Analysis, Design and Implementation of a Quasi-Proportional-Resonant Controller for Multifunctional Capacitive-Coupling Grid-Connected Inverter

Tao Ye¹, Ning Yi Dai¹, Chi-Seng Lam¹,², Josep M. Guerrero³

1. Electrical and Computer Engineering Department, University of Macau, Macao, P.R. China
2. State Key Laboratory of Analog and Mixed-Signal VLSI, University of Macau, Macao, China
3. Department of Energy Technology, Aalborg University, Denmark

Abstract—The capacitive-coupling grid-connected inverter (CGCI) is able to achieve reactive power compensation and active power transfer simultaneously with a low operational voltage. The CGCI is coupled to the point of common coupling (PCC) via a second-order LC circuit, which makes its modeling and current control characteristics differ from the conventional inductive-coupling grid-connected inverter. The direct current tracking with hysteresis pulse width modulation (PWM) was used in previous studies. However, this method suffers from widely varying switching frequency and large current ripples. A Quasi-proportional-resonant (Quasi-PR) current controller is designed for the CGCI in this paper. Its modeling and parameter selection are studied in detail. In contrast with proportional-integration (PI) current controller, the Quasi-PR controller reduces steady-state error. It also generates a voltage reference for applying the carrier-based PWM to improve output waveform quality.

Simulation results are provided to verify the Quasi-PR controller and comparison with the PI controller is also done. A lab-scale prototype is built. Experimental results are given to show the validity of the proposed control method and its design.

Keywords—Capacitive-coupling grid connected inverter; Quasi-PR controller; Proportional-integration controller; parameters design

I. INTRODUCTION

The increasing need for more effective and environmental friendly power electrical system plays an active role in the development of smart grid [1-4]. Grid-connected inverter is the key for efficient use of distributed energy resources. Recently, more and more attention has been paid to multifunctional grid-connected inverters, which provide auxiliary services on power quality enhancement [5-6]. Most of the inverter is coupled to the grid via L type, LC type or LCL filter. They are named as inductive-coupling grid-connected inverters (IGCI) in this paper [7-9].

The capacitive-coupling grid-connected inverter (CGCI) was proposed and used to achieve the same function [10-11]. The configuration of a single-phase CGCI is shown in Fig. 1. The CGCI is integrated to the grid via an inductor in series with a capacitor. This topology is first proposed with the name of hybrid filter [12-13]. It is named capacitive since the fundamental frequency impedance of its coupling branch is capacitive [14]. The capacitive coupling branch reduces the operational voltage of the CGCI when leading reactive power is injected to the grid [15]. By adding active power transfer capability to the CGCI, the previous research work shows that it is a promising low-cost alternative to existing IGCI.

The direct current tracking with hysteresis PWM was used to control CGCI in previous work since it is simple and easy to implement [10,11]. The hysteresis PWM method has the drawbacks of widely varying switching frequency and large current ripples. The carrier-based PWM is able to fix switching frequency and reduce output current distortion. A current controller is used to generate voltage reference for applying carrier-based PWM in conventional IGCI. Proportional-integrator (PI) and proportional-resonant (PR) are the two most widely used current controller [16-18]. However, conventional PI controller is not able to eliminate steady-state errors in current tracking [19]. Synchronous PI controller was proposed for three-phase IGCI, in which stationary-frame ac quantities are transformed to dc quantities [20]. In this way, to achieve theoretical zero steady-state errors is then possible. However, additional computations are required for coordinate transformation when this method is applied to single-phase IGCI.

Fig. 1 System configuration of a single-phase CGCI
A stationary-frame PR controller has the same operational principal as a synchronous-frame PI controller when it is applied to a single-phase IGCI [21,22]. Compared to a stationary-frame PR controller, Quasi-PR controller avoids the stability problems associated with an infinite gain and can reduce sensitivity towards slight frequency variation in a typical utility grid [23,24]. Through comparing pros and cons of different current controllers for IGCI, a Quasi-PR current controller is selected and will be applied to control CGCI in this paper. The frequency response of the grid-connected inverter is studied and parameters design of the Quasi-PR current controller for the CGCI will be proposed in this paper. In Section II, the operational principal of the CGCI is introduced and its mathematical model is built and analyzed. The parameter design of the Quasi-PR controller is presented in Section III. Comparison with the PI current controller is also given in this Section. Simulation results are given in Section IV. Experimental results are provided in Section V.

II. MODELING OF THE CGCI

A. Operational principal of the CGCI

The system configuration of CGCI is shown in Fig. 1. It is integrated to the grid via a inductor in series with a capacitor. The power flow between the inverter and the grid can be integrated to the grid via an inductor in series with a capacitor. The operational principal of the CGCI is expressed as follows.

\[ V_s = \frac{V \cos \theta - V_i}{Z} \cos \theta + \frac{V \sin \theta \sin \theta}{Z} \]  
\[ Q_s = \frac{V \cos \theta - V_i}{Z} \sin \theta - \frac{V \sin \theta \sin \theta}{Z} \sin \theta \cos \theta \]

In (1) and (2), \( V_{\text{avg}} \) is the output voltage of the inverter; and \( \theta \) represents the phase angle between \( V_s \) and \( V_{\text{avg}} \). The value of \( Z \) and \( \theta \) is determined by the coupling impedance of the grid-connected inverter. The impedance of coupling branch in CGCI is expressed as follows.

\[ X_c = Z \cos \theta = -j \frac{1}{X_c} \]

The power base is introduced as follows.

\[ S_{\text{base}} = V_s \cdot X_c \]

By combining (1) to (4), the normalized output voltage of the CGCI is calculated as shown in (5), and its variation in power flow is depicted in three dimensions (3D) in Fig. 2.

\[ V_s = \sqrt{\left( \frac{P_{\text{source}} - P_{\text{source}} \cdot \cos \theta} {S_{\text{base}}} \right)^2 + \left( \frac{Q_{\text{source}} - Q_{\text{source}} \cdot \sin \theta} {S_{\text{base}}} - 1 \right)^2} \]

It can be concluded from Fig. 2 that the operational voltage of the inverter is lower than the grid voltage when the reactive power is in the vicinity of \( S_{\text{base}} \). The CGCI is better to be connected to a PCC, where continuous reactive power compensation is required for inductive loadings, for example, pumps or air-conditioners installed on the roof top.
As mentioned in Section I, Quasi-PR controller is selected in this paper. Its transfer function, \( G_{\text{Quasi-PR}}(s) \) is as follows.

\[
G_{\text{Quasi-PR}}(s) = \frac{2K_p \omega_s}{s^2 + 2\omega_s s + \omega_s^2}
\]

(7)

- **PWM Unit**

In an average s-domain model, the PWM converter may be simplified to a unity gain. However, the computation time of the digital controller is not negligible [26-29]. In order to accurately describe the real effects of time delay, the sampler in this paper. Its transfer function, \( G_{\text{s}}(s) \) is chosen so that phase and magnitude steady state errors are eliminated.

On the basis of the power quality standard of Macau and Hong Kong (HKE and CLP supply rules of Hong Kong, CEM supply rules), the standard limit of frequency variation is ±2%. Assuming that the frequency variation margin is ±2%, then \( \omega_c = 2\pi \times 50 \times 2\% = 6.28 \).

\( K_p \) should be large enough to obtain high gain at the fundamental frequency and low-order harmonic frequency. However, larger \( K_p \) makes the system less stable. Using Quasi-PR controller can overcome this problem. It provides high value gain at the fundamental frequency and low-order harmonic frequency with relatively small \( K_p \) value.

The boundary of the \( K_p \) value is determined by using Routh’s stability criterion. The open loop transfer function of related closed-loop transfer function \( G_{\text{Quasi-PR}}(s)G_{\text{PWM}}(s)G_{\text{Imp}}(s) = \frac{N(s)}{D(s)} \). Then the characteristic equation can be obtained from:

\[
D(s) + KN(s) = 0
\]

(13)

It is assumed the delay time of the PWM unit is half a sampling period (0.5Ts), the corresponding boundary of \( K_p \) are deduced as follows:

\[
K_p \leq \left( \frac{8 \cdot L}{f_s \cdot (3 \cdot T_s)} \right)
\]

(14)

It can be concluded from (12) that the magnitude gain of \( G_{\text{Quasi-PR}}(s)G_{\text{PWM}}(s)G_{\text{Imp}}(s) \) needs to be higher than 100 in order to decrease the current tracking error to below 1%. The value of \( K_c \) and \( K_p \) are selected to satisfy this requirement and to guarantee stable operation and acceptable transient response at the same time.

The design procedure of the Quasi-PR controller for a CGCI is as following:

1) According to the power quality standard to select the value of \( \omega_c \).

\[
\omega_c = 2\pi \cdot f_s \cdot \Delta f
\]

(15)

Where \( f_s \) is the fundamental frequency and \( \Delta f \) is the standard limit of the frequency variation.

2) Calculate the upper boundary (106.7) of the controller’s proportional gain \( K_p \) according to (14). Select a value of \( K_p \) within this boundary.

3) Set a small value for \( K_c \) which can guarantee the magnitude response of the open-loop transfer function at the designed resonant frequency (50Hz) is above 40 dB.

4) Adjust \( K_p \) value within its boundary so that The magnitude response of closed-loop transfer function \( G_{\text{ref-c}}(s) \) would be.

III. PARAMETER DESIGN OF THE QUASI-PR CONTROLLER

A. Parameters design of Quasi-PR Controller in CGCI
approaches to 0 dB and its phase response curve approaches to 0 degree at fundamental frequency.

5) The value of $K_r$ and $\omega_c$ may need minor adjustments to fulfill the following requirements:
   - The magnitude response of the closed-loop transfer function $G_{ref,c}(s)$ should be close to the 0 dB for low-order harmonic frequencies.
   - $K_r$ is adjusted to make sure that the magnitude response at high frequency especially around 10 KHz are well suppressed.

B. Parameters design Verifications

Current controller model is analyzed by using Matlab. The system settings are given in Table I. The bode diagram of open-loop current controller and closed-loop current control system is shown in Figs. 5 and 6, respectively. Results indicate that $G_{ref,c}(s)$ has a unity gain with zero phase shifting at fundamental frequency with the selected parameters.

C. Comparison of PI controller controlled CGCI and Quasi-PR controller controlled CGCI system

Comparison with PI controller is done in this section. PI controller is one of the most widely use current tracking controller for grid-connected inverter. The PI controller is expressed as follows:

$$G_{pi}(s) = K_p + \frac{K_i}{s} \quad (16)$$

By using (16) to replace $G_{quasi-pr}(s)$ in (12), the S-domain transfer function with PI controller is obtained.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching frequency $f_i$</td>
<td>10kHz</td>
</tr>
<tr>
<td>Fundamental frequency</td>
<td>50Hz</td>
</tr>
<tr>
<td>Filter inductor $L_C$</td>
<td>4mH</td>
</tr>
<tr>
<td>Filter capacitor $C_C$</td>
<td>125uF</td>
</tr>
</tbody>
</table>

With the parameters in Table I, the Bode diagram of the closed-loop transfer functions $G_{ref,c}(s)$ are shown in Fig. 7. It can be concluded from Fig.7 that the magnitude response varies in the vicinity of zero at low frequency range when PI controller is used. Increasing the proportional gain of the PI controller may force the magnitude response approaching zero. However, large gain could cause the control system unstable. The current tracking capability is improved when the Quasi-PR controller is used. The steady state current tracking error is reduced by Quasi-PR controller according to the magnitude and phase response in Fig. 7. Thus, the Quasi-PR controller is a more suitable candidate than PI controller for the precisely controlled CGCI.

The Bode diagram of closed-loop transfer functions $G_{vs_c}(s)$ are shown in Fig. 8. Results indicate that both two control methods provide enough attenuation to the disturbance from the grid-side voltage. That is to say, the distortion component in the grid-side voltage will not be amplified by the CGCI. Even its coupling circuit is a second LC branch. While for the current control system using PI controller it cannot offer zero dB magnitude gain and zero degree phase shift.
IV. SIMULATION

A. Simulation setting

In order to verify the effectiveness of the Quasi-PR controller, a set of tests are carried out by PSCAD/EMTDC. For comparison, PI controller with carrier-based PWM is studied in the simulations. Table. II lists the simulated system parameters, and the DC-link voltage of the inverter is supplied by an ideal DC voltage source.

The comparison is mainly focused on the steady state performance. Thus the performances are conducted with respect to the following performance parameters:

- Source current THD at steady state situation.
- Active power error between the injecting active power and the reference active power.
- Reactive power error between the injecting reactive power and the reference reactive power based on the load reactive power.

According to Part II, a set of parameters of the Quasi-PR controller can be gotten: $K_p=50$, $\omega_c=5$, $K_r=5800$.

B. PSCAD Simulation results

Simulation results using PI controller with carrier-based PWM and using Quasi-PR controller with carrier-based PWM respectively are shown in Fig. 9 to Fig. 11, respectively. The system performance indexes are summarized in Table. III to Table. IV respectively. It can be concluded that PI controller with carrier-based PWM cannot eliminate the steady state current tracking errors. As a result, both active and reactive power output of the CGCI are not able to tracking the reference with high accuracy. In general, the parameters of the PI controller can be tuned to improve the performance. As shown in Fig. 7, tuning the parameters cannot reduce the source current THD and the power error at the same time.

When Quasi-PR controller with Carrier-based PWM is used, the active power from the external source can be injected to the grid and the load side reactive power is compensated. Both errors are very small. What’s more, the low source current THD (<2%) indicates that the Quasi-PR controller is more fit for applying to CGCI. It can be concluded that Quasi-PR controller with Carrier-based PWM is a better choice for the CGCI structure to achieve active power and reactive power injecting as well as harmonic compensation.

### Table III Steady State Performances Under Different Loads Situation Using PI Controller

<table>
<thead>
<tr>
<th>Time</th>
<th>THD_inj (%)</th>
<th>P_inj (W)</th>
<th>P (W)</th>
<th>Q_inj (VAR)</th>
<th>Q_load (VAR)</th>
<th>P_error (%)</th>
<th>Q_error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29s</td>
<td>0.95</td>
<td>439.3</td>
<td>9</td>
<td>500</td>
<td>1855.8</td>
<td>9</td>
<td>12.12</td>
</tr>
<tr>
<td>0.49s</td>
<td>1.09</td>
<td>659.0</td>
<td>7</td>
<td>500</td>
<td>2510.4</td>
<td>7</td>
<td>31.81</td>
</tr>
<tr>
<td>0.69s</td>
<td>1.10</td>
<td>214.1</td>
<td>7</td>
<td>500</td>
<td>1178.0</td>
<td>4</td>
<td>57.17</td>
</tr>
</tbody>
</table>

### Table IV Steady State Performances Under Different Loads Situation Using Quasi-PR Controller

<table>
<thead>
<tr>
<th>Time</th>
<th>THD_inj (%)</th>
<th>P_inj (W)</th>
<th>P (W)</th>
<th>Q_inj (VAR)</th>
<th>Q_load (VAR)</th>
<th>P_error (%)</th>
<th>Q_error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29s</td>
<td>0.84</td>
<td>500.12</td>
<td>500</td>
<td>2021.64</td>
<td>1202.3</td>
<td>0.02</td>
<td>0.97</td>
</tr>
<tr>
<td>0.49s</td>
<td>0.99</td>
<td>499.94</td>
<td>500</td>
<td>2761.53</td>
<td>2738.78</td>
<td>0.01</td>
<td>0.83</td>
</tr>
<tr>
<td>0.69s</td>
<td>1.02</td>
<td>482.22</td>
<td>500</td>
<td>1257.8</td>
<td>1227.64</td>
<td>3.56</td>
<td>2.42</td>
</tr>
</tbody>
</table>

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**Fig. 8 Bode diagram of closed-loop transfer functions $G_{t_c}(s)$ (solid line: use PI controller; dashed line: use Quasi-PR controller)**

**Fig. 9 to Fig. 11**

(a). PI controller

**Table. II System Setting in the Simulation**

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grid Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Grid Voltage $V_s$</td>
<td>220V</td>
</tr>
<tr>
<td>Fundamental frequency $f_0$</td>
<td>50Hz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>20 KHz</td>
</tr>
<tr>
<td>Source Inductor $L_s$</td>
<td>0.001mH</td>
</tr>
<tr>
<td><strong>Inverter parameters</strong></td>
<td></td>
</tr>
<tr>
<td>DC link capacitor $C_{DC}$</td>
<td>1mF</td>
</tr>
<tr>
<td>Filter inductor $L_C$</td>
<td>4mH</td>
</tr>
<tr>
<td>Filter capacitor $C_C$</td>
<td>125uF</td>
</tr>
<tr>
<td><strong>Linear load</strong></td>
<td></td>
</tr>
<tr>
<td>DC link voltage $V_{DC}$</td>
<td>170V</td>
</tr>
<tr>
<td>Linear Load 1 ($0.5s$-$0.7s$)</td>
<td>15 ohm; 0.12 H, 8 ohm</td>
</tr>
<tr>
<td>Linear Load 2 ($0.1s$-$0.3s$)</td>
<td>20 ohm; 0.06 H, 10 ohm</td>
</tr>
<tr>
<td>Linear Load 3 ($0.3s$-$0.5s$)</td>
<td>28 ohm; 0.04 H, 8 ohm</td>
</tr>
</tbody>
</table>
V. EXPERIMENT RESULTS

A lab-scale prototype, with parameters listed in Table. V, was constructed. The control algorithm was implemented in a DSP-TMS320F28335. The photo of the prototype is shown in Fig. 12. The grid-side voltage is drop to 110V due to the laboratory safety reason and the fact that 110V grid voltage is also used by many countries such as America, Japan and Canada. The parameters ($K_p=50$, $\omega_c=5$, $K_r=5800$) designed by the proposed parameters design approach was used in the experiment.

As stated previously, the comparison are mainly focused on the steady state performance. Thus the performances are conducted with respect to the following performance parameters:

- Source current THD at steady state situation.
- Active power error between the injecting active power and the reference active power.
- Reactive power error between the injecting reactive power and the reference reactive power based on the load reactive power, in the experiment, it can be simply regarded as the source side reactive power.

<table>
<thead>
<tr>
<th>TABLE. V EXPERIMENTAL SYSTEM SETTINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Items</strong></td>
</tr>
<tr>
<td>Capacitor $C_c$</td>
</tr>
<tr>
<td>Inductor $L_c$</td>
</tr>
<tr>
<td>Grid Voltage $V_s$</td>
</tr>
<tr>
<td>DC-link Voltage</td>
</tr>
<tr>
<td>Active power transfer</td>
</tr>
<tr>
<td>Linear Load</td>
</tr>
</tbody>
</table>

Table. VI EXPERIMENTAL RESULTS

<table>
<thead>
<tr>
<th>Load Side</th>
<th>Source Side</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF</td>
<td>Q (Var)</td>
<td>THD (Load)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>THD</td>
</tr>
<tr>
<td>P ref (W)</td>
<td>P actual (W)</td>
<td>P Error (W)</td>
</tr>
<tr>
<td>Q source (Var)</td>
<td>Q source (Var)</td>
<td></td>
</tr>
</tbody>
</table>

Table. VI EXPERIMENTAL RESULTS
When the DC voltage is 85V, which is lower than the grid side voltage 110V, and the active power range is set as 90W, a linear inductive load is used to verify the current control performance of Quasi-PR controller with carrier-based PWM. Comparison experiments are conducted by using PI controller with carrier-based PWM. The results are given in Table. VI. Fig. 13 shows the experimental results (load side, injecting side and source side) when PI controller with carrier-based PWM was used and $P_{source}$ equals to 90W. Fig. 14 shows the experimental results when Quasi-PR controller was applied and $P_{source}$ equals to 90W. It indicates from those experimental results that compared to PI controller with carrier-based PWM, power error between reference value and actual value can be reduced and source current THD can be decreased by using Quasi-PR controller with carrier-based PWM, which shows the effectiveness of the proposed current control model and parameters design of Quasi-PR controller.

VI. CONCLUSION

The capacitive-coupling grid-connected inverter can decrease DC-link voltage and running losses. While due to its second-order transfer function of capacitive-coupling interfacing branch, the mathematical model and current controller for traditional IGCI cannot be directly applied to CGCI. This paper proposes a Quasi-PR controller with carrier-based PWM for CGCI and its related Quasi-PR controller parameters design method. Both simulation and experimental results are provided. Comparison with PI controller is also given to validate the effectiveness of proposed model and parameters design approach. Results shows that the Quasi-PR controller with carrier-based PWM is a better choice to fulfill the requirements of active power and reactive power injecting as well as harmonic compensation.

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References


