CHAPTER 3

A STRATEGY OF OPTIMIZATION BASED ON RANDOM SEARCH

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Abstract
In this paper the most common methods of parameter-optimization are briefly reviewed with emphasis on the so-called random search techniques. In order to improve these methods a new strategy is defined and presented in detail.

The strategy is of the creeping random search type, which is supposed to be superior to pure random search. The purpose of the new strategy is to increase the probability of obtaining global rather than local optimal estimates by introducing both local and global steps in the strategy. In order to improve the convergence of the creeping random search the local steps are directional adapted. The directional adaptation is accomplished by introducing correlations between past successful steps and future steps. If a step during a random search is a success its direction is given a preference in connection with the random choice of the next step. Further the step length can be modified by a gain-dependency.

In opposition to the local steps the directions of the global steps are random and the lengths of the global steps are uniformly distributed over a sufficiently large interval. In this way the global steps cover the feasible region.

The success of the strategy is tested on some test functions used in earlier investigations and on some new test functions.

INTRODUCTION

One of the most difficult and important problems in parameter optimization technique is to obtain a global rather than a local optimum. The traditional hill climbing methods based on gradient technique will only guarantee a global optimum in a parameter optimization problem if the criterion function is unimodal. For multimodal functions there exists a risk that methods based on gradient technique only will disclose a local

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optimum, namely when the starting point for the hill climbing method is outside of the field of influence of the global optimum.

If the field of influence of the global optimum only is a small part of the feasible region for the criterion function the chance on obtaining the global optimum is corresponding small if no special precautions are taken. All this is well known and has inspired many research-workers in this field to construct new optimization techniques hoping that they are more powerful in solving optimization problems, which is they will handle a larger class of problems with success than the usual techniques.

In this paper such an attempt based on a random search technique is presented. Random search techniques seem to be superior to the usual gradient techniques with respect to the global optimum problem mentioned above. But it must be admitted that random search methods usually are more time-consuming than the usual methods at least for problems with few parameters. However, the steady increasing speed of the digital computers makes it so important to think of efficiency. In this paper first priority is given to the problem of finding global optimum having in mind that no optimization technique will be one hundred per cent effective.

OPTIMIZATION TECHNIQUES

A very great number of different methods of optimization are discussed in the literature. A systematic presentation of them all is not easy because they often overlap with regard to technique and with regard to applicability. Even when parameter optimization techniques only are considered there are several ways of making a division. Perhaps the most common one is the following division:

a. Random methods.
b. Non-random methods.

In this paper the non-random methods (e.g. steepest descent and conjugate direction methods) will not be treated. The random methods can be further divided into;

a. Pure random search.
b. Creeping random search.

The new strategy presented in this paper is of the creeping random type. All the above mentioned methods are discussed in several books and review articles [1] – [7]. The problem considered in this paper is to find minimum for the so-called criterion function \( F : M \subset R^n \rightarrow R \). The ordered set of \( n \) parameters \( x_0 \in M \) is called a global minimum for the function \( F \), if

\[
F(x) \geq F(x_0) \quad \text{for all} \ x \in M,
\]

where \( M \) is called the feasible region for the function \( F \). If there exists a neighbourhood \( \omega(x_0) \) of the point \( x_0 \) so that

\[
F(x) \geq F(x_0) \quad \text{for all} \ x \in M \cap \omega(x_0),
\]

then \( x \) is called a local minimum for the function \( F \).

In the pure random search method a number of points \( P_1, \ldots, P_m \) in the feasible region \( M \) are selected and the value of the function \( F \) in \( P_i \), \( i = 1, \ldots, m \), are calculated. The smallest value of \( F(P_i) \), \( i = 1, \ldots, m \), is then used as an approximation of the minimum value of \( F \) in \( M \). It can be shown that a very great number of points \( P_1, \ldots, P_m \)
must be selected if a good approximation for the minimum point $x_0 \in M$ is to be located. Sometimes, the pure random search method may be useful in determining a starting point for a creeping search method or as a procedure for choosing a feasible region for the criterion function.

In a creeping random search method a starting point $x_0 \in M$ is chosen in some way and the value $F(x_0)$ of the criterion function $F$ is calculated in $x_0$. Then by some procedure a step $\Delta x_1$ is determined and the value $F(x_1)$, where $x_1 = X_0 + \Delta x_1$, is calculated. If $F(x_1) < F(x_0)$ the step is called a success and $x_1$ is used as the starting point for a new step $\Delta x_2$ etc. If $F(x_1) \geq F(x_0)$ a new trial step $\Delta x_1$ is used until a success is accomplished.

From the simplified description above of the creeping random search method it is clear that the success of the method is for example depending on how the steps $\Delta x_i$, $i = 1, 2, \ldots$ are determined. Experience seems to show that both the length of the steps $|\Delta x|$ and the directions of the steps must be chosen in a very sophisticated way if global rather than local optimum with a fair probability is determined within a reasonable time. It is easy to see that the step-size $|\Delta x|$ is of very great importance. In earlier investigations several procedures for step-size adjustment have been used. Some authors increase $|\Delta x|$ if a success takes place and decrease $|\Delta x|$ if none occurs within some number of trials. Other authors reduce $|\Delta x|$ following a number of failures. There seems to be no universal answer to this problem.

The procedure for step-size must probably be correlated in some way to the procedure for the step-direction.

Not only the step-size but also the step-directions are of great importance for convergence of a creeping random search method. If the last step during a search is a success it is likely that its direction should effect the direction of the next trial step. Such a directional adaptation has been used in different ways. Some authors have used the so-called absolute positive and negative biasing, where the direction of the last success-step is used again for the next step, and the opposite direction of a step without success is used for the next step. But many other way of correlating past successful steps with future trial steps are presented in the literature.

**THE MAIN IDEA**

As the main purpose with the new strategy of optimization is to obtain a global minimum rather than a local minimum it is necessary to develop a method where the random search not is trapped by a local minimum. On the other hand the strategy must be so planned that the rate of convergence is high as soon as the field of influence of the global minimum is reached. It is obvious that there exist a conflict between these two purposes so that it probably not is possible to develop a method, which in a limited time with one hundred per cent certainty gives a global minimum.

The main idea behind the strategy presented in this paper is to try to satisfy the two purposes mentioned above as good as possible by using *local* and *global steps*. The intention with the local steps is to obtain a high degree of convergence to the nearest minimum not depending on whether it is a global or a local minimum. So the step-size for the local steps is rather "small". On the contrary the global steps are not limited in size as their purpose only is to escape from a local minimum by making sufficiently
large step. The global steps must be arranged in such a way that any point in the feasible region can be reached by a global step from the point in question.

The main idea is thus the division in local and global steps, which not only differ by step-size but also in the way the step-directions are chosen. The strategy is in this manner planned so that local and global steps occur in a ratio that can be arranged as one likes. If a rough knowledge to the criterion function suggests that it is a unimodal function only few global steps in relation to the local steps are necessary, but even in such a case use of global steps may be advantageous. On the other hand if the criterion function possess several local minima it is appropriate to use a high ratio of global steps to avoid being trapped by a local minimum.

THE STARTING POINT

A simple way of choosing a starting point is to use an orthogonal net with the same net width in all the parameter directions and then choose at random one of the net points as a starting point by use of a random number generator. Another possibility is of course to calculate the value of the criterion function \( F \) in all the net points and then choose a starting point the point with the lowest value of the function \( F \). If the last mentioned method is used it seems to be necessary to use a rather fine net if a reasonable return for the computer time can be expected. In such a case the procedure is like the strategy used in pure random methods. In the strategy presented here the first mentioned method to choose a starting point is the most logical because of the presence of global steps.

To increase the probability of obtaining global estimates it is advantageous to initiate searches from several starting points.

THE LOCAL STEPS

![Fig.1. Flow-chart and step-diagram for the random search strategy.](image)

After a starting point \( P_0 \) has been chosen the strategy is based on local and global steps as shown in the figure. First the following procedure for local steps is used. The step-size for the local steps is limited to a predetermined interval. The sizes of the first local step \( |\Delta x| \) can e.g. be half of the net width. The length \( |\Delta x| \) defines a hyper-sphere with \( |\Delta x| \) as the radius and \( P_0 \) as center. On this hyper-sphere a prescribed number of points are then chosen until a successful local step is found by some probability distribution in such a way that directional
adaptation is obtained. If no successful local step is found the length of the trial steps (and therefore the radius of the hyper-sphere) is bisected or doubled. In this way two sets of hyper-spheres with radius $|\Delta x|/2$, $|\Delta x|/4$, $|\Delta x|/8$ ... and $2|\Delta x|$, $4|\Delta x|$, $8|\Delta x|$ ... are defined and the tracking of a successful trial step is continued on these spheres one after each other. When a successful step $|\Delta x|$ located the same procedure continues in the new point $x + \Delta x$ but with a step length modified by some kind of gain-dependency. In this way a prescribed (small) number of successful local steps are carried out. Then a global trial step follows.

**THE GLOBAL STEPS**

As mentioned earlier the intention with the global steps is to be able to reach any point in the feasible region. It is important to notice here that no information on any "good" direction for global steps can be given in general. Therefore the step-size is chosen random in a predetermined interval with an upper bound greater than the length of the diagonal of the feasible region. By the same reason the directions of the global steps are chosen random. A pre-described number of global trial steps are made until an improvement may happen. In such a case the new point is used as starting point for local steps. Otherwise further local trial steps are made from the same point but now with new directions.

The procedure described above continues until a pre-described number of successful steps have occurred or until no further improvements by local steps can be found under the restrictions given on the number of trial steps.

**GENERATION OF RANDOM NUMBERS**

In the literature several methods of generating pseudo-random numbers (see Jansson [9]) are suggested. A simple method is the so-called mid-square method, which can be described in the following way. Take e.g. a four digit number $x_0$ as the first random number and square it $x_0^2$. The next random number $x_1$ is then the middle four digits of $x_0^2$. Then $x_1$ is squared, etc.

The so-called linear congruential method, due to Lehmar [10], is supposed to be superior to most other methods used. A sequence of integers is initiated with a value $x_0$ and continued by

$$x_{i+1} = a + \lambda x_i \pmod{P},$$

where $a$, $\lambda$, and $P$ are given integers. It follows that $0 \leq x_i < P$ for all $i$. This method has very important advantages (see Dieter [11]):

1. For an appropriate choice of $a$, $\lambda$, and $P$, the fraction $y_i = x_i/P$ are uniformly distributed in the interval $[0;1]$.
2. Subsequences of most linear congruential generators pass different statistical tests, i.e. frequency tests, run tests, poker tests.
3. The method is fast and easy to program.

In the practical use of the new strategy of optimization illustrated below the following linear congruential generator is used:

$$x_{r+1} = 9901 x_i \pmod{2^{39} + 1}.$$
TEST FUNCTIONS

The only way to test a new strategy for optimization is to use the strategy on careful chosen test functions. If the purpose of the strategy is to obtain global rather than local optimum multimodal functions must of course be used. If only the convergence of the methods to the nearest minimum is under consideration unimodal functions can be used.

A great number of test functions are described in the literature (see e.g. Kowalik & Osborne [12] or Oren [15]. The test functions are used in testing and comparing optimization methods, but in most cases the published results are not so detailed that a rational choice can be carried out. Usually only unimodal functions are used.

The strategy presented in this paper is tested by the following 5 test functions:

1. \[ F_1(x, y) = 100(x - y)^2 + (1 - x)^2 + 1 \]
2. \[ F_2(x, y) = 2 - \exp(-\frac{1}{4\pi}\sqrt{x^2 + y^2})\cos\sqrt{x^2 + y^2} \]
3. \[ F_3(x, y) = 2 - \exp(-\frac{1}{4\pi}\sqrt{x^2 + y^2})\cos x \cos y \]
4. \[ F_4(x, y, z) = 100[(z - 10\theta(x, y))^2 + (r(x, y) - 1)^2] + z^2 + 1 \]
   \[ \theta(x, y) = \begin{cases} \frac{1}{2\pi} \arctan \frac{y}{x} & \text{for } x > 0 \\ \frac{1 + \frac{1}{2\pi}}{2\pi} \arctan \frac{y}{x} & \text{for } x < 0 \end{cases} \]
   \[ r(x, y) = \sqrt{x^2 + y^2} \]
5. \[ F_5(x, y, z) = 2 - \exp(-\frac{1}{4\pi}\sqrt{x^2 + y^2 + z^2})\cos x \cos y \cos z \]

The function \( F_1 \): \( \mathbb{R}^2 \rightarrow \mathbb{R} \) is essentially Rosenbrock’s function [16] used by many authors. It is unimodal and has a minimum at (1,1) with \( F_1(1,1) = 1 \). Further, there is a deep valley along the parabola \( y = x^2 \).

The function \( F_2 \): \( \mathbb{R}^2 \rightarrow \mathbb{R} \) is a multimodal function with a global minimum at (0,0) and \( F_2(0,0) = 1 \). Further, the function \( F_2 \) has local minima on a sequence of concentric circles with centre in (0,0). The “first circle” with local minimum has the equation \( x^2 + y^2 = 6.20^2 \) and the value \( F_2 = 1.392 \). The field of influence of the global minimum is the circle \( x^2 + y^2 = 3.06^2 \). It is therefore fairly difficult to obtain the global minimum if the area of the feasible region \( M \) is much larger than the area of the circle. In the numerical results presented below \( M = [-10; 10] \times [-10; 10] \) so the probability of obtaining the global minimum is only about 7% if a starting point is chosen random and a method which do not permit passage of a crest is used. It will be shown that the strategy used here gives a much better result due to first of all the presence of global steps.

The multimodal function \( F_3 \): \( \mathbb{R}^2 \rightarrow \mathbb{R} \) also has a global minimum at (0,0) and \( F_3(0,0) = 1 \), but the local minimum are now placed at discrete points. Apart from this the function has similar properties regarding the relation between global and local minimum.

The function \( F_4 \): \( \mathbb{R}^3 \rightarrow \mathbb{R} \) is unimodal and has a helical valley (see Oren [15]) The minimum point is (1,0,0) and \( F_4(1,0,0) = 1 \).
A more interesting test function in three dimensions is the function \( F_5: \mathbb{R}^3 \rightarrow \mathbb{R} \), which is a generalization of the function \( F_3 \). A further generalization to any dimension is obvious. The global minimum point is \((0, 0, 0)\) and \( F_5(0, 0, 0) = 1 \). The local minima are placed at discrete points and the difference between the “best” local minimum and the global minimum is rather small, so again global minimum is not so easy to obtain.

**NUMERICAL EXPERIMENTS**

As mentioned above, the new strategy of optimization is tested on five different test functions, namely three functions with two parameters and two functions with three parameters. A very great number of experiments have been made with different combinations of local and global steps, with various minimum step-sizes and with different feasible regions. Further, experiments with test function with more than three parameters are now being carried out, but these experiments are not concluded yet. The results from experiments on test functions with \( n \) parameters, \( n \geq 4 \), will be published shortly.

Only results from the final versions of the computer programme will be presented here. A CDC CYBER 72 computer belonging to AALBORG UNIVERSITY is used for all the computer runs. The presentation below is made so short as possible, but with the intention to emphasize the importance of the division in local and global steps.

**TEST FUNCTIONS \( F_1, F_2, F_3 \)**

It is important for the convergence of the local steps to use some kind of directional adaptation. This can be done in various ways. In the experiments done with the two-parameter test function \( F_1, F_2, \) and \( F_3 \) it is carried out by letting the directional angle \( \varphi \) of a new trial step follow the normal probability distribution \( N[\alpha, \pi] \), that is with a mean equal to the angle \( \alpha \) for the last successful step and a standard deviation equal to \( \pi \). But this is of course not the only distribution that can be chosen. The angle \( \varphi = \alpha + v \) is determined by a simple method for generating normal variates (see Newman & Odell [14]) namely by evaluating the angle \( v \) by the formulae

\[
v = \pi \sqrt{-2 \ln r_1 \cos(2\pi r_2)},
\]

where \( r_1 \) and \( r_2 \) are random variables in the interval \([0;1]\) calculated by the random number generator described above. Only values of \( v \) in \([-\pi, \pi] \) are accepted.

The initial step-size of the local steps are modified by a multiplicative factor, namely the ratio between the numerical value of the last gain and the numerical value of the criterion function at the point in question. Again this way to modify the step-size is not the only possible one.

The direction of the global steps is chosen random and also the length of the steps is chosen random in a given interval large enough to cover the feasible region.

By 36 experiments with the unimodal function \( F_1 \) in the feasible region \([-10; 10] \times [-10; 10] \) global minimum is obtained in all cases. The average number of steps is 74 and the average computer-time is 10 sec. Although the function is unimodal 9% of all steps are global steps. The minimum values of the function \( F_1 \) determined by the 36 experiments belongs to the interval \([1.000; 1.004]\) and the parameters \( x \) and \( y \) for the (global) minimum1 are in the intervals \([0.995; 1.061]\) and \([0.995; 1.062]\) respectively.
The minimum step-size accepted is $\sqrt{8} \times 10^{-3}$. The last mentioned values can easily be improved by accepting a smaller step-size and using more steps.

The multimodal test function $F_2$ is used in 40 experiments where global minimum is obtained in 31 cases and local minimum in 9 cases. The average number of steps is only 31 and the average computer-time 4 sec. 15% of all steps is global steps. The importance of the global steps can be illustrated by the following figures from one of the experiments:

<table>
<thead>
<tr>
<th>Step no:</th>
<th>x</th>
<th>y</th>
<th>$F_2(x,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.000</td>
<td>-9.000</td>
<td>2.454</td>
</tr>
<tr>
<td>6</td>
<td>2.680</td>
<td>-5.770</td>
<td>1.399</td>
</tr>
<tr>
<td>11</td>
<td>3.071</td>
<td>-5.540</td>
<td>1.392</td>
</tr>
<tr>
<td>12</td>
<td>-3.330</td>
<td>-0.234</td>
<td>1.110</td>
</tr>
<tr>
<td>17</td>
<td>0.017</td>
<td>-0.042</td>
<td>1.005</td>
</tr>
<tr>
<td>29</td>
<td>0.000</td>
<td>0.001</td>
<td>1.000</td>
</tr>
</tbody>
</table>

After 6 steps the experiment is trapped by a local minimum and the next 5 steps are used to improve the local minimum value ($F_2 = 1.392$). If no global steps exist this would be the end of the experiment, but now the global step no. 12 makes it possible to escape from the local minimum and reach the global minimum. After 17 steps the value of the function is improved to 1.005 and the experiment is concluded after 29 steps with the global minimum point (0.000; 0.001) and $F_2 = 1.000$. This behaviour of the strategy is typical for all the successful experiments with the function $F_2$.

In all the 31 successful experiments with function $F_2$ the value of the function in the global minimum is determined to 1.000 and the parameters $x$ and $y$ for global minimum are both in the interval [-0.001; 0.001]. The minimum step-size accepted is $\sqrt{8} \times 10^{-3}$ and the feasible region is [-10;10] $\times$ [-10;10].

The results from 26 experiments with the multimodal function $F_3$ with the same minimum step-size and feasible region are very much like the results for the function $F_2$ but global minimum is obtained in all but one experiment. When taking the number of local minima in the feasible region in account these results must be considered satisfactory.

The experience with the use of the new strategy for two-parameter test functions is very encouraging, and even better results than presented here are most certainly possibly regarding the convergence against the minimum point.

**TEST FUNCTIONS $F_4, F_5$**

For the three-parameter tests with the functions $F_4$ and $F_5$ the directional adaptation for the local steps is made by use of a polar description of the steps. The direction of a new trial step is now given by two angles $\theta$ and $\varphi$ which are assumed to follow normal probability distributions with means equal to the corresponding angles of the last successful step with standard deviations equal to $\pi$. The procedure use here is a generalization of the procedure used for the two-parameter case. Using other standard deviations can change the degree of the directional adaptation. The same procedure as used for the two-parameter functions is use to obtain gain-dependency with regard to step-size.
The direction of the global steps is chosen random and also the length of the steps is chosen random in a given interval large enough to cover the feasible region.

By 26 experiments with the unimodal function $F_4$ in the feasible region $[-10; 10] \times [-10; 10] \times [-10; 10]$ global minimum is obtained in 25 cases and one experiment is rejected because of problems with regard to obtaining steps leading to points inside the feasible region within the accepted number of steps. The computer programme is made in such a way that when this occur information about it is printed out. With a minimum step-size accepted equal to $\sqrt{12} \times 10^{-4}$ the average number of steps are 173 and the computer-time 69 sec. 6 % of all steps are global steps. The minimum values of the function $F_4$ determined by the 25 successful experiments all belongs to the interval (1.000; 1.007) and the parameters $x$, $y$ and $z$ for the (global) minimum are in the intervals [0.999; 1.000], [-0.052; 0.031], [-0.083; 0.048], respectively. The determination of the minimum point can easily be improved by accepting more steps, but of course this will increase the computer -time.

23 experiments with the same test function $F_4$ are also performed in the feasible region $[0; 3] \times [0; 3] \times [0; 3]$ where the minimum now is on the boundary of the region. This is a rather serious test because the minimum can only be approached from one side. But the strategy is successful in 22 cases and only one experiment is rejected for the same reason as above. The average computer-time is 132 sec. and the average number of steps is 199. 8 % of all steps are global steps and the minimum value of $F_4$ is in the interval [1.000; 1.006]. The first coordinate $x$ for the minimum point is defined to the interval (0.998; 1.000), while $y$ and $z$ are in the intervals [0.001; 0.049] and [0.001; 0.078] respectively. The good determination of the first coordinate compared with the two others is in accordance with the fact that the minimum point (1, 0, 0) only is a boundary point with respect to the last two coordinates.

The multimodal test function $F_5$ is used in 24 experiments and one is rejected for the same reason as above. 15 experiments give the global minimum and 8 local minimums. The average computer-time is 21 sec. for an average number of steps on 71. 14 % of all steps are global steps and the value of the global minimum is determined to 1.000 in all 15 cases. The coordinates for the global minimum are in the intervals [-0.002; 0.001], [-0.001; 0.001] and [-0.001; 0.002] respectively. For this test function the probability of getting global minimum in just one experiment is about 2/3. If two experiments are made then the probability of finding the global minimum is 8/9. With three experiments it is increased to 26/27 etc.

The importance of the global steps is illustrated by the following figures from a typical experiment with the function $F_5$:

<table>
<thead>
<tr>
<th>Step no:</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$F_5(x,y,z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.000</td>
<td>10.000</td>
<td>10.000</td>
<td>1.877</td>
</tr>
<tr>
<td>8</td>
<td>-3.317</td>
<td>5.916</td>
<td>-2.771</td>
<td>1.522</td>
</tr>
<tr>
<td>18</td>
<td>-3.203</td>
<td>0.094</td>
<td>3.131</td>
<td>1.304</td>
</tr>
<tr>
<td>29</td>
<td>-3.099</td>
<td>0.007</td>
<td>3.071</td>
<td>1.296</td>
</tr>
<tr>
<td>30</td>
<td>-0.074</td>
<td>0.577</td>
<td>-0.448</td>
<td>1.289</td>
</tr>
<tr>
<td>53</td>
<td>0.000</td>
<td>0.005</td>
<td>0.015</td>
<td>1.001</td>
</tr>
<tr>
<td>60</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

After 8 steps the experiment is trapped by a local minimum and the next 10 steps are used to improve the local minimum value ($F_5 = 1.458$). Step no. 18 is a global step that
brings the experiment to a better local minimum that is reached by step no. 29 ($F_5 = 1.296$). Finally the global step no.30 makes it possible to reach the global minimum.

**n-PARAMETER FUNCTIONS**

Only experiments with test functions with two and three parameters are presented here. Experiments with an arbitrary number of parameters do not cause particular problems except with regard to computer-time.

The directional adaptation or the local steps can be accomplished in different ways. For instance the following method can be used. Let $\Delta x$ be the last successful step and let the coordinate $a$ of its normalized directional vector be $(\Delta x_1, \Delta x_2, ..., \Delta x_n)$. The direction of the next trial step can then be determined by a normalized vector with coordinates given by $n$ normal probability distributions with means equal to $\Delta x_i$, $i = 1, 2, ..., n$. The step-size can be established in the same way as for the two and three-dimensional cases. The direction of the global steps can be determined by using uniform probability distributions over a given interval for each of the parameters. The step-sizes of the global steps can also be assumed uniformly distributed over a given interval.

Intense research after these lines is going on now, but the results cannot be presented yet. Experience with regard to the best way to establish the directional adaptation of the local steps is needed.

**CONCLUSIONS**

It is always difficult to discuss the value of a new strategy and to compare it with other algorithms because its performance is different from one situation to the next. Further, conclusions can only be based on a limited amount of experience.

Much more practical experience with the new strategy presented here is needed before a final evaluation of it is possible. First of all the method must be used on test functions with a great number of parameters ($n > 50$). Especially for such functions the random search methods may be superior to non-random methods (see e.g. Rastrigin [17]).

The only tenable conclusion based on the results presented here is that the strategy for some test functions perform in an encouraging way especially to the very difficult and important problem of obtaining global rather than local optimal estimates. Further it is shown that this is due to the presence of local and global steps because the use of two different kinds of steps makes it possible to obtain this as well as a good convergence of the method.

**REFERENCES**


