{air} \quad (E.4) \]

Energy flow with flue gas out of furnace

\[ \dot{q}_{fg,1} = \dot{m}_{fg,1} \cdot h_{fg,1} = \dot{m}_{fg,1} \cdot c_{p,fg,1} \cdot T_{fg,1} \quad (E.5) \]

Energy flow by means of radiation in furnace

\[ \dot{q}_{rad \rightarrow p} = A_{fur} \cdot \alpha_{rad,fur} \cdot \left( T_{fur}^4 - T_p^4 \right) \quad (E.6) \]

Mass balance for furnace

\[ \dot{m}_{fg,1} = \dot{m}_{fuel} + \dot{m}_{air} \quad (E.7) \]

Energy flow by means of convection in furnace

\[ \dot{q}_{conv \rightarrow p} = A_{fur} \cdot \left( \frac{\dot{m}_{air}}{\dot{m}_{air,ref}} \right)^{0.8} \cdot \alpha_{conv,fur} \cdot \left( T_{fur}^4 - T_p^4 \right) \quad (E.8) \]

in E.8 it is presumed that: \( Nu \propto Re^{0.8} \) (see Appendix B).

Mean Temperature in furnace

\[ T_{fur} = \frac{T_{comb} + T_{fg,1}}{2} \quad (E.9) \]
Combustion temperature

\[ T_{comb} = \frac{\dot{q}_{fuel} + \dot{q}_{air}}{m_{fg,1} \cdot c_{p,fg,1}} \]  

(E.10)

Reformulating the equations, the differential equation for the furnace temperature \( T_{fur} \) can be written as:

\[ \frac{dT_{fur}}{dt} = \frac{\dot{q}_{fuel} + \dot{q}_{air} - \dot{q}_{rad} - \dot{q}_{conv} - \dot{q}_{fg,1}}{V_{fur} \cdot c_{p,fg,fur} \cdot \rho_{fg,fur}} \]  

(E.11)

E.2.2 Convection Zone

The equation system modelling the convection zone is composed of:

- the energy balance for the convection zone (differential equation)
- the mass balance for the convection zone (algebraic equation)

and constitutive equations for:

- \( U_{fg,cz}, \dot{q}_{fg,2}, \dot{m}_{fg,2}, \dot{q}_{fg-p,cz}, T_{fg,cz} \) and \( M_{fg,cz} \).

Energy Balance

\[ \dot{q}_{fg,1} - \dot{q}_{fg,2} - \dot{q}_{fg-p,cz} - \frac{dU_{fg,cz}}{dt} = 0 \]  

(E.12)

Internal energy

\[ U_{fg,cz} = M_{fg,cz} \cdot h_{fg,cz} = V_{cz} \cdot \rho_{fg,cz} \cdot c_{p,fg,cz} \cdot T_{fg,cz} \]  

(E.13)

Mean flue gas temperature in convection zone

\[ T_{fg,cz} = \frac{T_{fg,1} + T_{fg,2}}{2} \]  

(E.14)
Convective energy flow out of furnace

\[ \dot{q}_{fg,2} = \dot{m}_{fg,2} \cdot h_{fg,2} = \dot{m}_{fg,2} \cdot c_{p,f,g,2} \cdot T_{fg,2} \]  

(E.15)

Mass balance for convection zone

\[ \dot{m}_{fg,2} = \dot{m}_{fg,1} \]  

(E.16)

Energy flow by means of convection in convection zone\(^3\)

\[ \dot{q}_{fg-p,ez} = A_{fg,ez} \cdot \left( \frac{\dot{m}_{air}}{\dot{m}_{air,ref}} \right)^{0.8} \cdot \alpha_{conv,ez} \cdot (T_{fg,ez} - T_{p,ez}) \]  

(E.17)

Mass of flue gas in convection zone

\[ M_{fg,ez} = V_{fg,ez} \cdot \rho_{fg,ez} \]  

(E.18)

Reformulating these equations (E.13, E.14, E.15, E.16, E.17 and E.18), the differential equation for the mean flue gas temperature in the convection zone \( T_{fg,ez} \) can be written as:

\[ \frac{dT_{fg,ez}}{dt} = \frac{\dot{q}_{fg,1} - \dot{q}_{fg,2} - \dot{q}_{fg,ez}}{V_{fg,ez} \cdot c_{p,f,g,ez} \cdot \rho_{fg,ez}} \]  

(E.19)

E.2.3 Water/Steam Section

Complete Water/Steam Section

For modelling the dynamic behavior of the water/steam section the model in Figure E.3 has been applied.

\(^3\)It is presumed that the convective area is constant, i.e. the water submerged area is not corrected for fluctuations in water level.
Mass balance for the complete system

\[
\frac{d(\rho_s [V_t + V_s] + \rho_w V_w)}{dt} = \dot{m}_{fw} - \dot{m}_s
\]  

(E.20)

as \((V_t = V_w + V_s + V_b)\) (E.20) yields:

\[
\frac{d(\rho_s [V_t - V_w] + \rho_w V_w)}{dt} = \dot{m}_{fw} - \dot{m}_s
\]

i.e.

\[
\frac{d\rho_s}{dt} V_t - \frac{d\rho_s}{dt} V_w + \frac{dV_t}{dt} \rho_s - \frac{dV_w}{dt} \rho_s + \frac{d\rho_w}{dt} V_w + \frac{dV_w}{dt} \rho_w = \dot{m}_{fw} - \dot{m}_s
\]

i.e.

\[
\frac{d\rho_s}{dt} (V_t - V_w) + \frac{dV_w}{dt} (\rho_w - \rho_s) + \frac{d\rho_w}{dt} V_w = \dot{m}_{fw} - \dot{m}_s
\]
applying the chain rule for differentiation\(^4\) yields:

\[
\frac{dp}{dt} (V_t - V_w) + \frac{dV_w}{dt} (\rho_w - \rho_s) + \frac{d\rho_w}{dt} V_w = \dot{m}_w - \dot{m}_s
\]

which can be written as:

\[
\frac{dp}{dt} \left( \frac{d\rho_s}{dp} [V_t - V_w] + \frac{d\rho_w}{dp} V_w \right) + \frac{dV_w}{dt} (\rho_w - \rho_s) = m_w - m_s
\] (E.21)

**Energy balance\(^5\) for the complete system:**

\[
\frac{d}{dt} \left( U_w + U_b + U_s + U_p \right) = \dot{q}_{fg\rightarrow w/s} + \dot{q}_w - \dot{q}_s
\]

inserting the internal energy \(U = M \cdot u = \rho \cdot V \cdot (h - \nu \cdot p)\) yields:

\[
\frac{d}{dt} \left( \rho_w V_w (h_w - \nu_w p) + \rho_s (V_b + V_s) (h_s - \nu_s p) + \rho_p V_p c_p T_s \right) = \dot{q}_{fg\rightarrow w/s} + \dot{q}_w - \dot{q}_s
\] (E.22)

which after utilizing that \(\rho \cdot \nu = 1\) and \(V_b + V_s = V_t - V_w\) can be written as:

\[
\frac{d}{dt} \left( h_w \rho_w V_w + h_s \rho_s (V_t - V_w) - p \cdot V_t + \rho_p V_p c_p T_s \right) = \dot{q}_{fg\rightarrow w/s} + \dot{q}_w - \dot{q}_s
\] (E.23)

which can be written as:

\[
\frac{dh_w}{dt} \rho_w V_w + \frac{d\rho_w}{dt} h_w V_w + \frac{dV_w}{dt} \rho_w h_w + \frac{dh_s}{dt} \rho_s V_t + \frac{d\rho_s}{dt} h_s V_t - \frac{dh_s}{dt} \rho_s V_w -
\]

\[
\frac{d\rho_s}{dt} h_s V_w - \frac{dV_w}{dt} \rho_s h_s - \frac{dp}{dt} V_t + \frac{dT_s}{dt} \rho_p V_p c_p T_s
\] = \dot{q}_{fg\rightarrow w/s} + \dot{q}_w - \dot{q}_s
\] (E.24)

i.e.

\[
\frac{dh_w}{dt} \rho_w V_w + \frac{d\rho_w}{dt} h_w V_w + \frac{dV_w}{dt} \rho_w h_w + \frac{dh_s}{dt} \rho_s V_t - \frac{dp}{dt} V_t + \frac{dT_s}{dt} \rho_p V_p c_p T_s = \dot{q}_{fg\rightarrow w/s} + \dot{q}_w - \dot{q}_s
\] (E.25)

\(^4\) The chain rule for differentiation is applied to split the differentials in (i) differentials of water/steam properties and (ii) the pressure gradient with respect to time, i.e. \(dp/dt\).

\(^5\) Formulating the energy balance it has been presumed that: \(\dot{q}_{fg\rightarrow w/s} = \dot{q}_{\text{rad}} + \dot{q}_{\text{conv}} + \dot{q}_{fg\rightarrow p,cz}\) - see Figure E.2. Furthermore, it has been presumed that all energy transfer from the flue gas side to the water/steam side takes place below the water level.
applying the chain rule yields:

\[
\begin{align*}
\frac{dh_w}{dp} \frac{dp}{dt} \rho_w V_w + \frac{d\rho_w}{dp} \frac{dp}{dt} h_w V_w + \frac{dV_w}{dt} h_w \rho_w + \frac{dh_s}{dp} \frac{dp}{dt} \rho_s (V_t - V_w) + \\
\frac{d\rho_s}{dp} \frac{dp}{dt} h_s (V_t - V_w) - \frac{dV_w}{dt} \rho_s h_s - \frac{dp}{dt} V_t + \frac{dT_s}{dp} \frac{dp}{dt} \rho_c z c_{p,c} = \dot{q}_{fg \rightarrow w/s} + \dot{q}_{fw} - \dot{q}_s
\end{align*}
\] (E.26)

which after differentiation and rewriting can be reduced to:

\[
\begin{align*}
\frac{dp}{dt} \left( \frac{dh_w}{dp} \rho_w V_w + \frac{d\rho_w}{dp} h_w V_w + \frac{dh_s}{dp} \rho_s (V_t - V_w) + \frac{d\rho_s}{dp} h_s (V_t - V_w) - V_t + \frac{dT_s}{dp} \rho_P c_{p,P} \right) + \\
\frac{dV_w}{dt} (h_w \rho_w - h_s \rho_s) = \dot{q}_{fg \rightarrow w/s} + \dot{q}_{fw} - \dot{q}_s
\end{align*}
\] (E.27)

**Below Water Level**

For modelling the system below the water level (see Figure E.3) the following equation system has been developed:

**Mass balance**

\[
\frac{d}{dt} (\rho_w V_w + \rho_s V_b) = \dot{m}_{fw} - \dot{m}_{b \rightarrow s}
\] (E.28)

**Energy balance**

\[
\frac{d}{dt} (U_w + U_b + U_p) = \dot{q}_{fg \rightarrow w/s} + \dot{q}_{fw} - \dot{q}_{b \rightarrow s}
\] (E.29)

which after inserting the internal energy and rewriting yields:

\[
\frac{d}{dt} (\rho_w V_w (h_w - \nu_w p) + \rho_s V_b (h_s - \nu_s p) + \rho_P V_P c_{p,P} \cdot T_s) = \dot{q}_{fg \rightarrow w/s} + \dot{q}_{fw} - \dot{q}_{b \rightarrow s}
\] (E.30)

i.e.

\[
\frac{d}{dt} (h_w \rho_w V_w + h_s \rho_s V_b - p(V_w + V_b) + \rho_P V_P c_{p,P} \cdot T_s) = \\
\dot{q}_{fg \rightarrow w/s} + \dot{m}_{fw} h_{fw} - \dot{m}_{b \rightarrow s} h_s
\] (E.31)
and for the steam underneath the water level, the mass balance can be written as:

\[
\frac{d(\rho_s V_b)}{dt} = \dot{m}_{l-b} - \dot{m}_{b-s} \tag{E.32}
\]

and as a relation for the mass flow from the water to the bubbles and the bubbles to the steam space the following empirical equation [98] is applied:

\[
\dot{m}_{b-s} = \beta \frac{V_b}{V_w} + \gamma \cdot \dot{m}_{l-b} \tag{E.33}
\]

inserting the mass balance for the system underneath the water level (Equation E.28) in Equation E.33 yields:

\[
\dot{m}_{l-b} = \frac{1}{\gamma} \left( \dot{m}_{f_w} - \frac{d(\rho_w V_w + \rho_s V_s)}{dt} - \beta \frac{V_b}{V_w} \right) \tag{E.34}
\]

inserting Equation E.31 in Equation E.32 (eliminating \( \dot{m}_{b-s} \)) and rearranging yields:

\[
\dot{m}_{l-b} = \frac{1}{h_s} (\dot{q}_{fg-w/s} + h_{f_w} \dot{m}_{f_w} - h_w \frac{d(\rho_w V_w)}{dt} - \rho_w V_w \frac{d\rho_w}{dp} - \rho_s V_b \frac{dh_s}{dt} + V_w \frac{dV_w}{dp} + V_b \frac{dV_b}{dp} + p \frac{dV_b}{dp} - \rho_p \frac{dp}{dp} \frac{dT_s}{dt} ) \tag{E.35}
\]

combining Equation E.34 and Equation E.35 and rewriting yields:

\[
\frac{d\rho_s}{dp} \left( -h_w \frac{V_w d\rho_w}{dp} - \rho_w V_w \frac{d\rho_w}{dp} - \rho_s V_b \frac{dh_s}{dp} + V_w + V_b - \rho_p \frac{dp}{dp} \frac{dT_s}{dp} \right) \gamma + \left( V_w \frac{d\rho_w}{dp} + V_b \frac{d\rho_s}{dp} \right) h_s + \frac{dV_w}{dt} \left[ (-h_w \rho_u + p) \gamma + \rho_u h_s \right] + \frac{dV_b}{dt} \left[ \rho_s h_s + p \gamma \right] = - (\dot{q}_{fg-w/s} - h_{f_w} \dot{m}_{f_w}) \gamma + (\dot{m}_{f_w} - \beta \frac{V_b}{V_w}) h_s \tag{E.36}
\]

Combining Equation E.21, E.27 and E.36 yields:

\[
A \cdot y' = B, \quad i.e. \quad \left[ \begin{array}{ccc} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{array} \right] \cdot \left[ \begin{array}{c} \frac{dp}{dt} \\ \frac{dV_w}{dt} \\ \frac{dV_b}{dt} \end{array} \right] = \left[ \begin{array}{c} B_1 \\ B_2 \\ B_3 \end{array} \right]
\]

Which is an ordinary set of differential equations (ODE).

A more detailed description and derivation of the developed model can be found in [127].

\footnote{The philosophy behind E.33 is that the flow from the bubbles to the steam is: (i) proportional to the \textit{volume of the bubbles} and inverse proportional to the \textit{volume of the water} and (ii) proportional to the flow from the liquid phase to vapor phase (i.e. evaporation).}
Modelling, Simulation and Optimization of Boilers - State of the Art

Over the years boilers have been the main subject for several modelling and simulation projects. In general the development of digital computers has been a necessary pre-condition for boiler modelling and simulation. But even before the computers were developed to an acceptable level, dynamic models for boiler performance were formulated, and different simplification were made to be able to solve the equation systems.

Several authors have prepared summaries of the development within boiler modelling and simulation. Amongst the more comprehensive are [86] and [49]. [144] is a new reference summarizing the modelling and simulation of boilers. The main references highlighted in [144] are the works carried out by Prof. R. Doležal, IVD and Prof. W. Linzer and Prof. K. Ponweiser, TU-Wien. In the summaries results of the work carried out and future challenges are summarized.

One of the classical references within dynamic boiler modelling is [106], where the governing equations are formulated and special attention is paid to the control of boilers. [75] is another classical reference, which also focusses on control of boilers. A relatively new very interesting reference with boiler and energy utilization is [117], which also has a detailed chapter about dynamic modelling of boilers as an integrated part of an energy utilization plant.

An important era within boiler modelling and simulation started with Prof. Dr.-Ing. R. Doležal, who was the leader of IVD [173] until 1992.

By means of the de-coupled regenerative model (see: [43] and [109]), Prof. Doležal formulated a model, that still forms the basis of the modelling activities being carried out at the IVD. The work of Prof. Doležal has been applied as a basis of several Ph.D. studies. In the seventies and eighties, the work focussed on preparing and extending the models and on modifying the regenerative model

1Institute of Process Engineering and Power Plant Technology. In German: Institut für Verfahrenstechnik und Dampfkesselwesen - University of Stuttgart.
to simulate the true recuperative heat exchangers in the boilers. The activities within boiler modelling and simulation initiated by Prof. Dr.-Ing. R. Doležal now forms the basis of research activities within coupling of the flue gas and water/steam side in boiler modelling. The modelling and simulation of the flue gas side is carried out by means of CFD, where especially activities within furnace incl. burner modelling are carried out. The comprehensive activities at IVD has always been supplemented by experimental verification of the simulation results. Prof. Doležal’s most impressive list of publications includes items as:

- control of boilers [33]
- planning of boiler plants.
- time constants for once-through and natural circulation boilers [33] and [39].
- flow phenomenons for boilers (especially two-phase flow) [35] and [37].
- modelling of natural circulating boilers [33], [34] and [42].

Figure F.1: The *de-coupled regenerative model* developed by Prof. Dr.-Ing. R. Doležal at IVD [109].
Dynamic Boiler Performance

Figure F.2: The DNA model and examples of results from simulations developed by Ph.D. B. Elmegaard at Technical University of Denmark [49].

- start-up of boilers [29], [36], [37], [38] and [91].
- modelling of boilers by mean of the de-coupled regenerative model [31], [32], [41] and [43].

The work initiated by Prof. Doležal has been continued by Prof. Dr.-Ing. Klaus R. G. Hein and today several Ph.D. students are carrying out research within this and related areas at IVD.

One of the latest works from IVD is [146], where focus is on modelling and experimental work as a basis of economical optimization of the operation of boilers.

At the Department of Mechanical Engineering, Energy Engineering, DTU\(^2\) intensive research within boiler modelling and simulation has been carried out for many years. As an integrated part of this work special attention has been paid to development of numerical methods for solving the equation system. Program codes for solving the DAE systems, which typically is the result of dynamic boiler modelling has been developed [49] and [89], and several projects verifying the validity of the codes have been carried out [49].

The activities at DTU have had a broader view than the activities at IVD. Together with the activities within boiler technology, activities have been in, for example, refrigeration technology, engine technology and solar collectors. Furthermore, studies within the modelling process have been carried out at the Institute [51] and [52].

Technische Universität Braunschweig [163], leaded by Prof. Dr. techn. Reinhard Leithner, has also been very active within boiler modelling and simulation. Apart from the modelling and simulation

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\(^2\)The Technical University of Denmark (in Danish: Danmarks tekniske Universitet), see [180]
activities attention has been paid to optimization of the operation of boilers, a special technique: *evolutionary algorithm* has been applied, see [85], [28], [86] and [88].

The research at Technische Universität Braunschweig is based on the optimization-algorithm EVOBOX and the simulation-algorithm ENBIPRO - see [4]. The object function to be optimized is defined as a sum of all the energy flows, i.e. energy to be utilized and energy losses. Results from a power plant optimization carried out with the *evolutionary algorithm* can be seen in Figure F.4.

At the Technical University of Vienna [187] research work has been carried out within the area of modelling of natural circulating boilers, and the SIMPLER algorithm, which is a general programme for modelling heat affected pipe systems, e.g. boilers, has been developed [143] and [145].

A more practical approach to control of boilers is given in [47]. This reference does not give a very detailed theoretical background, but is very useful for the practical oriented engineer.

A popular topic for optimization is the optimization of operation of plants with complex configurations e.g. plants with more boilers, steam turbine, gas turbines etc. As the different elements of the total plant have different investment and operation costs, the optimization of the operation of the plants to fulfil the externally given requirements (*constraints*) with respect to heat and electricity is extremely important. During the years more and more complex constraints (i.e. external conditions) have been, and still are being implemented in the models [59], [78] and [119].

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3 *Evolutionary algorithm* has its origin within the biological science.
**Figure** F.4: Optimization of a power plant by means of the *evolutionary algorithm* and improvements in power plant efficiency by Prof. Leithner [4].

**Figure** F.5: The SIMPLER model developed by Dipl.-Ing. Dr. Heimo Walter and Prof. Dr. W. Linzer at the Technical University of Vienna [144].

Apart from the above mentioned research activities within boiler modelling and simulation the following interesting references\(^4\) have been found:

- [5] is an interesting model for simulation of evaporator circuits, the model is based upon the work of Tyssöe (1981).
- [48] and [90] describes the modelling and simulation of a complete power plant boiler. [90] uses MATLAB for the simulations.

\(^4\)It has not been possible to assess to what extent these references are the result of *stand-alone* activities or a part of a research project.
A new and very interesting reference is [80] and [107] describing a work carried out by ABB [160]. In this work the control and regulation of boiler has been optimized on the basis of the following objective function:

$$J = \int_{t=0}^{T_f} \left[ \frac{T_{LS}(t) - T_{set}}{\omega_T^2} + \frac{p_{LS}(t) - p_{set}}{\omega_p^2} + \frac{q_{mLS}(t) - q_{m, set}}{\omega_{q_m}^2} \right] dt \rightarrow \min_{q_{m,F}(t), Y_{HFB}(t)}$$  \hspace{1cm} (F.1)

The idea is on the basis of an existing boiler (i.e. a given geometry) to optimize the control algorithms to achieve the quickest start-up of the boiler plant. The optimum solution is the control algorithm maximizing F.1 where the deviation from the set point are weighted and summed - here live steam temperature, pressure and flow. The weighting parameters $\omega_T, \omega_p, \omega_{q_m}$ are used to rank the operation parameters according to the operational requirements. Furthermore a number of constraints related to thermal stresses in certain plant components (e.g. superheater headers), min/max flow etc. are defined.

Another of the classical references with in boiler dynamics is the work carried out by Prof. Karl J. Åström from the Department of Control Engineering at Lund University, Sweden. The work which has been going on since the early 70’ties is mainly focussed on the control of drum-boilers, and Åström’s dynamic boiler model has been developed (see Figure F.6)- this model is often referred to within control engineering. A detailed description of the model that over the years has evolved from a low order model to a fifth order model can be found in [48], [151] and [152] where references to Åström’s earlier works also can be found.

From the references found by the author, there seems to be the following main areas:

- activities within the modelling area - [99], [100], [148], [93], [149], [18] and [58].
• activities with focus on control of boilers - [94] and [96]

• activities with focus on procedures and algorithms for solving the equation systems.

• activities with focus on optimizing operation of plants with complex supply/demand configuration.

No references have been found within optimization of boiler design with respect to dynamic performance, where the optimization mainly is related to optimization of boiler design with respect to dynamic performance.