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## CHAPTER 13

### MODELLING STOCHASTIC SIGNALS FOR DYNAMIC EXPERIMENTS<sup>1</sup>

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#### 1. INTRODUCTION

When performing dynamic testing of models of structures it is often necessary to be able to apply forces that can be modelled as realizations of stochastic processes with given spectral densities. Here, for the sake of simplicity, only stationary and ergodic Gaussian processes are considered (see e.g. Lin [1]). In this paper we deal in particular with problems connected with generation of loads with given spectral densities on models. This task can be difficult to perform when an electrodynamic vibration exciter applies the load. This is due to the fact that the input signal for the exciter generated on the basis of a given spectrum will result in an output signal (load) from the exciter, which is significantly different from the signal intended. The reason for this is partly that the frequency characteristics of the exciter is not a constant, and partly that the exciter itself is not linear. The frequency characteristics of the exciter depend on the eigenfrequencies and damping ratios of the exciter and the structural model but also on the given spectrum. It is usually difficult by electronic means to modify the output from the exciter so that the intended spectrum is obtained. In this paper a method is suggested by which the intended spectrum can be obtained in some frequency intervals. The method is based on an interaction between generation and analysis of data by a computer and by experiments.

#### 2. SIMULATION TECHNIQUES

As mentioned above the input of the exciter are realizations of a stationary Gaussian stochastic process with given spectral density and mean functions. In this section a simulation technique (suggested by Wittig and Sinha [2]) is presented, by which realizations of a stationary Gaussian process  $\{X(t), t \in [0, T]\}$  characterized by

$$E[X(t)] \equiv 0 \quad (1)$$

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<sup>1</sup> Proceedings of the International Conference on “Computational Methods and Experimental Measurements”, July 1982 (eds. G.A. Keramidas & C.A. Brebbia), Springer-Verlag, pp.341-350.

$$\kappa_X(\tau) = \int_{-\infty}^{\infty} S_X(\omega) e^{i\omega\tau} d\omega \quad (2)$$

can be simulated. In equation (1)  $E[\cdot]$  denotes the expectation operation. In equation (2)  $\kappa_X$  is the auto covariance function and  $S_X$  is the spectral density. By simulation a process  $\{X_0(t), t \in [0, T]\}$  is obtained, which approximately satisfies the conditions (1) and (2) and which is approximately Gaussian. Only realizations of the stochastic variable  $X_0$  are necessary for the index values  $m \cdot \Delta t$ ,  $m \in \{0, 1, \dots, N-1\}$ . The process  $\{X_0(t)\}$  is defined by

$$X_0(m \cdot \Delta t) = \text{Re}(B(m)) \quad (3)$$

$$B(m) = \sum_{k=0}^{N-1} A(k) e^{ikm\Delta\omega\Delta t}, \quad m \in \{0, 1, \dots, N-1\} \quad (4)$$

$$A(k) = \sqrt{4S_X(k\Delta\omega)\Delta\omega} (R_k + iS_k), \quad k \in \{0, 1, \dots, N-1\} \quad (5)$$

where  $\Delta\omega$  is the frequency resolution and  $R_k$  and  $S_k$ ,  $k \in \{0, 1, \dots, N-1\}$  are mutually independent and identically distributed normal variables  $n(0, \sqrt{1/2})$ . By this definition  $X_0(m \cdot \Delta t)$  is normally distributed  $n(0, \sqrt{\kappa_X(0)})$ .  $\Delta\omega$  and  $\Delta t$  are chosen in such a way that

$$\Delta\omega \cdot \Delta t = \frac{2\pi}{N} \quad (6)$$

Note that  $B(m)$  and  $A(k)$  are mutual Fourier transforms. The calculation of (3) is significantly facilitated by an FFT-algorithm. From equations (3) - (5) it is seen that

$$E[X_0(t)] = 0 \quad (7)$$

and

$$\kappa_{X_0} = 2 \sum_{k=0}^{N-1} S_X(k \cdot \Delta\omega) \cos(k \cdot \Delta\omega \cdot \tau) \Delta\omega \quad (8)$$

Therefore,

$$\kappa_{X_0}(\tau) \rightarrow \kappa_X(\tau) \text{ for } N \rightarrow \infty \quad (9)$$

It follows directly from equation (5) that the first order distribution of  $\{X_0(t)\}$  is normal. However, Yang [3] has shown for  $N \rightarrow \infty$  that the process  $\{X_0(t)\}$  is Gaussian if the stochastic variables  $X_0(t_1), X_0(t_2), \dots, X_0(t_m)$  for arbitrary index values  $\{t_1, t_2, \dots, t_m\} \in t^m$  and arbitrary  $m$  are jointly normally distributed. Therefore, the simulated process  $\{X_0(t)\}$  is asymptotically Gaussian and it has asymptotically the intended second moment properties (1) and (2). Further it follows from (3)-(6) that  $X_0(t)$  is periodic with the period  $N \cdot \Delta t$ .

By this simulation technique, the maximum circular frequency in  $S_X(\omega)$  is given by the Nyquist-frequency

$$\omega_c = \frac{\pi}{\Delta t} \quad (10)$$

The parameters  $\Delta t$ ,  $\Delta\omega$  and  $N$  must be chosen in such a way that (6) and (10) are fulfilled and so that  $S_X$  is represented to a sufficient degree of accuracy.

### 3. DESCRIPTION OF TEST EQUIPMENT

The test equipment and the test procedure used in this investigation are schematically illustrated in figure 1. The test procedure can be divided into 3 steps (I, II and III in figure 1).

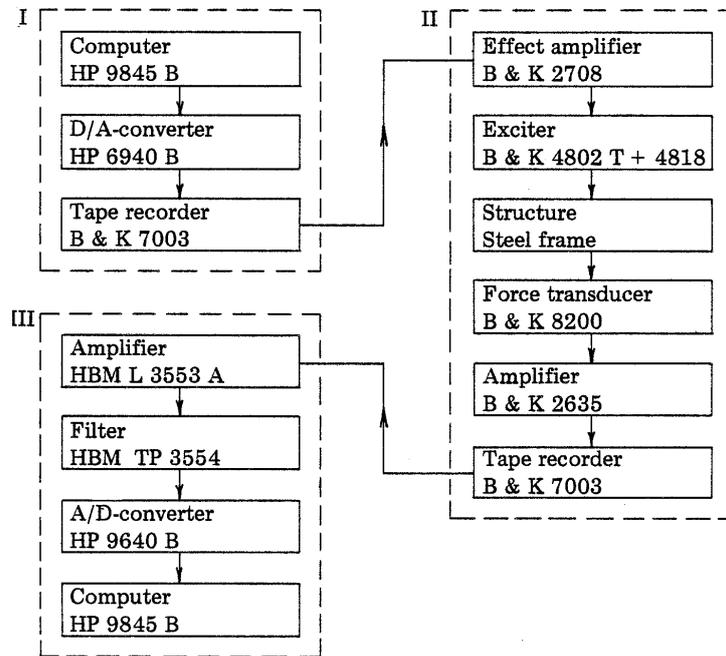


Figure 1.

In *step I* a realization of the input process for the exciter is generated. Realizations of such a process with a given spectral density and mean function are generated on the computer by the simulation technique described in section 2. Next, the D/A-converter converts the digital data into analog signals.

In *step II* the model testing is performed. The analog signal is used as input for the exciter. The output from the exciter (i.e. the loading on the structure) is

registered by a force transducer.

The structure used in this investigation is a model of a two-story steel frame (0.8 m × 0.4 m). The dimensions of all sections are 0.015 × 0.0005 m<sup>2</sup>. The frame is shown in figure 2. The 2 lowest eigenfrequencies are calculated by numerical analysis.

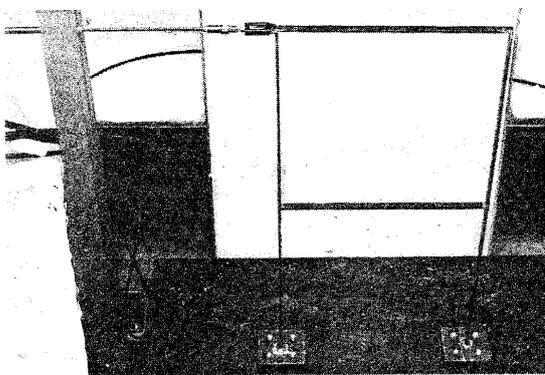


Figure 2.

They are 1004 Hz and 3500 Hz, respectively. The frame is supported in such a way that it can be assumed that it is restrained at the supports. The loading is a single horizontal load in the plane of the frame in the direction of the upper beam. This loading may e.g. be modelling a wind load.

In *step III* the measured signal first passes a filter, then it is converted into digital data, and finally it is analyzed in the computer.

The frequency analysis is performed as described by Bendat & Piersol [4], [5]. The measured data are divided into a number of smaller realizations with e.g. 1024 points per realization. These small realizations are then frequency analyzed. The length of such a realization and the number of points are chosen in such a way that the frequency resolution and the Nyquist-frequency are reasonable for the spectra in question. The result of these calculations is a number - say  $Nr$  - of spectra. Each estimate of the spectral density will under these assumptions get a standard

deviation, which is of the same order as the estimate. To reduce this unacceptable uncertainty the mean of the  $Nr$  spectral estimates is used. By this procedure a consistent estimate of the spectral value is obtained. The coefficient of variation of the estimate is inversely proportional to the square root of the number  $Nr$ .

By the filtering mentioned above that part of the spectrum that corresponds to frequencies greater than the Nyquist-frequency are removed.

#### 4. ITERATIVE METHOD

As mentioned in the introduction the spectrum of the input signal to the exciter will usually be significantly different from the spectrum of the output signal from the exciter (load signal for the structure). In this section a method is presented which iteratively produce an input signal for the exciter with corresponding spectrum so that the output signal for the exciter (load signal for the structure) has the intended spectrum.

Let the intended load spectrum be  $S_p(\omega)$ . The input spectrum  $S_{I,i+1}(\omega)$  for the exciter in step No.  $i+1$  is then defined by

$$S_{I,i+1}(\omega) = \frac{S_{I,i}(\omega)}{S_{O,i}(\omega)} S_p(\omega) \quad (11)$$

where  $S_{I,i}$  and  $S_{O,i}$  is input- and output spectra for the exciter for test No.  $i$ . The first input spectrum  $S_{O,1}$  is chosen as

$$S_{I,1} = S_p(\omega) \quad (12)$$

As a stop criterion for the iteration method (11) can be used

$$\varepsilon_S = \int_{\omega_L}^{\omega_U} \left( \frac{S_{O,i}(\omega) - S_p(\omega)}{S_{CU,i}(\omega) - S_{CL,i}(\omega)} \right)^2 d\omega \leq \varepsilon \quad (13)$$

where  $\omega_L$  and  $\omega_U$  is the lower and upper frequency for the frequency window in question.  $S_{CU,i}$  and  $S_{CL,i}$  is upper and lower limits for the  $(1 - \alpha)100\%$  confidence interval for  $S_{O,i}$ . The constant  $\varepsilon$  must be determined on the basis of the intended accuracy of  $S_I$ . The confidence intervals for  $S_{O,i}$  are determined from the fact that the single estimates of the spectrum at frequency No.  $j$  are  $\chi^2$  distributed with 2 degrees of freedom.

For specific input  $S_p$  it may be convenient to use other stop criteria than (13). For example, if  $S_p$  is the spectrum of a band-limited white-noise one may use as a stop criterion that  $S_{O,i}$  at a given level of significance  $S_{0,i}$  can pass a test with the following null hypothesis:  $S_{O,i}$  is a constant in the given frequency window against the alternative hypothesis:  $S_{O,i}$  is not a constant in the frequency window (see e.g. Lindgren [6]. 539). This test is based on the assumption that the estimate in each of the  $Nr$  spectra is normally distributed. However, the estimates of the spectrum are  $\chi^2$  distributed with 2 degrees of freedom. It is therefore necessary in each spectrum to use the average over at least 4 adjoining estimates so that an approximation to the needed normal distribution is

obtained. The estimates obtained by this procedure are  $\chi^2$  distributed with minimum 8 degrees of freedom.

## 5. EXAMPLE

In this section the iterative techniques described in section 4 are evaluated on the basis of specific experimental tests. The object is to determine an input spectrum for the exciter so that the output spectrum (load spectrum) has a given form - in this case the spectrum of a band-limited white noise signal. The structure is the two-storey frame shown in figure 2. The upper and lower limits of the spectrum are chosen so that the lowest eigenfrequency of the structure is between these limits and further so that the second lowest eigenvalue is greater than the upper limit. The intended load spectrum is shown in figure 3. In the digital generation of the input spectrum the following parameters are used:  $\Delta t = 0.01041$  sec and  $N = 4096$ . A total number of 10 realizations are generated by these parameters. These realizations are then linearly connected in a given time interval. This approximation is assumed to be of negligible importance for the form of the spectrum.

The measured analog output signal (load signal) is filtered at a limit frequency of 30 Hz. The characteristic of the filter is such that the spectrum will be unchanged below 20 Hz, when this limit frequency is used. The frequency analysis is then performed using the following parameters:  $\Delta t = 0.0178$  sec ( $\omega_c = 2\pi \cdot 28.1$  sec<sup>-1</sup>),  $N = 1024$ ,  $Nr = 20$ .

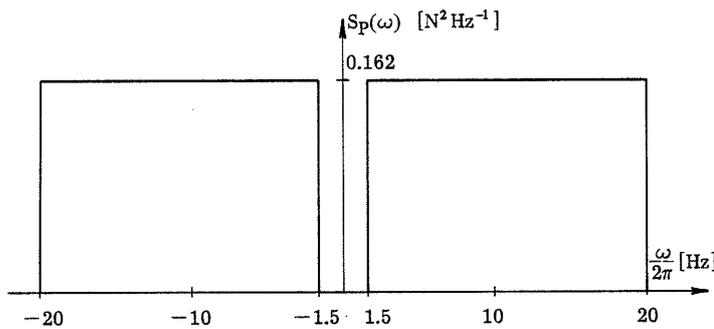


Figure 3. Load spectrum  $S_p(\omega)$ .

The stop criterion mentioned at the end of section 3 in connection with band-limited white-noise is used in this example and in each spectrum the average over 4 estimates is performed before the hypothesis test is made. The hypothesis  $H_0$ : the spectrum is constant in the interval  $[\omega_L, \omega_U] =$

$[2\pi \cdot 1.5$  sec<sup>-1</sup>,  $2\pi \cdot 20$  sec<sup>-1</sup>], is rejected at the confidence level 0.05, if the statistic (test variable)  $f$  is in the rejection interval  $[F_{0.95; 82; 1596}, \infty [$ , where  $F_{1-\alpha; \nu_1; \nu_2}$  is the  $(1-\alpha)$ 100% fractile in the  $F$ -distribution with  $\nu_1$  and  $\nu_2$  degrees of freedom.

i	1	2	3	4	i	1	2	3	4
f	30.5	3.13	1.70	1.20	$\epsilon_S$	$39.7 \cdot 10^3$	36.2	1.13	1.28

Table 1. Values of the statistics  $f$ . Table 2. Values of  $\epsilon_S$ .

Input spectra  $S_{I,i}$  and output spectra  $S_{O,i}$  for  $i = 1, 2, 3$ , and 4 from the iteration process described in section 3 are shown in figure 4. The corresponding values of the statistic  $f$  are shown in table 1. The iteration process is stopped after 4 iterations as  $F_{0.95; 82; 1596} =$

1.30. Therefore the input spectrum  $S_{I,4}$  can be used in the dynamic testing of the steel model frame. The corresponding values of  $\varepsilon_S$  (see (13)) with 95% confidence interval and  $\omega_L = 1.5 \cdot 2\pi \text{ sec}^{-1}$  and  $\omega_U = 20 \cdot 2\pi \text{ sec}^{-1}$  are shown in table 2. No significant difference in the form of variation of  $f$  and  $\varepsilon_S$  is found. However, the value of  $\varepsilon_S$  for  $i = 3$  is a little smaller than for  $i = 4$ . Therefore, if the last stop criterion is used the

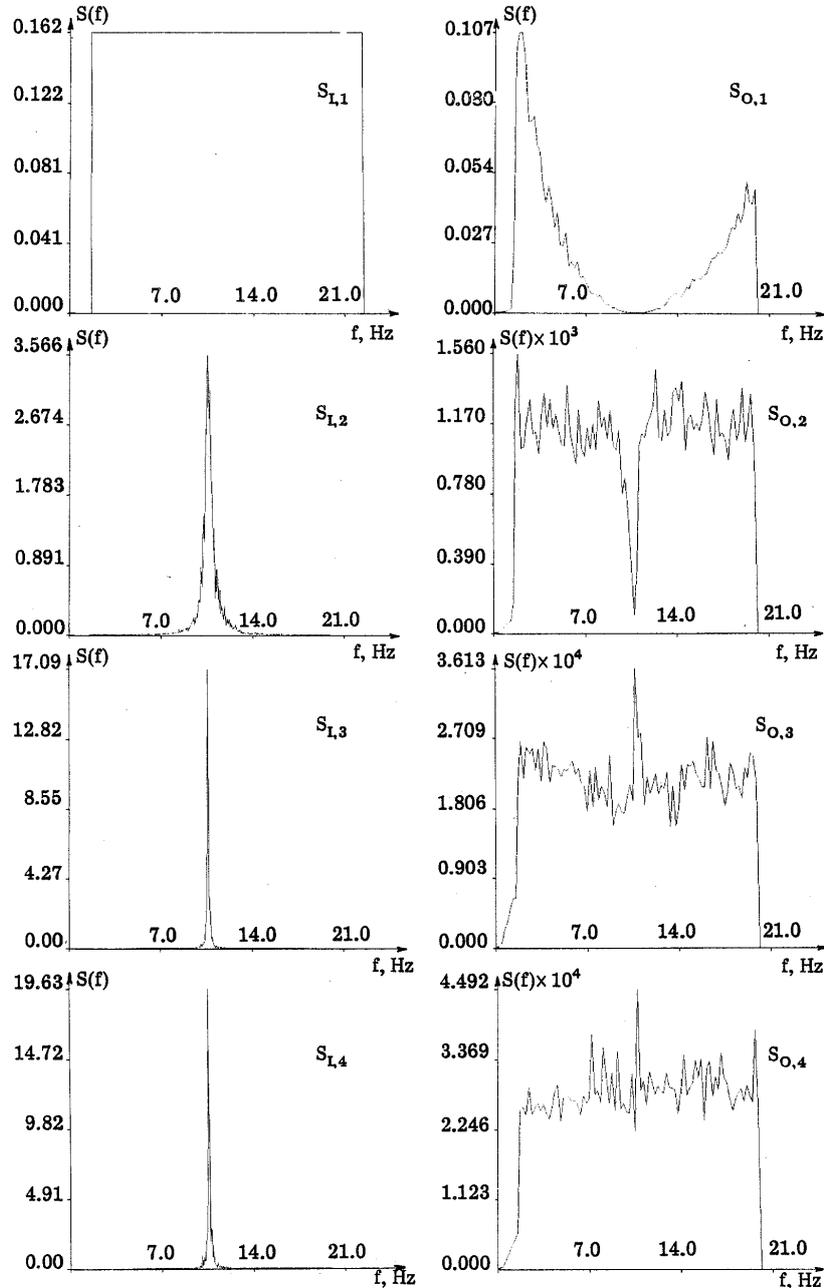


Figure 4. Input-spectra  $S_{I,i}$  (frequency resolution = 0.0488 Hz) and output spectra  $S_{O,i}$  (frequency resolution = 0.219 Hz) for  $i = 1, 2, 3$  and

iteration should be stopped already after 3 iterations. By use of (13) as stop criterion estimates for  $S_{O,i}(\omega)$  smaller than  $S_p(\omega)$  will contribute more to  $\varepsilon_S$  than estimates for

$S_{o,i}(\omega)$  greater than  $S_p(\omega)$ . This is due to the fact that the confidence intervals are proportional to the values of the estimates. It follows from figure 4 that this may be the reason for the trend of the statistics  $f$  and  $\varepsilon_s$  shown in tables 1 and 2.

It follows clearly from figure 4 that the resulting input spectrum  $S_{I,4}$  is completely different from the initial spectrum in figure 3. The frequency corresponding to the peak of the spectrum  $S_{I,4}$  is 10.7 Hz. The lowest eigenvalue of the frame has been determined by calculation of the frequency response function. The calculation is based on simultaneous measurement of the loading of the frame and the displacement of the upper beam (see Bendat & Piersol [5] p. 87). The result is that the lowest eigenvalue is 10.7 Hz, i.e. the same value as mentioned above. As shown earlier the lowest eigenvalue has been calculated with the result 10.4 Hz. The coincidence between the peak of  $S_{I,4}$  and the lowest eigenvalue is apparently a characteristic of the system because the same kind of tests has been performed with a number of different frames with other eigenvalues. All these tests confirm the above observation that the form of the input spectrum to a high degree is influenced by the lowest eigenvalue. The same conclusion is most likely true with respect to the damping ratio.

Only a minor part of the load spectrum will fall in the frequency interval  $[\omega_L, \omega_U]$ . To investigate this realizations are frequency analysed without using the filter. The spectral content at frequencies above the Nyquist-frequency  $\omega_c$  (see (10)) will be folded into frequencies below  $\omega_c$ . Therefore, one half of the variance  $\sigma_{o,i}^2$  of the load signal can be calculated by integration over the interval  $]0, \omega_c]$ . This has been done for the 4 tests shown in figure 4. The result is presented in table 3 where the values of  $\delta_i$ ,  $i = 1, 2, 3, 4$ , defined by

$$\delta_i = \frac{\int_{\omega_L}^{\omega_U} S_{o,i}(\omega) d\omega}{\int_0^{\omega_c} S_{o,i}(\omega) d\omega} = \frac{2}{\sigma_{o,i}^2} \int_{\omega_L}^{\omega_U} S_{o,i}(\omega) d\omega \quad (14)$$

are shown.

i	1	2	3	4
$\delta_i \cdot 100\%$	99.1	25.3	4.97	7.76

Table 3.

It is clear from table 3 that at the first iteration step almost the complete spectrum is inside the interval  $[\omega_L, \omega_U]$  but at the 3rd and 4th step only about 5% are in this interval. The reason for this drastic reduction may be non-linear performance of the exciter. It is clear from the spectra in tests without filter that the main part of the spectrum is concentrated on a few frequencies. This is a general conclusion from tests with several different frames. If the system is perfectly linear then the load spectrum will not have any components outside the interval  $[\omega_L, \omega_U]$  when the input spectrum generated on the computer is completely inside this interval. However, if the behaviour of the exciter is non-linear then the form of the input spectrum, the eigenvalues of the frame and presumably also the damping ratio will have influence on this matter. The undesirable load spectrum may also be due to an unfortunate frequency

characteristics of the combined system of exciter and structure, so that inevitable noise will have significant influence even if the system were linear.

## 6. CONCLUSIONS

In this paper a method to determine the input signal (input spectrum) for an electrodynamic vibration exciter is suggested, so that the output signal from the exciter - i.e. the load signal on the model - has a given spectrum. The method is based on an iterative process involving combined use of experimental tests and computer simulations. The method is illustrated by an example where the intended load signal is band-limited white noise. A stop criterion is suggested and with this stop criterion the result is obtained after 4 iterations. The input spectrum is significantly different from the load spectrum due to influence from the dynamic properties of the complete system. The input spectrum determined by the iteration method is therefore only usable if these dynamic properties are un- changed during the experimental tests. Therefore the method is of little value in stochastic testing of models beyond the elastic behaviour.

One important problem arises in some situations with this method, namely that only a minor part (in this example 5%) of the total spectral content will be found in the given frequency interval where e.g. the load spectrum is aimed to be constant. The rest is concentrated at frequencies beyond this interval. Therefore, by some types of test the resulting input spectrum is not usable. This is e.g. so if the load spectrum contains non-negligible components at frequencies where the uncontrollable peaks exist or if such peaks appear at frequencies where the response of the model will be significantly influenced.

The method seems to be of greatest value if a number of tests with the same load spectrum and the same model have to be performed. Then this method has a great advantage compared to an electrical modification of the output signal, because only one calculation of the input spectrum is necessary. By the electrical modification this has to be done at each test. Further the method suggested can be used in connection with any load spectrum with the limitations described.

Finally, it should be emphasized that the problems mentioned in constructing stochastic loads will not occur if e.g. a servo-controlled hydraulic exciter is used. However, such experimental equipment is rather expensive.

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