AN EXPLANATION OF INTERGENERATIONAL EDUCATIONAL MOBILITY USING THE CORRELATED MARE MODEL: A DANISH TEST CASE

A James McIntosh\textsuperscript{1} and Martin D. Munk\textsuperscript{2}

\textsuperscript{1}Economics Department
Concordia University
1455 De Maisonneuve Blvd. W.
Montreal Quebec, H3G 1M8, Canada
and
\textsuperscript{2}Centre for Mobility Research
Department of Political Science
Aalborg University
Frederikskaj, DK-2450 Copenhagen SV, Denmark

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E-mail Addresses and Telephone Numbers: jamesm@vax2.concordia.ca, +1-514-848 2424-3910 and mdm@dps.aau.dk. +45-9940 2335.

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Abstract

We examine Danish student participation in upper secondary and university education in 1985 and 2005 using a generalization of the famous Mare educational transition model. Factors not related to the observed characteristics of the household in which the respondent was raised have become more important relative to observable family background variables for upper secondary school success than they were a generation ago. This conclusion is based a new mobility index which we have developed to measure the relative dependence of respondent educational success on family background variables which in turn resolves a paradox in our empirical results. The conclusion is also confirmed by the application of the conventional methods that are usually used to measure mobility.

1 Introduction

The purpose of this paper is to determine whether there have been any changes in intergenerational educational mobility for recent cohorts of Danes, adding important methodological features to the study of Breen et al. 2009 and others, and a recent special issue in Research in Social Stratification and Mobility. Our
contribution to the stratification literature is to explain the pattern of increasing intergenerational educational mobility or declining inequality in educational opportunity by estimating Lillard-Willis (1994) bivariate probability models or correlated Mare models on each cohort for both males and females and comparing the degree of dependency of educational attainment outcomes on the respondent’s observable family background characteristics across these two cohorts. The approach used here is similar in spirit to Cameron and Heckman (1998) and many other researchers before them who also focused on this issue of dependency or ascription. It can be seen as a generalization of the seminal Mare (1980, 1981) model where the random components in the latent variables which determine each stage outcome are no longer independent. In addition, we also describe educational mobility in terms of conditional probabilities and odds ratios. We are also offering a new mobility index.

There are a large number of studies both within sociology and economics that attempt to relate individual performance as measured by educational attainment, earnings, or occupation to the characteristics of the household in which the individual grew up. These are seen as crucial in determining children’s outcomes as adults, both in the educational system and in the labour market. This literature is reviewed in Mare (1981), Haveman and Wolfe (1995), Dustmann (2004), Black, Devereux, and Salvanes (2005), Shavit, Yaish and Bar-Hakim (2007), McIntosh and Munk (2007), Breen et al. (2009), Marks (2009), Björklund, Lindahl, and Lindquist (2010), and Black and Devereux (2011).

In addition, there is a literature on changes in economic, educational, and social mobility over time and across cohorts including a literature on intergenerational changes in choice of secondary education (see Heath and Clifford (1990) going back to the tradition of Glass (1954)). Recent trends in European stratification and educational mobility are examined by Shavit et al. (2007), Breen et al. (2011). 

The effects of family background variables on educational attainment are examined in this paper. They appear to be smaller for Denmark than Belzil and Hansen (2003) found for US white males in the National Longitudinal Survey of Youth. They attribute 67% of the explained cross-sectional variation in educational attainments to family background variables. Whereas, McIntosh and Munk (2007) found that this was about 50% for Danish respondents.
Jonsson (2007), and Breen et al. (2009). Both studies report some change in educational mobility for some of countries considered. Breen and Jonsson (2007) point out that changes over time in social fluidity are more likely to be cohort-driven than period-driven. Overall Breen et al. 2009 find an overall decline in most of the European On the other hand Wiborg and Hansen (2009) do not find a decline in the importance of family background for selected European countries.

For Canada Wanner (1999) found that relative mobility rates are unchanged but these are reversed by the McIntosh (2010) study which employs statistical techniques to deal with the unobservability problems mentioned above. For the US, Hauser (2010) concluded that ‘there is no global trend in the intergenerational persistence of education from the 1960’s to the 1990’s’. Similar results were found by Lucas (2001:1679) who reported that his results were consistent with ‘maximally maintained inequality’ hypothesis, which he reformulated to the ‘effectively maintained inequality’ hypothesis, underlining the possible persistence in educational inequality at a more refined level. Cameron and Heckman (1998) report the effects of several family background variables on educational attainment. However, these are mixed with the effect of household income showing a slight decline in importance towards the end of their sample period. On the other hand, their parental education variables retain their importance. Mare (2011:240-241) summarizes other US studies going back to 1960s and onwards and responds also to the problem of unmeasured heterogeneity. Most of the US results showing stable patterns or even increasing inequality of educational opportunity except for a few studies reporting some or little change (Shavit, Yaish and, Bar-Haim 2007: 43; see also Morgan et al. 2006).

For France, Vallet (2004: 31) reports a decline over thirteen cohorts over the period 1908-1972 using log-linear models to examine changes in associations between social origin and educational destination. He also notes that ‘the decline in origin-destination education association in France therefore seems largely independent of major secondary school reforms introduced to promote equality of educational opportunity’.
For Italy, Shavit and Westerbeek (1998) found a decrease in educational inequality at the secondary level but not at the university level. For Sweden, in many ways close to Denmark, Jonsson (1993), Erikson and Jonsson (1996), Jonsson and Erikson (2000) tried to show a decline in the social inheritance effect on educational attainment, including low and intermediate transitions (see also Erikson and Rudolphi 2010; Breen et al. 2009).

Esping-Andersen (2004) examined educational mobility in several countries and found that the results depended on the country with increases in Scandinavia and declines in the UK and US. Marks and McMillan (2003) found a decline in the dependence of educational attainment on social background variables for Australia for cohorts born during the period 1961-1985. A later study found declines in both the effects of class background and parental education on having obtained a bachelor degree (Marks and McMillan 2007; see also Marks 2009).

Bynner and Joshi (2002) examined sample survey data from the 1958 and 1970 cohorts in Britain. They found no change in the response of the probability of leaving school at age sixteen to family or social origin variables. Blanden and Gregg (2004) found an increased dependence of tertiary educational attainments on household income over the period 1958 to 1970 using the British National Child Development and British Cohort Surveys. Individuals were aged 33 and 30, respectively. We also find an increase in household income dependence. In another British study Galindo-Rueda and Vignoles (2005: 352) also found an increase in the importance of some family background variables but a decline in the importance of cognitive ability as a determinant of educational attainment. This is in part due to the fact that low ability children from high economic status families experienced the largest increases in educational attainment. They also looked at secondary education, and discovered that ‘the reduction of secondary school selection on the basis of age 11 ability is likely to have reduced the role of early cognitive ability in determining a student’s eventual outcome.’

Blanden, Gregg and Machin (2005) examine educational mobility at both the secondary and tertiary levels and found first a rise and then a decline in cohort
educational inequality at the secondary level but an increase at the tertiary level. They attribute the latter change to increased financing constraints for higher education. However, it should be noted that most of these studies use a rather limited number of family background variables (see also Jackson et al. 2007 for mixed results for Britain).

Most of these studies have focused on final educational attainment. There closest study (change formulation) to our work on upper secondary education is a paper by Dustmann (2004) who uses the German Socioeconomic Panel data base to examine the secondary school outcomes of a sequence of cohorts the first of which was born in 1925 and the last in 1965. He finds using ordered probability models that the probability of completing German high school for respondents with ‘working class’ parents increases moderately over the ten cohorts and is higher for males than females. This is much lower for respondents with ‘academic’ parents whose probabilities also increase with females overtaking males by about 1960. The large gaps between these two probabilities leads him to conclude that considerable educational immobility still exists in Germany (see also Breen et al. 2009).

In summary, results differ by country and sometimes by type of procedure employed. Perhaps the most striking feature of the research on the evolution of intergenerational mobility is the almost uniform neglect of unobservable characteristics. Mare (1993) is an exception, which is also noted by Shavit, Yaish, and Bar-Hakim (2007: 46-47).

Here, we continue in the tradition of Halsey et al. (1980), Shavit (1984), Micklewright (1989), Gamoran and Mare (1989), Erikson and Jonsson (1996), Dustmann (2004), Galindo-Rueda and Vignoles (2007), Marks (2009), Erikson and Rudolphi (2010), Rudolphi (2011) by examining participation in upper secondary education of two cohorts of Danish males and females who were aged 23 in 1985 and 2005, respectively. We also look at participation rates in university at age 23 in 1985 and 2005 for these cohorts to see whether the trends that appeared at the upper secondary level are likely to continue on into post-secondary education.
The paper is organized in the following way. The next section discusses the declining importance of social origin on choices in the Danish Upper Secondary Educational System and also a new mobility index. The next section discusses some methodological problems that arise in educational mobility research. Section 3 reviews the relevant literature on changes in educational mobility. Section 4 outlines the data used in the study. Section 5 describes a new estimation procedure which is based on a generalization of the Mare transition model. The results appear in section 6 and are discussed in section 7. The analysis of upper secondary education is done first. We then briefly discuss what has happened at the tertiary level at the end of this section.

Declining importance of social origin on choices in the Danish Upper Secondary Educational System

Upper secondary education is the place to begin the analysis of educational mobility since, as Table 1 shows, there have been phenomenal changes in the way students have completed their upper secondary schooling. For example, as is shown in this table the proportion of females who had completed a gymnasium qualification by the age of 23 increased from 34.1% in 1985 to 54.9% in 2005 a huge increase for such a short period of time. Upper secondary educational attainments and their dependence on family background variables is an important issue in its own right. For individuals who do not continue in the educational system past upper secondary schooling their life-time earnings are much influenced by their lack of educational qualifications. But, of course, the most important reason for looking at upper secondary educational attainments is because what respondents achieve at this level largely determines what educational opportunities are available to them at the tertiary level, especially for gaining entrance to university.

To summarize our results we find that traditional family background variables continue to explain a significant amount of the variation in upper secondary educational attainments for both males and females. The educational attainment of the respondent’s parents and household income turned out to be the
most important variables. However, the occupations of both parents, the num-
ber of siblings that the respondent had and whether the respondent’s father was
unemployed or had a single mother were also significant explanatory variables.
Consequently, the dependence of upper secondary educational attainments on
the characteristics of the respondent’s family remains a prominent feature of
Danish society. But the degree of this dependence has become relatively less
important over the twenty years under consideration.

1.0.1 A NEW MOBILITY INDEX

We construct a new mobility index, which provides a relative measure of the de-
pendence of educational outcomes on traditional observable family background
variables. This index has increased significantly for both males and females in-
dicating an increase in mobility. Traditional family background variables have
become less important relative to the intercept terms which have increased sig-
nificantly in our statistical models. As is the case for the regression model, these
represent the mean effects of variables which are not included in the model and
increases in them are consistent with an increase in mobility.

Our index also allows us to make comparisons between situations where the
consequences of random effects are different in the two cohorts. This resolves a
long standing problem in the intergenerational mobility literature when compar-
isons across time periods or educational stages are complicated by unobservable
differences in the variances of the error terms. This is an additional contribution
of the paper.

The index that we employ gives results that agree with and help to explain
established notions of intergenerational educational mobility. The conditional
probabilities of going to gymnasium have improved for all respondents regardless
of their father’s level of education. Likewise the odds for going to gymnasium
have also increased for all respondents but respondents whose fathers have no
education have improved their relative position. At the same time, however, our
results show that the absolute values of the regression coefficients associated with
observable family background variables have actually increased on average over
the twenty year time period. This would appear to conflict with the claim that mobility has increased. This apparent paradox is resolved by the application of our new mobility index.

The large increases in the proportion of successful gymnasium completions by age 23 have been matched by an equally impressive increases in the rate of participation at University.

Gender plays an important role in the analysis. Girls have done much better in the [Danish, delete] educational system than boys (Buchman et al. 2008). They continue to do so at the university level; moreover, but their comparative advantage over boys has not increased over the twenty years under consideration. Issues concerning gender are discussed in section 7.

2 Methodological issues

The question that we address in this paper is whether there has been any change between 1985 and 2005 in the dependence of upper secondary educational choices on the family backgrounds of Danish students making them. While the question is quite straightforward providing a comprehensive answer to it is not. To illustrate what the problems are we will consider a number of simple examples beginning with the classical regression model and then go on to models which deal with discrete outcomes.

Suppose that we want to examine the dependence of the continuous variable $y_{it}$ on another continuous variable $x_{it}$ where the subscripts $i$ and $t$ refer to the respondent and the time period, respectively. We will assume that there are two times $t_1$ and $t_2$ with $t_1 < t_2$. Most researchers would examine this type of data in a regression context and estimate the model

$$y_{it} = \alpha_0 + \alpha_1 x_{it} + \epsilon_{it}$$

(1)

for both time periods and then determine whether the dependence of $y$ on $x$ had increased or decreased by looking at the ordinary least squares estimates of $(\alpha_0, \alpha_1)$ for the two time periods. In equation (1) $\epsilon_{it}$ is an error term with a
zero mean and is identically distributed over individuals and independent from
\( x_{it} \).

Dependence will have certainly decreased if the estimate of \( \alpha_1 \) is lower in
the second period. But it will have also decreased if the intercept term, \( \alpha_0 \), is
higher in the second period. The reason for this is because the intercept term
captures the effect on \( y \) of variables which have been omitted from equation
(1). To see why this is the case suppose that the true data generation process
is given by

\[
y_{it} = \alpha_1 x_{it} + \alpha_2 w_{it}
\]

(2)

where \( w_{it} \) is orthogonal to \( x_{it} \) but can not be observed by the researcher\(^2\).

This can be made to look like the standard zero mean error regression model
in equation (1) by letting \( \bar{w}_t \) be the sample mean of \( w_{it} \) and then adding
and subtracting \( \alpha_2 \bar{w}_t \) to and from the right hand side of equation (2) which
makes \( \alpha_0 = \alpha_2 \bar{w}_t \) which is the average effect of the unobservable variables
and \( \epsilon_{it} = \alpha_2 (w_{it} - \bar{w}_t) \) is the effect of the unobservables which are specific to
individual \( i \).

When the intercept term increases dependence has declined relatively in the
sense that \( x \) is less important relative to the omitted variables at \( t_2 \) than it was
at \( t_1 \) although \( \alpha_2 \) has remained constant over the two times.

There is a third case where dependence decreases over the two times and
that is when \( \sigma_t \), the standard deviation of \( \epsilon_{it} \) increases. When this happens
dependence decreases because the importance of random effects on \( y \) have in-
creased over the two time periods. This is also a relative comparison since more
of the variation in \( y \) is being explained by random effects relative to the effects
which are due to \( x \). Björklund and Jäntti (2000) have also noted this in their
analysis of intergenerational income mobility.

The situation becomes more complicated when there are many regressors.
This means that \( \alpha \) and \( X_{it} \) are now vectors with the first element of \( X_{it} \) being
a vector of ones making \( \alpha_0 \) the intercept term as in the case in equation (1).

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\(^2\)Some of the ideas in this section were inspired by the discussion of unobservables in
Cameron and Heckman (1998: 268)
This leads to

\[ y_{it} = \alpha X_{it} + \epsilon_{it} \]  

(3)

It is possible to compare the individual coefficients but often a summary statistic which depends on all of the regression parameters is needed. If an index, whose purpose is for making intertemporal comparisons, is going to be constructed the coefficients upon which the index is based have to reflect the importance of the regressors. Equation (3) does not do this but it can be replaced by

\[ y_{it} = \beta Z_{it} + \epsilon_{it} \]  

(4)

where \( Z_{it} \) is a vector of normalized (zero mean and unit variance) variables. Note that the inferences for \( \beta \) are the same as those for \( \alpha \) and the statistical properties of the model have not changed but now the size of the \( \beta \) coefficients, which are referred to as standardized coefficients in the statistical literature, reflect the importance of the variable\(^3\).

As an index which can be used for comparative purposes we propose  \( (\text{what is } K?) \)

\[ I(\beta) = \frac{|\beta_0|}{\sum_{k=1}^{K} |\beta_k|/K + |\beta_0|} \]  

(5)

Intergenerational educational mobility is about inheritance mechanisms or equivalently about how an individual’s educational success depends on the characteristics of the household in which he or she resided as a child and an adolescent. \( I(\beta) \) is a measure of this dependence and it takes the value zero if the intercept term is equal to zero and only family background variables are important in determining educational outcomes. When \( I(\beta) = 1 \) this describes a society which exhibits perfect mobility. For sociologists interested in mobility, a perfectly mobile society is one in which the odds associated with categories \( i \) and \( j \) in an outcome variable are the same for all social groups which occurs when family background variables are unimportant or when \( I(\beta) = 1 \). We will have more to say on this point in section 7.

This is an improvement on and generalization of an index first proposed by McIntosh (2010). It gives equal weighting to all of the regression coefficients\(^3\). 

\(^3\)On this point see Wooldridge (2006: 196).
once they have been normalized to take account of their importance. Using the average absolute value of the slope coefficients means that the index is not affected by the number of regressors. In section 6 the index will be expanded to take account of random effects in probability models (changes in $\sigma$ over time).

In addition to this mobility index we will also show how the coefficients associated with a particular family characteristic change relative to the intercept terms. These will be represented by subindexes which are similar to that described in equation (5) and reveal what is happening at a more disaggregated level.

The data that researchers have to use to get results in the economics and sociology of education are often not suitable for the application of regression methods. Educational choices are most often described by qualitative measures. In a classic paper Robert Mare (1980) laid the foundations for the analysis of individual progress through the educational system. In Mare type stage models the outcome variable for individual $i$ is a vector $D_i = (D_{i1}, D_{i2}, ..., D_{iS_i})$ which is a sequence of ones followed by a zero for the last component indicating successful completion of first $S_i - 1$ stages but a failure to complete stage $S_i$. Success at stage $s$ is modelled as a random process where success occurs when the latent variable

$$D_{is}^* = \beta_s Z_{is} + \gamma_s \theta_i + \epsilon_{is}$$

is non-negative. $\theta_i$ is the ability or some other unobservable attribute of individual $i$ and $\epsilon_{is}$ is a random effect whose distribution is usually assumed to be logistic with variance $\pi^2/3$. The $\epsilon_{is}$ are assumed to be independent across individuals and stages. When $\theta_i$ can be observed by the researcher the probability of being successful at stage $s$ is

$$\Pr\{D_{is}^* \geq 0\} = \frac{\exp(\beta_s Z_{is} + \gamma_s \theta_i)}{1 + \exp(\beta_s Z_{is} + \gamma_s \theta_i)}$$

These probabilities can be used to construct a likelihood function whose contribution from individual $i$ is
\[ L(D_i) = \prod_{s=1}^{S-1} \Pr\{D_{is}^* \geq 0\} \Pr\{D_{is}^* < 0\} \quad (8) \]

On the other hand when \( \theta_i \) can not be observed serious estimation problems arise. When there is no information on \( \theta_i \) the error term for stage \( s \) becomes \( \gamma_s \theta_i + \epsilon_{is} \). As Lillard and Willis (1994) noted these errors are correlated across stages because of the common dependence on \( \theta_i \) and while the probability of passing or failing the first is based on a univariate distribution, the probabilities involving second and third stage outcomes are described by a bivariate distribution or a trivariate distribution etc. Consequently, the likelihood function in equation (8) is not the correct one for this model and when the correct model is used the estimation of the parameters quickly becomes intractable as the number of stages increases. The correct likelihood function for the three stage model can be found in Lillard and Willis (1994: 1138).

What actually happens when researchers estimate Mare educational stage models and ignore the presence of unobservable factors like ability was first explored by Mare (1993: 365; see also Mare 2011) himself and later in a classic paper by Cameron and Heckman (1998). By simulating a simple stage model with one regressor and the unobservable \( \theta_i \) and \( \beta_s = 1 \) for all \( s \) Cameron and Heckman were able to show that the estimates, \( \hat{\beta}_s \), declined as the stage increased, although the true values of \( \beta_s \) were all equal to 1. The cause of this is what they refer to as dynamic selection bias. Even if \( \theta_i \) is normalized to have a zero mean and unit variance for the population that starts at the beginning of the simulation the successive application of the passing criterion to each stage will select on the more able as the less able fail to succeed to higher stages. After the first stage mean ability of those who remain in the system will be greater than zero and this will increase with each stage. In order to accommodate the increasingly positive mean of \( (\gamma_s \theta_i + \epsilon_{is}) \), \( \hat{\beta}_s \) will have to be smaller than the true parameter value and it will decline as \( s \) increases.

From these simulations Cameron and Heckman concluded that the declining ascription result that so many sociologist had found was based on a statistical
flaw in the model that was being used. The major claim that many sociologists have made is that the respondent’s dependence on family background variables (particularly, parental levels of education) on academic success diminishes the further the respondent advances in the schooling system. This may or may not be correct but when it is shown that this is what will happen when there are inadequate controls for the effects of unobservable variables the only alternative is to start afresh and develop statistical procedures which can deal with the problems that arise with this type of data.

They also claimed that the Mare model stage model was not non-parametrically identified unless there were some different variables for each stage. We have found using simulation methods that this is not the case when there are more variables than stages\(^4\).

The Danish data that will be used in this paper exhibit some of the complexities mentioned in this section. There are characteristics of Danish adolescents which are not observable to us. But we want to draw valid inferences about what matters in the decision making processes of our respondents and whether this has changed over time. Our statistical model has the same mathematical structure as the Mare model outlined above so that dealing with unobservables is a problem for us as well.

### 3 Data and variables

The data used in this study comes from Statistics Denmark register data on two cohorts of 23-olds. The two cohorts were born in 1962 and 1982, respectively. Danish registers are very comprehensive and contain a great deal of information on every individual\(^5\). Everyone who was aged 23 and was born in Denmark was included in the sample but parental background variables were collected when the respondents were aged 16. These registers contain the central population register numbers for the parents of each individual. Hence, for each cohort it

\(^4\)Anyone interested in these simulation results can obtain them from the authors upon request.

\(^5\)The construction of the dataset was built on ideas from an earlier research project about Education and Inequality (see Munk 2003b).
is possible to assemble a data set which contains personal information on the individual as well as a set of variables relating to his or her family background. This was done for each cohort. Register data for individuals born prior to 1962 is not as comprehensive and is characterized by large numbers of missing values for parental information so 1982 is the earliest cohort that could be used\textsuperscript{6}.  

For the dependent variable our choice is the three category variable: completed level of secondary education at age 23. In Denmark all students are compelled to complete primary education which is grade nine; but about 60\% go on to grade ten. After grade nine or ten there are two further educational choices at the upper secondary level in addition not continuing at all. The individual can elect to enroll in a vocational programme. Welding, carpentry, hair dressing, or being an electrician are typical options. Vocational programmes can take quite a long time to complete and involve apprenticeships. The entire programme can take up to five years to complete.

Gymnasiums, the other option for those to continue at the upper secondary level, offer four types of curricula: a general program with various theoretical programmes in the humanities, natural and social sciences, a technical program, a business program, and a preparatory programme for university. After grade nine or ten students can enroll in these programmes which typically last about three years and provide qualifications that are required for entrance to a university. Many programmes involving short or intermediate tertiary educational programmes also require a completed gymnasium certificate for entry. In the data set here an individual is in the designated category if the individual had completed the programme associated with it.

The numbers and percentage allocations for each cohort are displayed in Table 1. There are number of variables for parental characteristics. Parents education is a six category variable where the first category is no education past grade nine or ten. The second category is a vocational qualification, the third is gymnasium only and the next are three categories of tertiary education which

\textsuperscript{6}In what follows we refer to a cohort by the year when the respondent was aged 23 and not the year of birth.
in Denmark are characterized by their durations: short, medium and long, and the varying level of academic content in them. Examples for the three types are police training, primary school teacher training and university, respectively. The residual category is no education past grade nine or ten. In some of the analysis of the categories are grouped together when sample sizes are too small. There are eight parental occupations; the first three are white collar occupations starting with high level managerial, low level managerial and ordinary employee. Occupations four, five, and six are self-employed and skilled and unskilled blue collar workers and occupation seven is the missing category. For the first cohort there are many parents whose occupations are not known and it does not seem appropriate to combine them with the unemployed so they are represented a separate category for all of the cohorts although there is very little missing parental information for the last cohort. The residual category consists of those who are unemployed or not in the labour force.

The data set also contains the number of siblings, whether the father was unemployed, whether the respondent’s mother was a single mother, and household income. These were collected at various ages depending on the particular variable.

Table 4 contains information on education participation rates for the same two cohorts. Unlike the data in Table 1 these are not completions but simply enrollments in university education programmes. Parents with missing educational information were included at the estimation stage but sometimes excluded in later calculations where their inclusion would have lead to misleading results had they been used in the construction of Tables 4 to 6.

4 Estimation methods and models

The three choices open to Danish students who have completed lower secondary education can be achieved by making two decisions. The first decision is whether to be involved in any type of upper secondary education at all and the second is, given the decision to participate in further upper secondary education, whether
the student will attend a gymnasium or take a more practically oriented vocational type of training. This is, of course, a purely conceptual framework for analyzing these three decisions but we see this as a realistic characterization of this decision process and as will be shown later in section 6 this way of modelling these decisions is statistically superior to other alternatives which could be used to explain the data.

To go further in the educational system first requires a desire or some motivation to make the sacrifices necessary to turn an educational programme into a successful educational qualification. Success in this endeavor also requires some ability. Attitudes, desires and ambitions are in part determined by the environment in which the student resided when these decisions are made. So let us suppose that there is an index which depends on these environmental variables which represents the intensity of the desire to make a success of oneself. Let this be

\[ y_{iI} = \beta_0I + Z_{iI}\beta_I + \gamma_I\theta_i + \epsilon_{iI} \]  

where \( Z_{iI} \) is a set of normalized family background and other variables which describe the environment of the respondent and \( (\gamma_I\theta_i + \epsilon_{iI}) \) is an error term with two components. \( \theta_i \) is an individual specific random effect which could represent the respondent’s ability or some other unobservable attribute and \( \epsilon_{iI} \) measures other random effects that are important in the first decision. The sum of these two components will be assumed to be normally distributed with a variance \( \sigma^2_I \). The value of this latent variable will determine how the respondent makes his or her first choice. In keeping with traditions associated with latent variable models we will assume that the respondent decides to pursue some additional education past grade nine or ten if \( y_{iI} \geq 0 \).

If the respondent decides to continue in the educational system the decision about which type of upper secondary educational stream to pursue has to be made. We assume that this decision is also determined by the value of another latent variable

\[ y_{III} = \beta_{0II} + Z_{III}\beta_{II} + \gamma_{II}\theta_i + \epsilon_{III} \]  

\[ 16 \]
Here $y_{II}^*$ represents a number of attributes including preferences for occupations which depend more on academic and intellectual skills, analytical ability, and time preferences which can accommodate the longer durations of programmes of which attending a gymnasium is the first step. Individual $i$ will take the gymnasium option if $y_{II}^* \geq 0$ and the choice will be to go the a vocational school if $y_{II}^* < 0$. Like $y_{I}^*$, this will depend on the respondent’s environmental background and some random effects, but perhaps not in exactly the same way so that $\beta_I$ and $\beta_{II}$ will not be the same.

This two stage representation of the decision process reflects some of the realities of the way Danish students actually made their decisions. At the time the data was generated students who wanted to go on to a gymnasium were evaluated by their lower secondary school teachers who then made a recommendation to the gymnasium on their suitability for this option. (Stoceke 2007).

These latent variables generate the probabilities of the three decisions. The probability that respondent $i$ will go no further in the educational system is

$$
\Pr\{y_{II}^* \leq 0\} = \Phi(-Z_{ii}\beta_I/\sigma_I)
$$

where $\Phi()$ is the cumulative normal distribution function.

The probability the respondent $i$ takes a vocational course after completing grade 9 or 10 is more complicated because the error terms are not independent. In keeping with the assumption that both marginal distributions are normal we also assume that the joint distribution of $\{(\gamma_I \theta_i + \epsilon_{ii}), (\gamma_{II} \theta_i + \epsilon_{II})\}$ is bivariate normal with correlation coefficient $\rho$ as Lillard and Willis did. Here the reason for our use of normal (probit) marginal distributions rather than the more usual logistic (logit) model is because there is no suitable bivariate logistic distribution that could be used. Additionally, there are no bivariate distributions which are substantially different from the bivariate normal distribution and have, as well, the unrestricted correlation structure that the bivariate normal distribution has.

It is important to note that $\theta_i$ has a different parameter associated with it in each error term. This is required to allow for the possibility for a negative value of $\rho$ which can arise only when the signs of $\gamma_I$ and $\gamma_{II}$ are different. What
\( \theta_i \) actually could represent will depend on the sign of \( \rho \). This issue is discussed at the beginning of section 7.

The vocational choice occurs when \( y_{ii}^* \geq 0 \) and \( y_{ii}^* < 0 \). This probability is

\[
\Pr\{y_{ii}^* \geq 0, y_{ii}^* < 0\} = \int_{-\infty}^{\infty} \int_{-\infty}^{-\infty} \phi(z_1, z_{II}, \sigma_1, \sigma_{II}, \rho) \, dz_1 \, dz_{II} \quad (12)
\]

where \( \phi() \) is the bivariate normal density function. Likewise, going to a gymnasium occurs when \( y_{ii}^* \geq 0 \) and \( y_{ii}^* \geq 0 \), the probability of which is

\[
\Pr\{y_{ii}^* \geq 0, y_{ii}^* \geq 0\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(z_1, z_{II}, \sigma_1, \sigma_{II}, \rho) \, dz_1 \, dz_{II} \quad (13)
\]

As we have already mentioned, this model has the same formal structure as the Mare (1980) grade transition model. Although the stages here refer to a sequence of decisions rather than a sequence of grade transitions through the educational system the models share the same mathematical structure. The possibility of allowing for unobserved heterogeneity in these stage models by including the random effect, \( \theta_i \), in each error term was first proposed by Lillard and Willis (1994). Consequently our model is analogous to their model but it is also a correlated version of the Mare model.

It is interesting to note that Lucas et al. (2011) have recently discovered what Lillard and Willis proposed almost twenty years ago. Their model, like ours, is the Lillard-Willis model or as we have also described it: the correlated Mare stagemodel. Like us they also use standardized zero mean unit variance regressors in their analysis. Since the objective of this research is to compare the degree of educational mobility for two cohorts born twenty years apart an aggregate mobility index which measures this is required. The index that we are going to use is a version of the one proposed in section 2, which for probability models, become \( I(\beta/\sigma) \) and its purpose is to measure the importance of observable family background variables relative to all of the other variables which could affect respondent academic choices. This index involves all of the
parameters as well as the variance of the latent variable. Only parameters divided by the standard deviation of the error term, $\sigma_t$, are identifiable in probability models. In addition, the properties of the index mentioned in section 2 has another feature of the index which makes desirable for our purposes is that it does not depend on $\sigma_t$ (since they cancel out) so that comparisons across cohorts are not going to be contaminated by changes in the variance terms. This has been a major problem in much of the research on intergenerational educational mobility (including studies that rely on logistic models) and to our knowledge this is the first real solution to this problem.

On the other hand, there is nothing in the index which allows us to say anything about the relative importance of random effects and how these have changed over the two cohorts. In addition it is also possible for some of the omitted variables whose effects are being picked up by the intercept terms to represent variables associated with the household in which the respondent grew up. However, they must be the same for all households, hence our emphasis on the term ‘observable family background variables’. As shown in section 7 this possibility presents no problems for our index.

5 Results

The results for the model outlined in section 5 are shown in Table 2. The first and most important point to note is that all four of the mobility indices increase as the cohorts get younger. It is clear from the first two rows of this table that for males, the increases in the two indexes are $\Delta I(\beta_1/\sigma_1) = 0.092^{\ast\ast}(0.003)$ and $\Delta I(\beta_{11}/\sigma_{11}) = 0.349^{\ast\ast}(0.131)$. For females these increases are $\Delta I(\beta_1/\sigma_1) = 0.123^{\ast\ast}(0.004)$ and $\Delta I(\beta_{11}/\sigma_{11}) = 0.055^{\ast\ast}(0.011)$. For both males and females observable family background variables have, on average, become less important relative to the intercept terms in the determination of educational choices for both of the latent variable indexes. There also have been dramatic increases in the intercept terms so that models predict significant increases in the probabilities of the gymnasium choice for all respondents regardless of their socio-
conomic background. In particular, the increases in these intercept terms have lead to larger increases in the conditional probabilities of going to a gymnasium for respondents coming from disadvantaged households.

It is interesting to note that the responses by the males and females are different. For the males it is the preference for type of education that has exhibited the largest change, whereas, for the females it is the ambition variable.

In Table 7 sub-indexes are displayed for each type of family background variable. These are the same type of measure that is used in the index except only the coefficients associated with a particular attribute are used. They provide additional information about how the relative importance of specific family background variables have changed over the two time periods. For example, for males the sub-index for father’s occupation for the second latent variable rises significantly from 0.694** (0.117) to 0.898** (0.006). Of course, not all of the sub-indexes behave in this way. There are no significant changes in many of them but for males income has become more important both relatively and absolutely for both of the latent variables.

All of the variables representing the characteristics of the respondent’s parents are highly significant for both cohorts. Household income, the number of siblings and whether the respondent was brought up by a single mother have somewhat larger coefficients associated with them than the other regressors. In Table 2 the effects parent education and occupation are captured by the averages of the coefficients of the category dummy variables in order to keep the tables from being too large. For example, for males the estimate for the average effect of father’s education is 0.049** (0.004)\(^7\).

Changes in the individual parameter estimates across the two cohorts are not particularly large for some of the regressors but there are some major differences across the two cohorts and some of the parameter estimates, like those

\(^7\)For father’s education in 1985 the 6 \(\beta_I\) coefficients and their standard errors are 0.050** (0.006), 0.064** (0.006), 0.034** (0.006), 0.009** (0.004), 0.099** (0.006), and 0.037** (0.004). The average of these 6 coefficients is 0.049** (0.004) as reported in table 2.

Readers interested in the detailed coefficients associated with the other variables can obtain them from the authors.
associated with parent educational attainments or household income, are actually significantly larger for the younger cohort. On average the absolute values of the slope coefficients have increased over the period. However, the reason why the mobility indices sometimes increase is because the intercept terms are much larger for the 2005 cohorts. These are displayed in the last two rows of Table 2. Mobility indexes and averages of absolute coefficient can increase simultaneously when the increases in the intercept terms are larger than those associated with the coefficients. There are other consequences of the increases in the intercept terms and these will be discussed later in this section.

Our approach to mobility is to examine the dependence of educational outcomes on observable environmental and family background variables. However, this is not how other researchers look at intergenerational educational mobility so it is of some interest to contrast what we have found with the results of more traditional procedures like those used by sociologists interested in mobility.

In Table 3 the conditional probabilities of the three upper secondary educational outcomes are compared across the three categories of father’s education.8 Mobility researchers are interested in how these probabilities or ratios of these probabilities change over time. For both genders the conditional probability of getting a gymnasium qualification was higher in 2005 than in 1985 for all types of father. But for all males the conditional probability of getting no upper secondary education was also higher in 2005.

In the analysis of mobility it is important to distinguish the difference between absolute and relative changes in mobility; but in our view, both are important. There is a huge literature which distinguishes between absolute and relative mobility. See Beller (2009) for a discussion of these two concepts. Current sociological mobility analysis focuses more on relative changes. Unlike the absolute measures mentioned above, the odds of graduating from a gymnasium as opposed to not participating in upper secondary education is a relative

8Fathers education was chosen because it was slightly more significant than mother’s education. However, similar results hold when the education of the respondent’s mother is used. Conditional rather than cell probabilities are used since they do not depend on the margins as represented by the row probabilities.
measure. These are higher for all educational categories of the father for both males and females. Furthermore, the odds of getting a gymnasium qualification as opposed to getting no further education have improved more for respondents whose father had no education than they did for those respondents whose father had a gymnasium qualification. Using the data in Tables 3 and 5 it is possible to see exactly what this means. Note that for 1985 the odds of a male respondent getting a gymnasium qualification as opposed to no further education given that his father had no education is \(0.063/0.182 = 0.346\). The same odds for a male respondent whose father had at least a gymnasium qualification is \(0.083/0.022 = 3.772\). Forming the odds ratio \((3.772/0.346 = 10.902)\) gives a measure of relative performance which can be compared with other time periods. This odds ratio falls to 6.564 in 2005 indicating that the relative chances of getting a gymnasium qualifications opposed to no education at all have moved in favour of respondents whose father had no education. Similar results hold for females. Sociological mobility theorists measure relative mobility by these odds ratios.

Thus Danish society has not only become more mobile in terms of relative odds but it is better off since a much larger proportion of individuals is going further in the educational system than was the case in 1985. That the proportion males with a gymnasium qualification has increased from 24.0% in 1985 to 40.6% in 2005, a 69.2% increase, is quite remarkable for the rather short time span under consideration. The only cloud in the Danish educational sky is the increase in the male conditional probabilities of getting no secondary education at all.

Our models provide some insight as to why these events have occurred. The reasons why these probabilities of getting a gymnasium qualification have increased so dramatically over this period is because of the large absolute increases in the intercept terms, \(\beta_{0I}/\sigma_I\) and \(\beta_{0II}/\sigma_{II}\). When these increase relative to the regression coefficients the mobility indices \(I(\beta_I)\) and \(I(\beta_{II})\) increase indicating an increase in mobility. But when they increase absolutely, increased mobility

\footnote{These results are based on the sample where there is no missing information on fathers education. They are, however, to results based on the data in Table 1.}
is accompanied by lower drop out (non-participation) probabilities, higher success rates at the gymnasium level, or both. Later we will see that our models help explain why males and females have such different success rates. It is also clear that mobility can increase even when the regression coefficients increase because what is important is their size relative to the intercept terms. Thus our index can be useful in reconciling these apparently contradictory or paradoxical results that occur when increases in mobility as measured by increases in odds and conditional probabilities are accompanied by increases in the size of the regression coefficients.

Turning now to some statistical issues, as we mentioned earlier, our model is one of several alternatives that could be applied to this data but that it was preferred to other possible alternatives on statistical grounds. The three educational categories could be explained by a single index logistic ordered probability model which is generated by the latent variable crossing two threshold points\textsuperscript{10}. The two index multinomial logit model can also be used to explain these choices. The values of the maximized ln-likelihood function and the (number of parameters) for 2005 males for the ordered logit, multinomial logit, Mare and correlated Mare model are, respectively, -27074.284 (31), -26469.166 (60), -26437.273 (57) and -26427.217 (58). The multinomial logit model dominates the ordered logit model in terms of both AIC and BIC criteria. A Vuong (1989: 320) test indicates that the Mare model has a significantly higher ln-likelihood function than the multinomial logit model, and the correlated Mare model is superior to the Mare model when $\rho$ is significantly different from zero. Similar results hold for females, hence the correlated Mare or Lillard-Willis model is the vehicle of choice for the data used in this study. It should also be pointed out that all of these models are parametrically identified in the sense of Bowden (1973).

\textsuperscript{10}We also tried to estimate latent class (mixture) ordered probability models but serious convergence difficulties were encountered.
6 Discussion and Conclusions

We have stressed the importance of including a random effect, $\theta_i$, for individual $i$ in the model; now, given the actual estimates, we can provide some insight as to what this effect really measures. Usually, in educational attainment models the unobservable variable, $\theta_i$, is viewed as some measure of ability. This could be the case here but because, as shown in Table 2, the estimated correlations between the two error terms are negative, it is likely that there are components in $\theta_i$ that represent other attributes of the respondent which dominate the effects of unobserved ability on educational choices. This variable could include, for example, individual preferences for doing something practical as opposed to something esoteric or theoretical or preferences for programmes which take less time to complete than those that are available with a gymnasium qualification. If $\theta_i$ actually represents preferences for practical or short duration programmes then individuals with high values of $\theta_i$ will want to make something of themselves and be successful so that $\gamma_I > 0$, but they also want fulfillment in a practical sense and do not want to be involved with abstract or theoretical issues which they would have to deal with if they went to a gymnasium so that $\gamma_{II} < 0$. As a result they take the vocational option. If this is what $\theta_i$ represents then $\rho$ will be negative. On the other hand if $\theta_i$ just represented unobservable ability or ambition then then $\gamma_I$ would be positive. In addition to wanting to get some some upper secondary education he or she would also be likely to want to go to a gymnasium so that $\gamma_{II}$ would also be positive. But then $\rho$ would be positive contradicting the results in Table 2 where the estimated values of $\rho$ are negative. This, of course, does not mean that ability is unimportant in educational decisions; it is just not well represented by $\theta_i$. It is always an open question as to what unobservables represent. We think the example above is plausible but there are many other possibilities.

Turning to more general issues, the main result of this research is that mobility has improved because other factors have replaced the more traditional

\footnote{Lucas et al (2011) also find significant negative correlations in their stage model for some cohorts.}
variables that represent the characteristics of the households in which the respondents grew up in explaining Danish upper secondary schooling outcomes. And, as we showed, these new factors that have come to play such an important role in educational success are captured by the intercept terms in our probability models. Since these intercept terms, by definition, are picking up the average effects of the variables that have been omitted from the statistical models, knowing what they actually represent is crucial in understanding the causes of mobility change in Denmark.

There are many plausible factors. A short list would probably include the following items. Recent changes to the gymnasium system like the introduction of business and technical options have given students more choice. These can very well have made the gymnasium choice more attractive to members of the younger cohort. In addition, there has been a change in attitudes or perceptions about the value of going to a gymnasium in terms of the options it gives to attendees for acquiring tertiary education as noted by Andersen (2004: 60-61). The abandonment of tracking in lower secondary education which occurs when students are allocated to streams based on potential, may also be part of the reason why the cohorts who went through the school system after 1980 were able to be more successful\textsuperscript{12}.

Social programmes including welfare support and unemployment insurance programmes which had been established prior to the 1990’s lead to major reductions in poverty. These have been financed by high tax rates which generated a high level of living standards for the average Dane. After 1993 there were considerable changes in Danish social policy and there were some new policies that could have affected educational decisions. Reduced entitlements to welfare programmes (Rosdahl 2003: 123) and the tying of benefits to schooling decisions made the costs of not getting more education much higher (Munk 2001: 94, 2003a). Esping-Andersen (2004: 131) has suggested, somewhat controversially, that the cohorts who were making educational choices in 1990’s were

\textsuperscript{12}There is a very large literature on tracking. See Hanushek and Wössman (2006) for the effects of this for a sample of European countries.
the first to fully benefit from the expansion of the day-care programmes at the end of the 1970’s and thus were in a better position to participate is secondary education. Yet another plausible explanation is that children benefits from increasingly better educated mothers (Mare and Maralani 2006). Finally, it could be that non-cognitive or personal traits have become more important Bowles et al. (2001), Borghans et al. (2008), Diprete and Jennings 2012.

As we noted earlier it is possible that some of these omitted variables are also family background variables. However, this should not affect how our index should be interpreted because it is observable variables like parent education, income, or occupation that are the focus of interest in mobility research. Suppose, for example, in the determination of respondent educational success, what parents do for their children has become much more important than who the parents are, as represented by their characteristics\textsuperscript{13}. Any mobility calculation, like our index or computing odds or odds ratios from mobility tables, which compare parent and respondent educational attainments will show an increase in mobility because respondent educational outcomes have become less dependent on parent characteristics. Unfortunately, the reason why mobility has increased will not be apparent to the researcher if it caused by changes in variables which are not observable. Nonetheless, the extent to which respondents inherit similar characteristics to those of their parents is still an issue of great interest to both the research community and to civil servants and politicians charged with the responsibility of implementing and evaluating social and educational policies, a position which has been recently emphasized by Heckman (2008).

While there have been dramatic increases in gymnasium participation should they encourage us to believe that similar results will eventually be found at the university level? Looking at Table 4 the percentage of males who had started a university education programme rises from 8.6% in 1985 to 17.1% in

\textsuperscript{13}Continuing with this example, Danish parents on average appear to be doing more for their children now than was the case twenty years ago so this variable can also be seen as part of an accepted social norm rather than being a variable that is exclusively associated with the family. This point is also addressed by Breen and Jonsson (2007) underlining a universalistic norm. See also Mare and Maralani (2006) and their work.
2005. This is truly a remarkable change and an even larger change holds for females. As Table 4 shows these gains have been generated by the participation of respondents regardless of their father’s educational attainment. Conditional probabilities are higher in 2005 for both genders for all types of father than they were in 1985 as are the odds. Odds ratios have also moved in favour of respondents whose fathers had no education for both genders. Thus the results that were found for upper secondary school are repeated at the university level for the sample of respondents who obtained a gymnasium qualification.

Of course, this result should be interpreted with some caution since some of the attendees will drop out and there will be entrants at ages above 23 who will eventually enroll in university educational programmes. However, these changes in attendance behaviour patterns appear to be of fairly recent origin so a comprehensive analysis of university completion rates will have to be deferred until these cohorts have had the opportunity to complete the programmes they have started.

One of the more important variables which matters in educational choices is household income. Like Blanden and Gregg (2004) and Blanden, Gregg and MacMillan (2011) we also find highly significant parameter estimates associated with this variable. This is consistent with our results using sample survey data in McIntosh and Munk (2007). In Denmark, like Britain, the effect of household income on choices is very important and has actually increased over the period. We interpreted high household incomes as proxies for parental competence rather than something which eases credit constraints since secondary education is free in Denmark. However, it is possible that higher income households are able to provide more of the things for their children that matter in the human capital accumulation process like access to personal computers, reading materials in the home, choice of high quality day-care, living in a good neighbourhood etc. But, whatever the causal mechanism happens to be household income has a dramatically increased impact on upper secondary school outcomes.

However, it should not be forgotten that in spite of this increase in the importance of household income; when all factors are considered the dependence
of secondary educational choices on observable family background variables has actually declined relative to other factors over the period. It would be interesting to see whether this is what has happened in Britain, and other countries where household income is important, when a full selection of family background variables is used in addition to household income.

Finally, there are some major results concerning gender. First, females appear to have experienced the biggest change in the rates of completing gymnasium. Our models provide some insight here. The first latent variable is an indicator of overall enthusiasm, ambition, or a desire for self-betterment; the second indicates preferences for the gymnasium option. There was a significant increase in one of the indexes for both genders: gymnasium preference for males and enthusiasm or ambition for females. The increases in the intercept terms were also much larger for females than for males which explains why female respondents from educationally disadvantaged households were more successful than males with similar backgrounds.

Secondly, the trends in dropping out of the school system are different for the two genders. As can be seen from the row and column sums in Table 3, fathers with no education declined from 46.7% in 1985 to 26.7% in 2005 but this process has come to a halt for the next generation since the dropout rate for sons is 29.5% in 1985 and only slightly lower in 2005 at 28.5%. For females there is a similar large decline in the drop out rate for the fathers of daughters in 1985. But this decline is still continuing for females since the dropout rate for females declines from 34.4% in 1985 to 23.1% in 2005.

One of the more interesting features of Table 3 is the simultaneous increase in the conditional probabilities of both getting no upper secondary education as well as getting a gymnasium qualification for males. This suggest to us that there are conflicting influences which determine how sixteen year old boys make this important decision. Since this phenomenon involves all boys regardless of their fathers education it is probably more related to broader issues than those associated with disadvantage. Unfortunately, the data in our sample provides no information on this topic. However, it is important and deserves further
examination.

At the university level females continue to outperform males but the performance gap is much less than it was at the upper secondary school level. Gymnasium is an entry requirement for most university education programmes. There is a much higher proportion of females with a gymnasium qualification, 57.6% as opposed to 40.6% for males. But 42.1% of males with a gymnasium qualification start university whereas only 33.7% percent of women with a gymnasium qualification start university. Thus, female educational advantage is still present but decreases as respondents advance through the educational system. As is the case in many countries\textsuperscript{14}, the gender revolution continues.

7 Acknowledgment

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\textsuperscript{14}Denmark is not the only country to experience this. In the United States, Buchmann, DiPrete and McDaniel (2008: 326) report women earn more university degrees than men. Women also earn their degrees more quickly, and they reported that 66% of women who enrolled in college in 1995–1996 had completed a bachelor’s degree by 2001, compared with only 59% of men. The Danish experience, in spite of the differences in social structure and welfare system, is similar to that of the United states (see also Mare and Maralani 2006).
References


[43] Mare 2011: x 2 demo RSSM


### Table 1
The Distribution of Completed Upper Secondary Educational Attainments (Proportions) by Cohort.

<table>
<thead>
<tr>
<th>Type of Education</th>
<th>Males 1985</th>
<th>Females 1985</th>
<th>Males 2005</th>
<th>Females 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gymnasium</td>
<td>8914 (0.233)</td>
<td>10792 (0.388)</td>
<td>12587 (0.341)</td>
<td>14685 (0.549)</td>
</tr>
<tr>
<td>Vocational</td>
<td>16959 (0.442)</td>
<td>8144 (0.292)</td>
<td>10681 (0.289)</td>
<td>5012 (0.187)</td>
</tr>
<tr>
<td>None</td>
<td>12257 (0.320)</td>
<td>8793 (0.316)</td>
<td>13481 (0.365)</td>
<td>6956 (0.260)</td>
</tr>
<tr>
<td>Total</td>
<td>38130 (1.000)</td>
<td>27737 (1.000)</td>
<td>36749 (1.000)</td>
<td>26653 (1.000)</td>
</tr>
</tbody>
</table>

### Table 2
Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1(β₁₁)</td>
<td>0.894**(0.003)</td>
<td>0.986**(0.001)</td>
<td>0.861**(0.004)</td>
<td>0.984**(0.001)</td>
</tr>
<tr>
<td>2(β₁₂₁)</td>
<td>0.636**(0.129)</td>
<td>0.985**(0.001)</td>
<td>0.926**(0.010)</td>
<td>0.961**(0.001)</td>
</tr>
<tr>
<td>3(β₁₃₁)</td>
<td>0.076**(0.002)</td>
<td>0.064**(0.002)</td>
<td>0.061**(0.002)</td>
<td>0.066**(0.002)</td>
</tr>
<tr>
<td>4(β₁₄₁)</td>
<td>0.045**(0.003)</td>
<td>0.062**(0.003)</td>
<td>0.038**(0.003)</td>
<td>0.074**(0.006)</td>
</tr>
<tr>
<td>5(β₁₅₁)</td>
<td>-0.738**(0.001)</td>
<td>-0.707**(0.062)</td>
<td>-0.785**(0.057)</td>
<td>-0.306**(0.187)</td>
</tr>
<tr>
<td>Father’s Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6(β₁₆₁)</td>
<td>0.049**(0.004)</td>
<td>0.057**(0.003)</td>
<td>0.035**(0.004)</td>
<td>0.065**(0.007)</td>
</tr>
<tr>
<td>7(β₁₈₁)</td>
<td>0.124**(0.009)</td>
<td>0.149**(0.011)</td>
<td>0.078**(0.008)</td>
<td>0.147**(0.013)</td>
</tr>
<tr>
<td>8(β₁₉₁)</td>
<td>0.064**(0.007)</td>
<td>0.079**(0.006)</td>
<td>0.032**(0.007)</td>
<td>0.059**(0.010)</td>
</tr>
<tr>
<td>9(β₁₁₀)</td>
<td>0.038**(0.002)</td>
<td>0.044**(0.003)</td>
<td>0.048**(0.004)</td>
<td>0.038**(0.006)</td>
</tr>
<tr>
<td>10(β₁₁₁)</td>
<td>0.991**(0.004)</td>
<td>0.117**(0.006)</td>
<td>0.083**(0.005)</td>
<td>0.169**(0.008)</td>
</tr>
<tr>
<td>11(β₁₁₂)</td>
<td>0.038**(0.002)</td>
<td>0.055**(0.003)</td>
<td>0.033**(0.004)</td>
<td>0.063**(0.007)</td>
</tr>
<tr>
<td>12(β₁₁₃)</td>
<td>0.091**(0.004)</td>
<td>0.054**(0.006)</td>
<td>0.010**(0.004)</td>
<td>0.059**(0.006)</td>
</tr>
<tr>
<td>13(β₁₁₄)</td>
<td>-0.148**(0.006)</td>
<td>-0.086**(0.009)</td>
<td>-0.129**(0.006)</td>
<td>-0.093**(0.010)</td>
</tr>
<tr>
<td>14(β₁₁₅)</td>
<td>-0.018 (0.011)</td>
<td>0.006 (0.010)</td>
<td>-0.040**(0.008)</td>
<td>-0.024**(0.012)</td>
</tr>
<tr>
<td>15(β₁₁₆)</td>
<td>0.126**(0.007)</td>
<td>0.218**(0.014)</td>
<td>0.112**(0.009)</td>
<td>0.193**(0.016)</td>
</tr>
<tr>
<td>16(β₁₁₇)</td>
<td>0.058**(0.007)</td>
<td>0.027**(0.011)</td>
<td>0.052**(0.009)</td>
<td>0.122**(0.022)</td>
</tr>
<tr>
<td>17(β₁₁₈)</td>
<td>-0.041**(0.006)</td>
<td>-0.045**(0.008)</td>
<td>-0.037**(0.006)</td>
<td>-0.035**(0.009)</td>
</tr>
<tr>
<td>18(β₁₁₉)</td>
<td>0.019*(0.008)</td>
<td>0.049**(0.011)</td>
<td>0.013**(0.011)</td>
<td>0.002 (0.002)</td>
</tr>
<tr>
<td>Father Unemployed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19(β₁₂₁)</td>
<td>-0.116**(0.006)</td>
<td>-0.117**(0.008)</td>
<td>-0.087**(0.006)</td>
<td>-0.137**(0.009)</td>
</tr>
<tr>
<td>20(β₁₂₂)</td>
<td>0.031**(0.009)</td>
<td>0.045**(0.010)</td>
<td>-0.047**(0.008)</td>
<td>0.035**(0.014)</td>
</tr>
<tr>
<td>Intercept terms</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>21(β₀₁₁)</td>
<td>0.507**(0.007)</td>
<td>0.534**(0.008)</td>
<td>0.379**(0.007)</td>
<td>0.721**(0.009)</td>
</tr>
<tr>
<td>22(β₀₁₂)</td>
<td>-0.079**(0.034)</td>
<td>0.475**(0.040)</td>
<td>0.473**(0.020)</td>
<td>0.809**(0.067)</td>
</tr>
</tbody>
</table>

*, ** indicate significant at 10, 5, and 1 percent levels, respectively.
### Table 3

**Cell Probabilities and Conditional Cell Probabilities For Respondent’s Upper Secondary Education by Father’s Educational Attainment, 1985 and 2005.**

<table>
<thead>
<tr>
<th>Father’s Education</th>
<th>1985 Males</th>
<th></th>
<th>2005 Males</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
<td>Voc.</td>
<td>Gym.</td>
<td>Row Sum</td>
<td>None</td>
</tr>
<tr>
<td>None</td>
<td>0.182</td>
<td>0.222</td>
<td>0.063</td>
<td>0.467</td>
<td>0.110</td>
</tr>
<tr>
<td>Vocational</td>
<td>0.091</td>
<td>0.200</td>
<td>0.094</td>
<td>0.385</td>
<td>0.126</td>
</tr>
<tr>
<td>Gymnasium+</td>
<td>0.022</td>
<td>0.042</td>
<td>0.083</td>
<td>0.147</td>
<td>0.049</td>
</tr>
<tr>
<td>Column Sum</td>
<td>0.295</td>
<td>0.464</td>
<td>0.240</td>
<td>1.0</td>
<td>0.285</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Father’s Education</th>
<th>1985 Females</th>
<th></th>
<th>2005 Females</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
<td>Voc.</td>
<td>Gym.</td>
<td>Row Sum</td>
<td>None</td>
</tr>
<tr>
<td>None</td>
<td>0.201</td>
<td>0.151</td>
<td>0.120</td>
<td>0.472</td>
<td>0.095</td>
</tr>
<tr>
<td>Vocational</td>
<td>0.119</td>
<td>0.126</td>
<td>0.139</td>
<td>0.384</td>
<td>0.101</td>
</tr>
<tr>
<td>Gymnasium+</td>
<td>0.024</td>
<td>0.025</td>
<td>0.095</td>
<td>0.144</td>
<td>0.035</td>
</tr>
<tr>
<td>Column Sum</td>
<td>0.344</td>
<td>0.302</td>
<td>0.354</td>
<td>1.0</td>
<td>0.231</td>
</tr>
</tbody>
</table>

*Notes for Tables 3 and 4:* The entry at the top of the cell is the cell probability and the lower entry is the conditional probability which is the cell probability divided by the row sum. Fathers whose educational qualification are missing are excluded. Gymnasium+ means a gymnasium qualification or some tertiary qualification.
### TABLE 4


<table>
<thead>
<tr>
<th>Father’s Education</th>
<th>1985</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not in University</td>
<td>In University</td>
</tr>
<tr>
<td>None</td>
<td>0.451</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>0.958</td>
<td>0.042</td>
</tr>
<tr>
<td>Vocational</td>
<td>0.353</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>0.927</td>
<td>0.073</td>
</tr>
<tr>
<td>Gymnasium⁺</td>
<td>0.109</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>0.741</td>
<td>0.256</td>
</tr>
<tr>
<td>Column Sum</td>
<td>0.913</td>
<td>0.086</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Father’s Education</th>
<th>1985</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not in University</td>
<td>In University</td>
</tr>
<tr>
<td>None</td>
<td>0.461</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>0.970</td>
<td>0.029</td>
</tr>
<tr>
<td>Vocational</td>
<td>0.362</td>
<td>0.020</td>
</tr>
<tr>
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<td>0.948</td>
<td>0.052</td>
</tr>
<tr>
<td>Gymnasium⁺</td>
<td>0.115</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>0.799</td>
<td>0.194</td>
</tr>
<tr>
<td>Column Sum</td>
<td>0.938</td>
<td>0.062</td>
</tr>
</tbody>
</table>
Table 5

Selected Odds and Odds Ratios for Males and Females 1985 2005

Upper Secondary Education.

<table>
<thead>
<tr>
<th></th>
<th>1985</th>
<th></th>
<th>2005</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Odds</td>
<td>Odds Ratio</td>
<td>Odds</td>
<td>Odds Ratio</td>
</tr>
<tr>
<td>I</td>
<td>Gym vs. None</td>
<td>None</td>
<td>0.346</td>
<td>0.582</td>
</tr>
<tr>
<td>II</td>
<td>Gym vs. None</td>
<td>Voc</td>
<td>1.032</td>
<td>1.230</td>
</tr>
<tr>
<td>III</td>
<td>Gym vs. None</td>
<td>Gym</td>
<td>3.772</td>
<td>III/I</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>II/1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.902</td>
<td>3.820</td>
<td>6.564</td>
</tr>
</tbody>
</table>

Females

<table>
<thead>
<tr>
<th></th>
<th>1985</th>
<th></th>
<th>2005</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Odds</td>
<td>Odds Ratio</td>
<td>Odds</td>
<td>Odds Ratio</td>
</tr>
<tr>
<td>I</td>
<td>Gym vs. None</td>
<td>None</td>
<td>0.597</td>
<td>1.147</td>
</tr>
<tr>
<td>II</td>
<td>Gym vs. None</td>
<td>Voc</td>
<td>1.168</td>
<td>2.495</td>
</tr>
<tr>
<td>III</td>
<td>Gym vs. None</td>
<td>Gym</td>
<td>3.952</td>
<td>III/I</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>II/1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.620</td>
<td>6.142</td>
<td>2.175</td>
</tr>
</tbody>
</table>
Table 6

Selected Odds and Odds Ratios for Males and Females 1985 2005

University Participation.

<table>
<thead>
<tr>
<th></th>
<th>Males</th>
<th></th>
<th>Males</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1985</td>
<td></td>
<td>2005</td>
<td></td>
</tr>
<tr>
<td>Odds</td>
<td>Odds Ratio</td>
<td>Odds</td>
<td>Odds Ratio</td>
<td></td>
</tr>
<tr>
<td>I In vs. Not In</td>
<td>None</td>
<td>0.044</td>
<td>In vs. Not In</td>
<td>None</td>
</tr>
<tr>
<td>II In vs. Not In</td>
<td>Voc</td>
<td>0.111</td>
<td>In vs. Not In</td>
<td>Voc</td>
</tr>
<tr>
<td>III In vs. Not In</td>
<td>Gym</td>
<td>0.349</td>
<td>III/I</td>
<td>7.932</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1985</td>
<td></td>
<td>2005</td>
<td></td>
</tr>
<tr>
<td>Odds</td>
<td>Odds Ratio</td>
<td>Odds</td>
<td>Odds Ratio</td>
<td></td>
</tr>
<tr>
<td>I In vs. Not In</td>
<td>None</td>
<td>0.030</td>
<td>In vs. Not In</td>
<td>None</td>
</tr>
<tr>
<td>II In vs. Not In</td>
<td>Voc</td>
<td>0.055</td>
<td>In vs. Not In</td>
<td>Voc</td>
</tr>
<tr>
<td>III In vs. Not In</td>
<td>Gym</td>
<td>0.243</td>
<td>III/I</td>
<td>8.100</td>
</tr>
</tbody>
</table>

Table 7

Sub-Index estimates

Males | Sub-Index estimates | Males | Females | Females |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1985</td>
<td>2005</td>
<td>1985</td>
<td>2005</td>
</tr>
<tr>
<td>Father's Education</td>
<td>$\beta_{10}/\beta_{11}(FE)$</td>
<td>0.912**(0.008)</td>
<td>0.905**(0.009)</td>
<td>0.915**(0.011)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{110}/\beta_{111}(FE)$</td>
<td>0.497**(0.141)</td>
<td>$\uparrow$</td>
<td>0.810**(0.015)</td>
</tr>
<tr>
<td>Father's Occupation</td>
<td>$\beta_{10}/\beta_{11}(FO)$</td>
<td>0.858**(0.008)</td>
<td>$\uparrow$</td>
<td>0.897**(0.008)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{110}/\beta_{111}(FO)$</td>
<td>0.695**(0.117)</td>
<td>$\uparrow$</td>
<td>0.898**(0.006)</td>
</tr>
<tr>
<td>Mother's Education</td>
<td>$\beta_{10}/\beta_{11}(ME)$</td>
<td>0.924**(0.007)</td>
<td>0.907**(0.007)</td>
<td>0.887**(0.010)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{110}/\beta_{111}(ME)$</td>
<td>0.550**(0.140)</td>
<td>$\downarrow$</td>
<td>0.845**(0.008)</td>
</tr>
<tr>
<td>Number of Siblings</td>
<td>$\beta_{10}/\beta_{11}(HI)$</td>
<td>0.938**(0.007)</td>
<td>$\downarrow$</td>
<td>0.909**(0.005)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{110}/\beta_{111}(HI)$</td>
<td>0.809**(0.095)</td>
<td>0.952**(0.009)</td>
<td>0.935**(0.022)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(FUC)$</td>
<td>0.813**(0.012)</td>
<td>$\downarrow$</td>
<td>0.866**(0.012)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(FUC)$</td>
<td>0.817**(0.085)</td>
<td>$\downarrow$</td>
<td>0.988**(0.018)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(ME)$</td>
<td>0.801**(0.012)</td>
<td>0.712**(0.008)</td>
<td>0.775**(0.014)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(ME)$</td>
<td>0.588**(0.132)</td>
<td>$\downarrow$</td>
<td>0.957**(0.015)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(MO)$</td>
<td>0.938**(0.007)</td>
<td>$\downarrow$</td>
<td>0.909**(0.005)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(MO)$</td>
<td>0.809**(0.095)</td>
<td>0.952**(0.009)</td>
<td>0.935**(0.022)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(NS)$</td>
<td>0.813**(0.012)</td>
<td>$\downarrow$</td>
<td>0.866**(0.012)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(NS)$</td>
<td>0.817**(0.085)</td>
<td>$\downarrow$</td>
<td>0.988**(0.018)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(HI)$</td>
<td>0.801**(0.012)</td>
<td>0.712**(0.008)</td>
<td>0.775**(0.014)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(HI)$</td>
<td>0.588**(0.132)</td>
<td>$\downarrow$</td>
<td>0.957**(0.015)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(FU)$</td>
<td>0.925**(0.012)</td>
<td>0.923**(0.011)</td>
<td>0.912**(0.016)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(FU)$</td>
<td>0.809**(0.134)</td>
<td>$\downarrow$</td>
<td>0.905**(0.014)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j0}/\beta_{j1}(SM)$</td>
<td>0.803**(0.010)</td>
<td>0.813**(0.009)</td>
<td>0.807**(0.013)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{j10}/\beta_{j11}(SM)$</td>
<td>0.718**(0.170)</td>
<td>$\downarrow$</td>
<td>0.913**(0.015)</td>
</tr>
<tr>
<td></td>
<td>Intercept terms</td>
<td>$\beta_{01}$</td>
<td>0.507**(0.007)</td>
<td>0.534**(0.008)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{01}$</td>
<td>-0.079**(0.034)</td>
<td>$\downarrow$</td>
<td>0.475**(0.040)</td>
</tr>
</tbody>
</table>

Sub-indexes are of the form $|\beta_{j0}|/\sum |\beta_{kj}|/K_j + |\beta_{k0}|$ where $k = I, II$ and $j = FE, FO, etc. \uparrow$ and $\downarrow$ indicate significant increase or decrease in the index at the 5 percent level.

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