Wind Farm Active Power Dispatch for Output Power Maximizing Based on a Wind Turbine Control Strategy for Load Minimizing

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Abstract—Inclusion of the wake effect in the wind farm control design (WF) can increase the total captured power by wind turbines (WTs), which is usually implemented by derating upwind WTs. However, derating the WT without a proper control strategy will increase the structural loads, caused by operation in stall mode. Therefore, the WT control strategy for derating operation should be considered in the attempt at maximizing the total captured power while reducing structural loads. Moreover, electrical power loss on the transmission system inside a WF is also not negligible for maximizing the total output power of the WF. In this paper, an optimal active power dispatch strategy based on a WT derating strategy and considering the transmission loss is proposed for maximizing the total output power. The active power reference of each WT is chosen as the optimization variable. A partial swarm optimizing algorithm is used for solving the problem. Simulation results show the effectiveness of the proposed strategy.

Keywords—wind farm active power dispatch; power maximizing; wake effect; transmission loss; load minimizing

I. INTRODUCTION

Wakes in a wind farm (WF) may bring a significant loss of wind power due to the reduction of wind speed at downwind wind turbines (WTs). However, many research works have claimed that derating the power captured by the upwind WTs can reduce the wake effect and increase the total power captured in the WF [1-8].

These works maximize the total captured power of WFs by optimizing the control settings to each WT. In [1] and [2], a model free approach and a data-driven adaptive scheme are proposed to adjust the control settings of each WT. The control variables used in these works are axial induction factors. In [3], both the axial induction factor and the yaw offset angle are adopted as control variables and a steepest descent algorithm is applied to find the optimal combination. In [4], the control parameters are the thrust coefficient of individual turbines as a function of time, and a reeding-horizon predictive control setting was employed in solving the optimization problem. The above methods did not include the WT model into the optimization problem. In [5], a static WT model was adopted to represent the WT aerodynamics and the pitch angle was chosen as the optimization variable. In [6], the pitch angle of each WT is chosen as the control variable and blade element momentum theory and the eddy viscosity model are used to describe the WT aerodynamics and the wakes. The pitch angle and rotational speed of each WT are chosen as the optimization variables in [7], and Jensen model is used to present the wake effect. In [8], the pitch angle and tip speed ratio are used as the optimization variable. However, using the tip speed ratio has the same effect with using the rotational speed, because one can be easily transformed to the other if the wind speed is known.

In the above works, the optimizing variables are chosen as the axial induction factor, the yaw offset angle, the thrust coefficient, the pitch angle, the rotational speed (tip speed ratio), or the combination of two of them. However, they tried to find the maximum power by derating the upwind WTs, without giving much attention to the control strategy of each WT. Furthermore, derating the WT without a proper control strategy may cause more load on the mechanical structure and may risk the WT running into the stall region, where more turbulence will be caused and more loads will be produced on the downwind WTs. Therefore, the WT control strategy for derating operation should be considered in maximizing the total captured power in the WF. A Max-Ω control strategy for WT derating control is proposed in [9], which claimed that the strategy can reduce the load on the WT structure, decrease the risk of going to stall region and bring bigger inertia in the rotor for the WT to provide inertial response. Besides the WT derating control strategy, the power loss in the transmission system was also not considered in the above works.

In this paper, an optimal active power dispatch strategy for maximizing the total output power is proposed, which considers the WT derating control strategy and the electrical loss on the transmission cables. The active power reference of each WT is chosen as the optimization variable and the Max-Ω control strategy is used for WT derating control. The WT active power is modeled by static lookup tables and the wakes
are modelled by Jensen wake model. A full Newton–Raphson method is adopted to calculate the electrical loss on the transmission system. The whole optimization problem is solved by a partial swarm optimizing (PSO) algorithm.

This paper is organized as follows: Section II describes the WF model. Section III shows the formulation of the optimization problem and the method used to solve the problem. The effect of the new strategy is illustrated in Section IV, and finally conclusions are drawn in Section V.

II. WIND FARM MODEL

The wake model and the power loss model on transmission cables are specified in this section. The WT model using traditional control strategy with no derating is also illustrated and will be used as a baseline for comparison.

A. Wind Turbine Model

The WT model used in this paper is a static model, which is based on the look-up tables of the power coefficient $C_p$ and the thrust coefficient $C_t$. Then, the WT mechanical power $P_{mech}$ and the thrust force $F_t$ can be calculated using the following equations [10]:

$$P_{mech} = \frac{\pi}{2} \rho R^3 v^3 C_p(\beta, \lambda)$$  \hspace{1cm} (1)

$$F_t = \frac{\pi}{2} \rho R^3 v^2 C_t(\beta, \lambda)$$  \hspace{1cm} (2)

where $\rho$ is the air density, $R$ is the rotor radius, $v$ is the wind speed, $\beta$ is the blade pitch angle and $\lambda$ is the tip-speed ratio.

The power coefficient $C_p$ and the thrust coefficient $C_t$ vary with the pitch angle $\beta$ and tip-speed ratio $\lambda$. Their relations are provided by look-up tables, which are shown in Fig. 1 and Fig. 2. The WT operating point can be determined under a certain $\beta$ and $\lambda$. The control of a WT in steady state is actually the process of choosing $\beta$ and $\lambda$.

As the tip speed ratio can be expressed by

$$\lambda = \omega R/v,$$  \hspace{1cm} (3)

the choice of $\lambda$ is equivalent to the choice of rotational speed $\omega$ under a certain wind speed $v$.

The normal control of WT in the whole wind speed region can be divided into five regions, which are shown in Fig. 3. Region 1 is the region where wind speed is below the cut-in wind speed and no power is produced. Region 2 is the region for maximum power capturing by actively choosing the rotational speed to reach the maximum $C_p$, while keeping the blade pitch angle to zero. Region 3 is when the wind speed exceeds the rated wind speed and the blade pitch angle should be increased to keep the captured power at rated value. In transition regions 1½ and 2½, the rotational speed is kept constant at the lower limit and the higher limit respectively. However the tip speed ratio is not the optimal value for the maximum $C_p$, because the tip speed ratio is changing with wind speed, according to (3).

![Fig. 1. The power coefficient of the NREL 5 MW wind turbine [11]](image1)

![Fig. 2. The thrust coefficient of the NREL 5 MW wind turbine [11]](image2)

![Fig. 3. Wind turbine control regions [12]](image3)

However, the control strategies in these regions are just for normal operation. Under derating operation, the control strategies need to be modified. In Region 2, the control target is no longer maximizing the captured power, but maintaining the captured power at the reference value. Under a certain wind speed, the power coefficient can be determined by (1). Fig. 4...
shows the level curve when $C_p$ is 0.3. It can be observed there are many options for choosing $\beta$ and $\lambda$. In Region 2, the rotational speed $\omega$ can be changed between its lower and upper limits, which gives a wide range of $\lambda$.

The Max-$\Omega$ control strategy for derating operation proposed in [9] maximizes the rotational speed for each wind speed under each demanded power. Fig. 5 shows the operating points on the $C_p$ level curve for Max-$\Omega$ control strategy when the demanded power is 2.5MW in the whole wind region. The red circles are the operating points. It can be seen the tip speed ratio $\lambda$ is maximized rather than being kept at the optimal value 7.55 in normal Region 2 control strategy. The Max-$\Omega$ control strategy can reduce the load on the WT structure, decrease the risk of going to stall region and bring bigger inertia in the rotor for providing inertial response for the grid. Therefore, it is used in this paper for the WT derating control.

The power loss in cable $ij$ is the algebraic sum of the active and reactive parts of the complex powers $S_y$ from bus $i$ and $j$ to and from bus $j$ and $i$, $S_y^{\text{loss}} = S_y + S_y^* = V_i I_j + V_j I_i^*$. So, the active power loss in cable $ij$ is:

$$P_y^{\text{loss}} = \text{real} (S_y^{\text{loss}})$$

C. Power loss model on transmission cables

Consider the cable connecting the two buses $i$ and $j$ in Fig. 6, where $y$ and $I$ is the admittance and current of each cable, and $V$ is the voltage on each bus. The cable current, $I_y$, measured at bus $i$ and $j$ defined positive in the direction $i \rightarrow j$ is given by

$$I_y = I_i + I_{i0} = y_{ij} (V_j - V_i) + y_{ij} V_j.$$  

Similarly, the cable current $I_{ij}$ is given by

$$I_{ij} = -I_i + I_{i0} = y_{ij} (V_j - V_i) + y_{ij} V_j.$$  

$$I_y = \begin{bmatrix} V_i \\ I_i \\ y_{ij} \end{bmatrix}, \quad I_{ij} = \begin{bmatrix} V_j \\ I_j \\ y_{ij} \end{bmatrix}$$

The power loss in cable $ij$ is:

$$P_y^{\text{loss}} = \text{real} (S_y^{\text{loss}})$$

III. PROBLEM FORMULATION AND OPTIMIZATION

The optimization problem including the objective function and constraints are formulated in this section, and is solve by an improved PSO algorithm that is adopted as the optimization method.

A. Application of Max-$\Omega$ control strategy

As discussed in section II, the power extracted by each WT is based on the $C_p$ value and wind speed, while the wind speed at downwind WT's is determined by the $C_T$ value at upwind WTs. Both $C_p$ and $C_T$ are functions of $\beta$ and $\lambda$; so $\beta$ and $\lambda$ can be chosen as the optimization variables. However, in this paper, the reference power of each WT is chosen as the optimization variable and the Max-$\Omega$ control strategy is chosen as the WT control strategy. Therefore, $\beta$ and $\lambda$ are
determined by solving the following optimization problem for each wind speed in Region 2.

Maximize

\[ \omega(\lambda, \beta) \]  

Subject to

\[ \omega_{\text{min}} \leq \omega \leq \omega_{\text{rated}} \]  

\[ C_p(\lambda, \beta) = \frac{P_{\text{ref}}}{P_w} \]  

where \( \omega_{\text{min}} \) and \( \omega_{\text{rated}} \) are the minimum and rated rotational speed, \( P_{\text{ref}} \) and \( P_w \) are the reference power and the total available power on the rotor disc.

This optimization problem can be solved offline to generate look-up tables for \( \beta \) and \( \lambda \) with respect to wind speed and \( P_{\text{ref}} \). Therefore, when the wind speed and \( P_{\text{ref}} \) is determined, \( \beta \) and \( \lambda \) of upwind WTs by searching the \( \beta(v, P_{\text{ref}}) \) and the \( \lambda(v, P_{\text{ref}}) \) lookup tables. Then \( C_p \) can be determined by searching the \( C_p(\lambda, \beta) \) lookup table. Thus the wind speed at the downwind WTs can be calculated using the wake model (4)–(6).

B. Optimization problem formulation

The total output power of the WF can be calculated by:

\[ P_{\text{out}}^k = \sum_{i=1}^{N_w} P_{\text{out}}^i - \sum_{j=1}^{N_w} P_{\text{loss}}^j \]  

where \( P_{\text{out}}^k \) is the captured power from WT \( k \), \( N_w \) is the number of WTs, \( P_{\text{loss}}^j \) is the active power loss in cable \( ij \), \( N_b \) is the number of buses.

The objective function of the problem can be expressed as:

\[ \max_{P_{\text{out}}} \sum_{k=1}^{N_w} P_{\text{out}}^k - \sum_{j=1}^{N_w} P_{\text{loss}}^j \]  

Constraints:

\[ P_j = V_j \left( \sum_{i=1}^{N_b} |V_{ji}| \cos(\theta_j - \delta_i + \delta_j) \right) \]  

\[ Q_j = V_j \left( \sum_{i=1}^{N_b} |V_{ji}| \sin(\theta_j - \delta_i + \delta_j) \right) \]  

\[ V_{\text{min}} \leq V' \leq V_{\text{max}} \]  

\[ \delta_{\text{min}} \leq \delta' \leq \delta_{\text{max}} \]  

\[ P_{\text{min}} \leq P_{\text{ref}} \leq P_{\text{rated}} \]  

\[ \frac{\partial C_p(\beta, \lambda)}{\partial \lambda} \leq 0 \]  

\[ \frac{\partial C_p(\beta, \lambda)}{\partial \beta} \leq 0 \]  

where \( P_j \) and \( Q_j \) are the active power and reactive power injected at bus \( j \), \( V' \) and \( \delta' \) are the voltage and angle of each bus, \( Y_{ji} \) is the entry in the \( j \)th row \( i \)th column of the admittance matrix.

The reviewed papers take the axial induction factors, the yaw offset angle, the thrust coefficient, the pitch angle, the rotational speed (tip speed ratio), or the combination of two of them as the control variables. However for the optimal power flow problems in power system engineering, it is more usual to use the active power of each WT as the optimizing variable. Therefore, the active power reference of each WT is chosen as the control variable and the other variables as the dependent variables, such as the pitch angles, the rotational speed, the thrust coefficient, the bus voltage and bus angles. In the power flow problem, the point of common coupling is treated as slack bus and all the other buses are treated as PQ buses. A full Newton–Raphson method is used to solve the power flow equations. The power flow constraints include the power flow balance limits (16), (17), and the bus voltage and angle limits (18), (19). In this paper, the voltage range is assumed as [0.95; 1.05], and the angle is assumed in the range of [-\( \pi/2 \); \( \pi/2 \)]. The control variable should within zero and the rated power \( P_{\text{rated}} \) and the pitch angle should be within its minimum and maximum value, which are chosen as 0° and 10°. In order to ensure the WT not to fall into stall region, \( \lambda \) and \( \beta \) has to be limited to the right side of \( \lambda-C_p \) curve and \( \beta-C_p \) curve, respectively. The constraints are expressed as (22), (23).

C. Optimization method

Since the problem is nonlinear and non-convex. Heuristic algorithm should be a suitable choice to solve this problem. Based on the social behavior of fish schooling and bird flocking, Kennedy and Eberhart [17] proposed the PSO algorithm which has a good performance of solving non-linear optimization problem. In this paper, the PSO algorithm is adopted to solve the optimization problem.

IV. CASE STUDY

In this paper, a WF with 5 turbines in 5 rows is chosen to test the proposed strategy. The WF is in a rectangular pattern with 882 m (seven times the WT diameter) between the turbines. The layout of the WF is shown in Fig. 7.
The cables in the WF are 95, 150 or 240 mm² (chosen by load, corresponding to cables between row 1 and row 3, between row 3 and row 5 and between row 5 and the transformer, respectively) XLPE-Cu, operated at 34 kV nominal voltage [18]. The parameters of the cables are shown in Table I. The WT used here is the 5 MW NERL WT, whose parameters are listed in Table II.

### TABLE I. \[\text{PARAMETERS OF CABLES [18]}\]

<table>
<thead>
<tr>
<th>Cross section mm²</th>
<th>Resistance Ω/km</th>
<th>Capacitance μF/km</th>
<th>Inductance mH/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>0.1842</td>
<td>0.18</td>
<td>0.44</td>
</tr>
<tr>
<td>150</td>
<td>0.1167</td>
<td>0.21</td>
<td>0.41</td>
</tr>
<tr>
<td>240</td>
<td>0.0729</td>
<td>0.24</td>
<td>0.38</td>
</tr>
</tbody>
</table>

### TABLE II. \[\text{NERL 5MW WIND TURBINE SPECIFICATION [11]}\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>5 MW NERL Wind Turbine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut-in, Rated, Cut-out Wind Speed</td>
<td>3 m/s, 11.4 m/s, 25m/s</td>
</tr>
<tr>
<td>Rotor, Hub Diameter</td>
<td>120 m, 3m</td>
</tr>
<tr>
<td>Cut-In, Rated Rotor Speed</td>
<td>6.9 rpm, 12.1 rpm</td>
</tr>
<tr>
<td>Gearbox ratio</td>
<td>97:1</td>
</tr>
</tbody>
</table>

In normal operations, the WTs are controlled using the traditional five region control strategy, which is specified in section II. In this section, the effects of the proposed strategy and the traditional strategy are compared when wind speed is 10 m/s and wind direction is 0°, 90°, 180° and 270°. The total captured power by WTs, the transmission cable loss, and the total output power of the WF are listed in Table III. It can be seen, the proposed dispatch strategy can maximize the total output power at every working conditions. The transmission cable loss is decreased at 180° and 270°, whereas is increased at 0° and 90°. The reason is that when wind direction is 0° or 90°, the WTs with higher wind speed are nearer to the PCC, thus the active power circulation distance is almost the smallest, which means smaller loss on cables. Therefore, if upwind WTs are derailed to increase the total captured power, it will increase the circulation distance of active power, resulting in higher cable loss.

### TABLE III. \[\text{COMPARISON OF THE RESULTS BEFORE AND AFTER OPTIMIZATION}\]

<table>
<thead>
<tr>
<th>Wind direction</th>
<th>0°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total captured power (MW)</td>
<td>49.872</td>
<td>49.872</td>
<td>49.872</td>
<td>49.872</td>
</tr>
<tr>
<td>Cable loss (MW)</td>
<td>0.117</td>
<td>0.133</td>
<td>0.167</td>
<td>0.199</td>
</tr>
<tr>
<td>Total output power (MW)</td>
<td>49.755</td>
<td>49.739</td>
<td>49.705</td>
<td>49.673</td>
</tr>
<tr>
<td><strong>Proposed strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total captured power (MW)</td>
<td>52.135</td>
<td>52.431</td>
<td>52.887</td>
<td>52.852</td>
</tr>
<tr>
<td>Cable loss (MW)</td>
<td>0.136</td>
<td>0.147</td>
<td>0.166</td>
<td>0.188</td>
</tr>
<tr>
<td>Total output power (MW)</td>
<td>51.999</td>
<td>52.284</td>
<td>52.721</td>
<td>52.664</td>
</tr>
<tr>
<td>Increase percentage</td>
<td>4.51%</td>
<td>5.12%</td>
<td>6.07%</td>
<td>6.02%</td>
</tr>
</tbody>
</table>

The optimal active power dispatch strategy proposed in this paper shows the ability to improve the total output power of a WF. Comparing with previous work, the control variables in this paper are the active power references of each WT, which can be received as a command signal by commercial WTs. The proposed dispatch also considered the power loss on cables. The results show that the cable loss is reduced at some wind directions, but may be increased at other wind directions. However, the total output power is always increased. The WT derating control strategy used in this paper is the Max-Ω strategy, which can reduce the load on the WT structure, decrease the risk of going to stall region and bring bigger inertia in the rotor for providing inertial response for the grid. The optimal active power dispatch strategy proposed in this paper can be used to generate look-up tables for the active power dispatch in a WF. The look-up table can be

![Fig. 8. Active power set points of each WT in the wind farm before and after optimization when wind direction is 270°](image)

![Fig. 9. Tip-speed ratios of each WT in the wind farm before and after optimization when wind direction is 270°](image)

### V. CONCLUSION

The active power set points and the tip-speed ratio of each WT when wind direction is 270° are shown in Fig. 8 and Fig. 9 respectively. The red points give the values after optimization. It can be seen from Fig. 8 that the upwind WTs are derated, especially the first column, whereas the power of downwind WTs is increased. From Fig. 9 we can see that the tip-speed ratios of each WT are increased by using the Max-Ω control strategy, which is in accordance with the theory.
implemented in WF energy management systems or wind power dispatch centers for real-time operation.

REFERENCES


