The significance of structural transformation to productivity growth

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The significance of structural transformation in productivity growth:
How to account for different levels in economic selection

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Abstract

This paper critically discusses the most common methodology for decomposing productivity change into inter- and intra-firm effects. It is argued that the methodology can be improved to explicitly take the role of structural transformation into account, and by so doing, a potential source of bias in the results is corrected. This requires the use of a tool from theoretical evolutionary biology: Price’s equation.

Reviewing a sample of studies applying decomposition analyses shows that the methodology is best suited for studies of the evolution of labour productivity and the reallocation of labour.

Based on Danish data for 1992-2010 it is then demonstrated how the results of decomposition analyses can be considerably improved by the explicit inclusion of levels in the selection process. In the specific analysis of the current paper economic selection among industries is included. It is found that the structural transformation of the economy has a large impact on the results of decomposition studies; not least on the magnitude of the inter-firm selection effect. Structural transformation from capital intensive and thus high labour productivity manufacturing towards labour intensive and thus low labour productivity services entails that the traditional methodology is biased downwards in its measure of economic selection.

Finally it is demonstrated that the length of the interval being studied, while often determined by data limitations, has a significant but predictable effect on the results, and it is demonstrated tentatively that economic selection tends to be stronger in the trough of the business cycle.

Keywords: Structural change; Productivity growth; Price’s equation; Economic selection; Business cycle

Jel Codes: L16; O47; B52
1 Introduction

Decomposing productivity change into the effects of intra-firm productivity growth, reallocation of productive resources to the most profitable use and net entry has been a relatively common tool for studying the micro dynamics of aggregate growth for some time. Most studies tend to find that inter-firm reallocation (i.e., economic selection) contributes relatively little to productivity growth. But results from different authors are not generally comparable because of methodological differences (compare for example Bottazzi et al. (2010); Foster et al. (2008); Cantner and Krüger (2008); Krüger (2008); Baldwin and Gu (2006b); Bartelsman et al. (2004); Disney et al. (2003); Foster et al. (2002, 1998)). The most common methodology for decomposing change is from Foster et al. (1998). Despite implicit agreement on using this technique results still lack comparability as researchers tend to study changes over different time spans and to use disparate approaches when computing aggregate results from sub-populations (such as industries). This variability in time and aggregation is to some extent driven by data limitations. But in fact it should not be necessary to discuss the methodology for aggregating results as evolution, which in this type of studies is the change in a population mean; e.g., productivity, is inherently a process taking place simultaneously at multiple levels (Morris and Lundberg (2011), ch. 5; Rice (2004), ch. 10). Decomposition of evolution should incorporate this rather than apply weighting schemes to control for dynamics in sub-populations.

One benefit from correctly accounting for the multiple levels in the evolution of productivity comes from re-examining the varying, and often low, effect ascribed to reallocation of resources. This result is in contrast to basic theories regarding the market mechanism for resource allocation as applied in modeling in the tradition of Nelson and Winter (1982) (see the overviews in Windrum (2007) and in Silverberg (1988)). Resources are generally expected to be allocated to the activities (firms) where they are the most productive. It is generally assumed in models that physical efficiency in production translates into a cost advantage and thus increased market shares through lower prices; or it translates into higher profits and thus greater scope for investment in imperfect capital markets.

Recent contributions (e.g., Coad and Teruel (2013); Bottazzi et al. (2010); Coad (2007)) have argued that firms’ management may pursue other strategies than growth even when the firm has relatively high productivity, and thus that the effect of productivity on growth depends on a general parameter representing, for example, the propensity to re-invest profits.

The most common technique for decomposing productivity growth is ascribed to Foster et al. (1998), who develop it based on earlier contributions. However it is also a slight generalisation of a common technique from biology called Price’s equation. It has been argued that economists can learn a great deal from insights thus developed in biology, though empirical application in economics it still lacking (Andersen, 2004; Metcalfe, 1994).

The primary advantage of applying Price’s equation compared to the me-
The methodology of Foster and colleagues is that it provides a coherent technique for accounting for sub-populations; whereas earlier studies in economics tend to report averages with a relatively arbitrary weighting system.

The various contributions decomposing the evolution of productivity also differ in the time horizon considered. Some of this diversity is driven by data limitations but it is necessary to consider the effect of the overall economic conditions on the contribution of reallocation to productivity growth. Periods where resources are abundant allow for various mutations to exist in biology but when the supply of resources drops only the fittest (i.e. most adaptive) survive. As a starting point a similar process can be expected in economics where there is larger room for experimentation and for taking chances during an economic expansion. This expectation fits well with empirical studies in the management literature (Navarro et al., 2010; Navarro, 2005; Mascarenhas and Aaker, 1989).

The paper is organised as follows. In section 2 the evolutionary perspective on productivity growth is related to the use of decomposition techniques thus providing the link between decomposition studies of productivity growth in economics and theoretical evolutionary biology. In section 3 the mathematics of Price’s equation and its relation to the technique of Foster and colleagues are presented, and it is discussed how the equation may be generalised to account for levels in populations. Section 4 presents the databases where Price’s equation will be applied and in section 5 the expected results of applying a two-level Price’s equation rather than the standard one-level technique are discussed along with the expected consequences of varying the timespan of the decompositions. Section 6 presents the results while section 7 sums up and concludes.

2 Productivity growth and economic selection

A relatively narrow definition of evolution is by the change in the mean characteristic of a population (Andersen, 2004). Economic growth, i.e. the aggregate change in real output per person, is the consequence of increasing productivity of the factors of production and of technological change in a very wide sense. For a constant participation rate it can be modelled as the change in firm level mean real output per employee weighted by the firms’ employment share in the population of firms of the economy. I will refer to this as the evolution of labour productivity.

Equation 1 is a stylised depiction of economic evolution. Economic evolution is an open ended process of novelty generation and reallocation of resources. Selection is the sorting of a population of agents (firms) which is implicit to their differential growth rates. Firms perform innovations and develop knowledge in attempts to gain decisive competitive advantages over competitors, but firms are only intentionally rational agents with limited information and innovation, or more generally: learning, may thus also lead to decreased productivity. Firms prosper or decline as a result of the interaction between their own learning.

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1 Potentials for widening the definition of evolution when using Price’s equation are discussed in Andersen and Holm (2014).
activities, the learning activities of competitors and the external factors setting
the premises for the interaction (Dosi and Nelson, 2010; Metcalfe, 1998). The
evolution of productivity or any other characteristic in a population of firms may
thus be described as the sum of two effects which are referred to by different
names in the literature: The inter-firm or reallocation or selection effect, and the
intra-firm or learning or innovation effect. It is necessary to also add the effects
of entry and exit, but as far as entry is the introduction of new knowledge by
entrepreneurs and exit is the disappearance of inferior firm, these effects are also
learning and selection. Notice that any of the effects may contribute negatively.
I will return to interpretation of the effects in section 3.

\[ \text{Evolution} = \text{Selection} + \text{Learning} + \text{Entry} + \text{Exit} \] (1)

In section 3 a decomposition technique will be suggested that splits up the
selection term of equation 1 into inter-industry selection and intra-industry
(inter-firm) selection. Whereas inter-firm selection is driven by the process of
competition as described above, inter-industry selection is the process of struc-
tural change which is somewhat different. Productivity understood as physical
efficiency is important in competition among firms producing a homogeneous
output, for example within industries. But less so when considering heteroge-
neous outputs. As the composition of demand changes over time, not least as a
consequence of economic growth in itself, relative prices change too and this af-
fects inter-industry selection: with constant productivity across industries but
changes in relative prices it can be expected that industries producing goods
or services for which prices are increasing will be allocated a larger share of
resources. Changes in productivity can in itself also affect relative prices: if
competition among firms eliminates profits within an industry then increased
mean productivity in an industry will drive down prices of goods produced in
the industry relative to other goods. Thus, even though physical efficiency
has increased, relative productivity has not grown and the industry should not
be expected to be favoured in the reallocation of resources.\(^2\) There are excep-
tions to these arguments—the growth of industries producing an internationally
tradable output will depend on physical efficiency—but in general physical ef-
ficiency in production is less important in selection between industries than in
selection between firms. Relative prices thus contain important information for
the process of structural transformation, and this entails that they should not
be confounded by the use of industry specific price indices in empirical analyses.

2.1 Measuring selection

When decomposing evolution into the elements of equation 1 two related issues
must be resolved: one is the measure of firm size—i.e. the resource for which
firms compete—the other is the characteristic expected to determine the sorting
of firms with respect to the reallocation of this resource. Evolution is change in

---

\(^2\) A process of this nature is for example seen in manufacturing of computers where the
consumer price index shows 80% deflation in Denmark for the period 2000-2013.
the size-weighted mean characteristic of the population. For the case of labour productivity, evolution is decomposed into the effects of intra-firm change in productivity, the reallocation of labour towards the more productive firms and the contributions of entry and exit.

An alternative size measure, which is often employed in other studies, is firms’ output market shares. When simulating industrial dynamics it is common to let firms acquire output market shares when they are capable of undercutting the prices of their competitors; that is, when they have higher physical efficiency. Sutton (2007) explores these dynamics and finds that changes in firms’ output market shares are independent of each other in most industries while Coad and Teruel (2013) explore the question with a clever econometric technique and show that, when endogeneity is sufficiently accounted for, the negative expected correlation among changes in firms’ output market shares can indeed be identified. A theory of selection based on output market shares suggests that the size measure in decomposition analysis is sales or production. However, empirically delimiting a population of firms in competition for output markers shares is hardly possible, as firms will be engaging in multiple output markets, will be competing with imports and will also themselves be engaged in competition in foreign markets. These problems are aggravated when using data for small open economies, such as the data for Denmark presented in section 4. Studies that compare the results from decomposing indices weighted by input and output find that using output weights results in very weak selection effects, as would be expected (Bottazzi et al, 2010; Krüger, 2008; Cantner and Krüger, 2008; Foster et al, 1998). Baldwin and Gu (2006a) suggest a highly unique decomposition technique for studying the evolution of labour productivity with labour weights but still taking into account that firms face selection in both labour and output markets. This means that even if employment at all firms remains exactly the same, some share of aggregate labour productivity growth will still be ascribed to selection as output market shares will shift in correspondence to firm level productivity changes. The result is a dominating role for selection in all reported results. This is a very interesting technique for studying labour productivity but it is much less general than the Foster et al (1998) techniques and Price’s equation.

The selection effects identified in decomposition analyses are mathematically based on the relationship between growth of firms over the period and the value of the characteristic at the beginning of the period. If there is to be selection over the period based on initial productivity it is necessary that productivity is not a wildly fluctuating characteristic of firms. Rather, productivity needs to be a characteristic by which firms can be credibly distinguished from each other. Persistence in the characteristic is a necessity for selection and one way such persistence would make itself seen would be through persistent differences in productivity in a population. There is much evidence of such large, persistent heterogeneity in productivity (Syverson, 2011; Dosi, 2007; Bartelsman and Doms, 2000). Of course, persistent firm level differences are a sufficient but not a necessary condition for population heterogeneity. If firm productivity is random then population heterogeneity would still be persistent. However, none
of the above references find indication that productivity is random.

## 2.2 The importance of selection

The focus in economics has been on isolating the effect of change within firms (learning) and the technique of Foster et al (1998) therefore includes a “cross level” effect in addition to the four effects of equation 1. But in the current paper, as in biology (see Endler (1986), ch. 1), focus is on the selection element and how its quantification is improved by taking into account that populations have sub-populations; rather than reporting weighted results as aggregate results. The intra-firm effect is here considered a residual and the generalisation by Foster and colleagues is not applied.

The effect of economic selection is generally found to be of minor relative importance in the studies mentioned earlier. The current pervasive use of decompositions was however preceded by more diverse approaches as reviewed in Caves (1998), which found that economic selection (or “turnover”) is an important force in aggregate productivity growth. The comprehensive analysis of Baldwin (1995), for example, finds that economic selection is important in productivity growth based on comparisons of the productivity of exiting plants, new plants, growing plants and declining plants, as well as a study of the productivity development of new surviving plants over time. In addition, Caves (1998) argues that there is evidence that the stage of the business cycle as well as the timespan of the analysis can affect the role of selection. Specifically, the relative importance of selection is countercyclical: adverse economic conditions entail increased creative destruction of low productivity firms. These mediating effects of time have not been studied systematically in decomposition studies but the current paper makes some progress in this respect.

There are a small number of exceptions to the general finding that selection is unimportant in decompositions. These suggest the area in which decomposition studies are most likely to be fruitful. In Foster et al (1998) a large number of decompositions are undertaken: the authors decompose the change in both total factor productivity (TFP) and labour productivity using three different measure of size: sales, employment in persons and employment as hours. The authors also compare five and ten year changes in productivity. Foster and colleagues generally find that economic selection contributes very little but in one case, the case of the evolution of labour productivity in US manufacturing from 1977 to 1982 with size measured by man-hours, they find that selection accounts for 85% of evolution. In the follow-up analysis Foster et al (2002) the authors again find that selection plays a dominating role for the evolution of labour productivity weighted by man-hours over a five year period. For both studies, however, the authors generally find low selection when decomposing the evolution of TFP, when applying output weights to either productivity measure or when studying evolution over ten years. In Foster et al (2008) three different measures of TFP are studied with output weights and selection is consistently weak.

The studies by Foster and colleagues rely on very idiosyncratic data: plant
level surveys undertaken at five year intervals. In particular one should be careful when interpreting the selection effect as reallocation among plants from competitive interaction. It will also have an element of managerial decision making. Baldwin and Gu (2006b), studying the Canadian manufacturing sector, also study the evolution of labour productivity and reallocation of labour at the plant level. They find a dominating selection effect for 1973 to 1979 but a negligible selection effect for 1979 to 1988 and 1988 to 1997. The focus of Baldwin and Gu (2006b) is on different approaches to identifying the effects of entry and they do not discuss the varying role of selection. The results presented in the current paper suggest that the difference may be associated with the early period being the shortest, and potentially with variation in the overall economic conditions.

Studies relying on firm level data also tend to find stronger selection when studying labour productivity weighted by employment than otherwise; but not as strong as the results by Baldwin, Foster and colleagues. In Disney et al (2003) and Bartelsman et al (2004) the selection effect identified by application of the Foster et al. technique is very small and so it generally is in Cantner and Krüger (2008). For some German manufacturing industries analysed by Cantner and Krüger the selection effect does exceed 30% of total evolution for 1990-1998 while it is generally lower for 1981-1989. This suggests that the overall economic conditions might have an effect on the results of decompositions. Disney et al (2003), however, report that their results are robust to choice of time period indicating that the economic conditions do not affect their results. Bartelsman et al (2004) decompose the evolution of labour productivity for 8 countries over two time intervals (1987-1992 and 1992-1997) and it appears that the variation in results across countries is much larger than the difference between the two intervals within countries. This suggests that relatively inert differences (e.g. institutions) matter more for results than relatively transient factors such as the business cycle. Bottazzi et al (2010) decompose labour productivity for 1989-2004 but not in one step. Bottazzi and colleagues decompose the evolution one year at a time and report the average over the period. The result is a relatively large selection effect when they use employment as size. But the magnitude of the selection effect relative to total evolution is not comparable to the other studies, as Bottazzi and colleagues lack data on entry and exit. Variation in the size threshold for firms to be included in the analyses is a general caveat when comparing results of different studies. As in the other studies Bottazzi and colleagues find a very weak selection effect when measuring firm size by sales.

The sample of earlier studies discussed above point in a general direction for using the decomposition technique of Foster and colleagues. The method cannot capture the competitive dynamics of firms stealing each other’s market shares.\(^3\) The method is more apt for describing the dynamics of labour reallo-

\(^3\) Schumpeter (1947), pp. 84-85 argued that reallocation of market shares through price cutting is as effective relative to innovation, as “forcing a door” is effective relative to “a bombardment”. And that thus the evolution of the economic system is comparatively indifferent to whether price competition functions or not. The results of Coad and Teruel (2013) show
cation to the most productive means as measured by labour productivity. The studies also point to methodological differences of which the consequences are not clear. Several studies decompose productivity at a more disaggregate level and reported weighted averages. This is definitely warranted both as a crude control for capital intensity and to mitigate the consequences of labour not being a homogeneous resource. Consensus is however needed regarding the weighting and here Price’s equation can help. It allows for the decomposition equation to remain an identity while still accounting for dynamics in sub-populations.

3 Price’s equation

The use of Price’s equation as an accounting device for evolutionary processes is not new to economics (see e.g. Metcalfe (2008); Andersen (2004); Metcalfe (1994)) but whereas methodological contributions abound, empirical applications do not. The classic reference for Price’s equation is Price (1970) but this “preliminary communication” is quite short and directed at biology, whereas the posthumously published Price (1995) elevates the discussion to a much more general level.\(^5\)

I will adhere to the following notation, which comes very close to the notations of Andersen (2004) and Frank (1995): Upper case letters denote population level aggregates at the highest level of aggregation and lower case letters denote firm level or sub-population values. As evolution is defined as the change in mean characteristic, then the initial value of the population characteristic can be seen as measured by a census of the pre-evolution population and the final value is achieved from a census of the post-evolution population.\(^6\) Adding a prime to a variable denotes the value in the post-evolution population, as opposed to the pre-evolution population, and a prefixed \(\Delta\) denotes the difference between pre- and post-evolution populations. The subscript \(i\) denotes firms and subscript \(j\) denotes sub-populations (i.e. industries). \(\text{Cov}(a_i, b_j)\) is the population covariance between random variables \(a\) and \(b\) weighted by firm size and \(E(a_i)\) is the population mean of \(a\) weighted by firm size. Table 1 contains the descriptions as well as the mathematical definitions of the components of Price’s equation.

Equation 2 is Price’s equation in the form where it can be used to decompose the change in productivity into an inter-firm and an intra-firm component.

---

\(^4\) Coad and Teruel (2013) find stronger indication that firms compete for labour than for market shares. The view that firms compete for labour is also supported by Sørensen (2004).

\(^5\) It is not that Price did not publish more on his work during his lifetime. But the exposition in Price (1995) with the accompanying paper by Steven Frank (Frank, 1995) is a very thorough exposition of the selection mathematics developed by George Price.

\(^6\) The use of pre- and post-evolution is not intended to signal that evolution starts and stops. It is intended to distinguish between the population before and after the change in mean characteristic.
Tab. 1: Elements of Price’s equation

<table>
<thead>
<tr>
<th>Formal</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>Firm size</td>
</tr>
<tr>
<td>$z_i$</td>
<td>Firm productivity</td>
</tr>
<tr>
<td>$X$</td>
<td>$\sum_i x_i$ Population size</td>
</tr>
<tr>
<td>$s_i$</td>
<td>$x_i/X$ Firm share</td>
</tr>
<tr>
<td>$w_i$</td>
<td>$x_i' / x_i$ Firm fitness (growth rate plus one)</td>
</tr>
<tr>
<td>$\Delta z_i$</td>
<td>$z_i' - z_i$ The evolution of productivity</td>
</tr>
<tr>
<td>$Z$</td>
<td>$\sum_i s_i z_i = E(z_i)$ Population (mean) productivity</td>
</tr>
<tr>
<td>$\Delta Z$</td>
<td>$Z' - Z$ The evolution of pop. productivity</td>
</tr>
<tr>
<td>$W$</td>
<td>$X'/X$ Population fitness</td>
</tr>
<tr>
<td>$Cov(w_i, z_i)$</td>
<td>$\sum_i s_i (w_i - W)(z_i - Z)$ Covariance of fitness and productivity</td>
</tr>
</tbody>
</table>

$$\Delta Z = \frac{Cov(w_i, z_i)}{W} + \frac{E(w_i \Delta z_i)}{W}$$ \hspace{1cm} (2)

$\Delta Z$ is evolution. The evolution of population level productivity from the pre-evolution population to the post-evolution population is equal to the sum of two terms.\(^7\) The first term is based on the covariance between growth and productivity and is named the selection effect, the inter-firm effect or reallocation. It indicates to what degree the growth of aggregate productivity can be attributed to firms with productivity above the population mean level in the census of the pre-evolution population growing more than other firms. The second term is the innovation effect, the learning effect or the intra-firm effect. It is the part of productivity growth that may be attributed to processes internal to firms. These processes include both the creation of novelty as well as adaptation and may be positive as well as negative.

An alternative specification of Price’s equation is reached by multiplying by $W$ and is given in equation 3.

$$W \Delta Z = Cov(w_i, z_i) + E(w_i \Delta z_i)$$ \hspace{1cm} (3)

The interpretation of what exactly is being decomposed is a bit more tricky in equation 3 than in equation 2. However, notice that the expression within the expectations operator on the right is the firm level equivalent of the population level term on the left. This means that Price’s equation can be substituted into itself as many times as the researcher may desire—and thus allow for a

\(^7\) Proof that the change in a weighted mean may be decomposed in this manner—and that Price’s equation is thus an identity—is given in the appendix to Andersen (2004) and will not be repeated here.
Price’s equation

3 Price’s equation

multilevel study of evolution (Andersen, 2004). It is thus possible to describe each agent of the population as a population in its own right.

3.1 Application to a population of firms

Price’s equation as specified above can be rewritten to be identical to the decomposition technique developed by Foster et al (1998). The first thing to notice is that Equation 3 cannot be applied directly to firm data, as two crucial phenomena in the evolution of a population of firms is not taken into account in this form. These are entry and exit. In order to take these phenomena into account it is necessary to distinguish between three sets of firms: Those that exist in both the pre- and post-evolution populations, those that exist in only the post-evolution population and those that exist in only the pre-evolution population. These sets will be labelled the C, N and X sets respectively (Continuing, eNtering and eXiting).

The two terms of equation 2 (and 3) refer to the contribution of the C-set to productivity growth. In order to specify the contributions of the N and X-sets it helps to expand the covariance and expectation terms and make explicit which set of firms are included in the computations.

The covariance and expectation operators in equation 2 can be expanded to

\[
\Delta Z = \sum_i s_i (w_i - W)(z_i - Z) + \sum_i s_i w_i \Delta z_i
\]

and so

\[
\Delta Z = \sum_i s_i (w_i/W - 1)(z_i - Z) + \sum_i s_i (w_i/W) \Delta z_i
\]

and as \( s_i w_i/W = \frac{x_i}{X} \cdot \frac{s_i}{X'} X' = s_i' \)

\[
\Delta Z = \sum_i (s_i' - s_i)(z_i - Z) + \sum_i s_i' \Delta z_i
\]

in order to indicate that this only refers to firms present in both the pre- and post-evolution populations.

\[
\Delta Z = \sum_{i \in C} \Delta s_i (z_i - Z) + \sum_{i \in C} s_i' \Delta z_i
\]

and adding the contributions of the N- and X-sets

\[
\Delta Z = \sum_{i \in C} \Delta s_i (z_i - Z) + \sum_{i \in C} s_i' \Delta z_i + \sum_{i \in N} s_i'(z_i' - Z) - \sum_{i \in X} s_i (z_i - Z)
\]

The four terms of equation 4 thus correspond to the selection effect, the learning effect, the entry effect and the exit effect. The entry effect will contribute positively (negatively) to productivity growth when the productivity of new firms is higher (lower) than population productivity in the pre-evolution
population. The exit effect will contribute positively (negatively) when firms exiting the population have productivity lower (higher) than population productivity in the pre-evolution population.

The only difference between equation 4 and the one developed by Foster and colleagues is that they divide the learning effect up into two different terms, as in equation 5.

$$\sum_{i \in C} s'_i \Delta z_i = \sum_{i \in C} s_i \Delta z_i + \sum_{i \in C} \Delta s_i \Delta z_i$$

The first term on the right hand side of equation 5 is termed the within effect (as opposed to the between effect, which is the label given to the selection effect) and the second term the cross level effect. The justification for separating the cross level effect from the intra-firm effect is to isolate the effect of productivity change for a given $s_i$. As the present paper focusses on the selection effect, the learning effect is not expanded in the decompositions.

### 3.2 A multilevel decomposition technique

Single level decomposition of evolution by means of Price’s equation takes the form of equation 6, which is a repetition of equation 4.

$$\Delta Z = \sum_{i \in C} \Delta s_i (z_i - Z) \quad (Selection\,\, effect)$$

$$+ \sum_{i \in C} s'_i \Delta z_i \quad (Learning\,\, effect)$$

$$+ \sum_{i \in N} s'_i (z'_i - Z) \quad (Entry\,\, effect)$$

$$- \sum_{i \in X} s_i (z_i - Z) \quad (Exit\,\, effect)$$

In order to expand this to a multilevel technique I return to equation 3 and replace the subscript $i$s (firms) with subscript $j$s for industries and the equation becomes:

$$W \Delta Z = Cov(w_j, z_j) + E(w_j \Delta z_j)$$

By adding subscript $j$s to equation 6 and substituting for $\Delta z_j$ in equation 7 the second term on the right may also be written as:

$$E(w_j \Delta z_j) = \sum_j s_j w_j \left[ \sum_{i \in C_j} \Delta s_{ij} (z_{ij} - z_j) + \sum_{i \in C_j} s'_{ij} \Delta z_{ij} + \sum_{i \in N_j} s'_{ij} (z'_{ij} - z_j) - \sum_{i \in X_j} s_{ij} (z_{ij} - z_j) \right]$$
And thus, substituting equation 8 into equation 7 and multiplying by $W^{-1}$, the equation used for the two level decomposition becomes (cf. the derivation of equation 4 page 9):\(^8\)

\[
\Delta Z = \sum_j \Delta s_j (z_j - Z) \quad \text{(Industry selection effect)}
\]

\[
+ \sum_j s'_j \sum_{i \in C_j} \Delta s_{ij} (z_{ij} - z_j) \quad \text{(Firm selection effect)}
\]

\[
+ \sum_j s'_j \sum_{i \in C_j} s'_{ij} \Delta z_{ij} \quad \text{(Learning effect)}
\]

\[
+ \sum_j s'_j \sum_{i \in N_j} s'_{ij} (z'_{ij} - z_j) \quad \text{(Entry effect)}
\]

\[
- \sum_j s'_j \sum_{i \in X_j} s_{ij} (z_{ij} - z_j) \quad \text{(Exit effect)}
\]

The first term of this decomposition is the industry selection effect or the inter-industry effect. It captures selection among industries and will contribute positively to aggregate productivity growth if industries with above mean productivity tend to expand at the expense of other industries. The second term is the firm selection effect or the inter-firm, intra-industry effect. As with the selection effect in equation 6 it captures selection among firms. The third effect is the learning or intra-firm effect, which has the same interpretation and definition as above. The fourth and fifth terms are the entry and exit effects respectively. Just as any other decomposition technique equation 9 is an identity. But in equation 9 selection among firms is implicitly assumed to take place only within industries $(j)$. Thus it is no longer necessary to apply the technique separately to industries and report weighted results. A potential issue with equation 9 is that entry and exit are evaluated relative to the average incumbent competitor; whereas it may be more reasonable to evaluate them relative to each other (Baldwin and Gu, 2006b). That is, entering firms contribute to increasing aggregate productivity if they are more productive than exiting firms rather than the overall average. However, the specification in equation 9 is the most common and will be used here.

### 3.3 Census years

An important source of discrepancy between the results of earlier studies is that they consider evolution over different time periods. From up to 42 years in the case of Krüger (2008) to just one year for Bottazzi et al (2010) in the sample of earlier studies discussed above. $\Delta Z$ is the change in population mean characteristic from the pre-evolution population to the post-evolution population, i.e. from time $t$ to $t'$. If there is a third census in between the two points in time,\(^8\) A similar decomposition technique is derived and applied in the appendix to Krüger (2008). The only substantial differences are caused by Krüger’s study being at a more aggregate level.

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\(^8\) A similar decomposition technique is derived and applied in the appendix to Krüger (2008). The only substantial differences are caused by Krüger’s study being at a more aggregate level.
t < t^* < t', then the evolution $\Delta Z$ may be broken up into two separate processes of evolution, $\Delta Z = \Delta Z_1 + \Delta Z_2$. $\Delta Z_1$ is the evolution from $t$ to $t^*$ and $\Delta Z_2$ is the evolution from $t^*$ to $t'$. The sum of the selection effects from decomposing $\Delta Z_1$ and $\Delta Z_2$ separately will generally not be equal to the selection effect from just decomposing $\Delta Z$. The relatively strong selection effect found by Bottazzi et al (2010) is likely to be at least partially driven by the researches reporting the sum of up to 15 one year processes rather than a single 15 year evolutionary process. This is discussed further in section 5 where also the consequences for the other elements of the decomposition equation are discussed.

4 Data material

For the empirical analysis in this paper two different databases are used; one covering the period 1992-1999, and another for the period 2000-2010. Data for the period 1992 to 1999 come from the Company and Industry Statistics database\textsuperscript{9} of Statistics Denmark, while data for 2000 to 2010 come from the General Business Statistics database\textsuperscript{10} of Statistics Denmark. The databases are similar in that they contain various financial data from the balance sheets and annual accounts of all private firms in Denmark as well as a number of estimated key indicators. The change of data source between 1999 and 2000 is necessary, as the former database was discontinued and replaced by the latter. Three variables from the databases are used: Total value added for the firm (VA), total full time equivalent employment for the firm over the year (FTE) and industry classification, which is NACE 1.1 in the early database and NACE 2 in the latter database. All industries with available data are used in the decompositions and firms' industry is determined by NACE code in the pre-evolution census.

The two databases contain more or less the same variables but they differ in the methodology used to compute VA and are thus not directly comparable. Therefore separate analyses are undertaken for the two periods corresponding to the respective coverage of the databases. The lack of a direct mapping between NACE 1.1 and 2 also means that the two databases cannot be merged and that two separate analyses are necessary.

As discussed earlier, relative prices contain important information for the process of structural transformation and therefore the standard approach of deflating the data using industry level price indices is not used. A common price index, the consumer price index as obtained from Statistics Denmark\textsuperscript{11}, is used to account for general inflation.

The databases contain sufficient information on individual firms for TFP to be estimated. This is not done, however, as the necessary assumptions for the theory of production are very comprehensive and since earlier studies have not been able to establish preference for TFP over labour productivity (Bottazzi

\textsuperscript{9} Original Danish name: Firma- og Ressourceområdestatistik.
\textsuperscript{10} Danish: General Erhvervsstatistik.
\textsuperscript{11} Available from www.statistikbanken.dk
et al, 2010; Syverson, 2011).

VA is not available for every firm in the databases; in particular, there are no data for agriculture, fishing and forestry, and a number of service industries (utilities, financial services, telecommunication services (not including IT), and a few entertainment industries such as theatre and radio/television).

Industry codes will be used to delimit sub-populations of the Danish economy. Experimenting with two, three and four digit industry codes has shown that the inter-firm, intra-industry reallocation effect tends to be stronger when the sub-population are defined at a more disaggregate level. This was to be expected as measuring firm size by labour input in connection with populations delimited by industry codes has stronger merit the lower the level of aggregation. The more similar firms are with regards to their production processes, the more they can be expected to require qualitatively similar labour services.

Firms of less than eight FTEs will be excluded from the analyses, and so will industries which in any one year have less than two firms. After this censoring, and combined with the earlier described problems of data availability, the data includes more than 90% of private sector employment in each year of the 1992-1999 period and about 80% in the 2000-2010 period. Removing small firms (< 8 FTEs) means that firms are considered to enter or exit when they add or remove the 8th FTE and thus the effect of entry and exit on aggregate productivity evolution must be interpreted with caution. Small firms are nevertheless removed as the determinants of growth among small, and especially young, firms are expected to by idiosyncratic, cf. Section 2. The cut-off value at 8 FTEs was determined by observing a scatter plot of size by productivity and choosing a cut-off that would remove the most extreme outliers in terms of productivity. Small firms have low weight \( s_i \) in decomposition analysis and, as expected, changing the cut-off point to 7 or 9 FTEs does not substantially affect the results reported in this paper.

### 4.1 The stages of the business cycle

The two databases span 19 years when combined and this period spans several business cycle stages. Figure 1 plots the evolution of labour productivity \( Z \), its two components real VA and FTE, and the number of firms over the two databases. For the 1992-1999 data all variables equal 100 in 1992 and for the 2000-2010 data all variables equal 100 in 2000. The periods are similar in that they both cover the early stage of an upswing where aggregate output has started to increase while employment has not, and the end of the upswing where both output and employment decline. But the periods are also different. In the 1990s the upswing was without productivity growth while in the 2000s the upswing was accompanied by both periods of productivity growth and periods of stagnation. The latter period seems to have been much more dramatic in terms of the decline in employment and firms in the downturn.

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\(^{12}\) The results reported in the paper all rely on four digit codes. Results for two and three digits are available upon request.
5 Expected consequences

Adding the inter-industry selection term means that the magnitudes of the other effects will change. Arguably the measurement of the other effects is biased without the industry selection term, and the measurement will become improved with the multilevel technique.

With the single level technique an entering firm ($i$) will contribute positively to productivity growth if its productivity in the end year is higher than the overall average productivity in the base year ($z_i' > Z$). The new entrant may be the most efficient producer of real estate services in the economy but if real estate services generally has low labour productivity, then the contribution of the firm to the overall average will be negative.

With the multilevel technique the contribution of an entering firm is instead relative to the average productivity of the sub-population that it enters ($j$). Thus if the productivity of the firm is higher than the industry average, then the firm contributes positively to growth even if its productivity is lower than the overall average ($z_j < z'_{ij} < Z$). The difference between the two techniques for quantifying the entry effect is one of the factors contributing to the industry selection effect. If firms tend to enter industries with low average productivity but which are increasing in relative size ($z_j < z'_{ij} < Z$ and $\Delta s_j > 0$), then the single level technique would quantify this as a negative entry effect, while the multilevel technique would account for it with a positive entry effect and a negative industry selection effect.

The correction achieved for the exit effect by application of the multilevel technique is similar: if exit tends to be from, for example, capital intensive
manufacturing industries with high labour productivity, then the single level effect would add a negative contribution to exit. The multilevel technique, on the other hand, would add a positive contribution to the exit effect and a negative contribution to the industry selection effect (assuming that $z_j > z'_j > Z$ and $\Delta s_j < 0$).

The learning effect will be exactly the same regardless of which technique is employed but the firm selection effect will not. As with the entry and exit effect, the learning firm effect will now be positive as long as resources are reallocated towards firms that have high productivity relative to others in the same industry; rather than relative to the entire economy. Thus if resources are being reallocated towards the most productive firms in some labour intensive, relatively low productivity service industry, then the single level technique will produce a negative firm selection effect, while the multilevel technique will produce a positive firm selection effect and a negative industry selection effect.

The benefit of applying the multilevel technique in equation 9 rather than the single level equation 6 is that the industrial dynamics of the general structural transformation away from capital intensive and high labour productivity manufacturing towards less capital intensive and therefore typically low labour productivity services is captured more intuitively correct. The general trend towards services will be captured by a negative industry selection effect, while exit of relatively low productivity manufacturing and entry of relatively high productivity services adds positively to productivity growth.

5.1 Time

Demonstrating the effect of applying the multilevel decomposition technique rather than the single level technique is the main contribution of this paper. However, when the data has been collected and the programme has been written, it is straightforward to also explore the effect on the results of decomposing one long period of evolution compared to summing over several shorter periods.

The industry selection and firm selection effects are based on the assumption that reallocation of resources over the period is determined by productivity in the pre-evolution census. That is, that reallocation between $t$ and $t'$ is determined by productivity in $t$. The longer the interval between $t$ and $t'$ the more questionable this assumption becomes. Thus if the inter-industry and inter-firm effects are summed up over sub-periods they are likely to be larger compared to the results of a single decomposition.

On the other hand, the exit effect and especially the entry effect are likely to become smaller when reported as sums over sub-periods. Firms that enter are likely to have varying levels of productivity (Jovanovic, 1982) but only those that survive until the end year are included in the entry effect. Thus a longer interval between $t$ and $t'$ will result in a survivor effect meaning that all of the low productivity entrants have exited before the end year and they are thus not considered in the decomposition.

When summing over several shorter periods of evolution it may be relevant to consider the variation in the business cycle over the periods. Firms’ strategies
and the success of different strategies vary over the business cycle (Navarro et al, 2010; Navarro, 2005; Mascarenhas and Aaker, 1989) and thus so may the decomposition of economic evolution. In addition, differences in capacity utilisation over the business cycle entails that selection between firms can be expected to be stronger during a downturn, as firms must maintain idle fixed capital. Such differences will only be approached exploratively in the current paper.

6 Results

In order to ascertain the benefits of applying the multilevel version of Price’s equation rather than the single level decomposition technique, productivity changes over the entire early period (1992-1999) and the entire latter period (2000-2010) have been analysed using both equations 6 and 9. Decompositions of productivity change for the two periods 1993-1997 and 2003-2007 have also been compared as these two periods are relatively comparable in terms of the business cycle stage (cf. figure 1) and are of equal length.

To compare the results of decomposing evolution over a given period with the result from splitting up the period and reporting sums of effects over sub-periods, the data for 1992-1998 and 2000-2008 have been utilised. The 1992-1998 data allows for comparison of the effects of studying 1 period of 6 years versus 2 periods of 3 years each, 3 periods of 2 years and 6 one year periods. The 2000-2008 data allows for 1 period of 8 years versus 2 periods of 4 years each, 4 periods of 2 years and versus 8 periods of one year each.

The final set of results are an explorative study of the evolution of economic selection over the business cycle. The period 2000-2010 is divided into three stages: The trough 2000-2003, where aggregate output is stagnating and employment is slightly increasing, the expansion 2003-2007, where both output and employment are increasing, and the contraction 2007-2010 where both output and employment decrease.

I order to enhance the interpretability of the results all decompositions will be scaled by \( \frac{Z}{100} \) and multiplied by 100. Thus it is the growth rate and the percentage point contributions of the various effects that are presented. When comparing results based on different methods focus will be on the share of total growth attributed to the various effects. The full results are presented in tables 3, 4 and 5 in the appendix. The main results are presented in bar charts in the following.

6.1 The effect of levels

Table 3 compares the results of decomposing the evolution of labour productivity with equations 6 and 9 for four different periods. The main results are illustrated in figure 2. From 1992 to 1999 labour productivity grew by 3.42 per cent. According to equation 9 this was driven by three effects: entrants had, or achieved, high productivity (34 per cent), low productivity firms were forced to
exit (30 per cent of the change), and labour was reallocated to the most productive firms (the selection effect; 64 per cent). The total role of competition among firms is thus arguably 30 + 64 = 94 per cent of productivity growth. However, at the industry level selection favoured industries with low labour productivity and this contributed −22 per cent of total growth.

Comparing the above to decomposition by equation 6, cf. figure 2, it is seen that the selection, entry and exit effects are all lower. This is consistent with a process of structural transformation where entry is into service industries with low productivity because they are relatively labour intensive. In terms of employment these industries are growing in Denmark, hence the negative industry selection effect. Arguably the single level decomposition underestimates the effects of reallocation, entry and exit since it does not take the structural transformation of the economy into consideration.

The results for 1993-1997 and 2003-2007 show the same pattern as the 1992-1999 results and are not included in figure 2. The result for 2000-2010, however, is different. It shows a positive industry selection effect meaning that from 2000 to 2010 labour was reallocated towards the more productive industries. As expected this means that the single level technique overestimates the effect of selection and of exit but the effect of entry is still too low. This means that, despite industry selection favouring high productivity industries, entrants tended to set up in low productivity industries. The period 2000-2010 is longer than any other period studied and contains a mixture of economic environments (cf. figure 1) and this is likely to be a source of this period’s unique result. This is discussed further in section 6.3 below.
6.2 The role of time

As already discussed, the selection effect quantified in decomposition studies assumes that the growth from pre-evolution population to post-evolution population is determined by the productivity in the pre-evolution population. This assumption becomes problematic as the interval between the two censuses is lengthened.

From 1992 to 1998 real value added per full time equivalent worker grew by 2.52 per cent. Decomposing this change with equation 9 is illustrated by the lighter bars in the left part of figure 3. If instead the change is decomposed separately for the six periods of one year each 1992-1993, ..., 1997-1998, and the resulting six sets of effects are added together, the result is the darker bars in the left part of figure 3. The right half of the figure shows the results for the period 2000-2008. These results can also be seen in table 4 along with results that are arrived at by splitting up the periods in intervals of intermediate length.

Some results stand out very clearly. It is no surprise that the entry effect becomes smaller when decompositions are undertaken for shorter periods. However, the effect does not appear to go towards zero as the interval between the censuses is shortened: in both the 1992-1998 and 2000-2008 results the effect becomes negative. This indicates that entrants’ productivity is not randomly distributed around the mean of the industry they are entering, but rather that it tends to be lower than the mean. A similar tendency is observed for the exit effect: the shorter the period the larger the effect. This means that exiting firms tend to have low productivity. However firms’ productivity in the pre-evolution census becomes a weaker indicator of which firms will exit when longer periods
are considered.

The exit effect is to some degree determined by the same selection mechanism that determines the firm selection effect and, as with the exit effect, the tendency for selection to be stronger when decomposing shorter periods is also seen in the firm selection effect. This was also to some degree seen in the sample of earlier studies discussed in section 2.2: studies where the two censuses are closer to each other in time tend to find stronger firm selection effects. In contrast, it is difficult to say anything general about the relationship between the industry selection effect and time. In the 1992-1998 period it seems that the structural transformation of the economy from manufacturing towards services is more closely captured when the time period is shorter. For the 2000-2008 period, on the other hand, the effect seems to go towards zero as short periods are studied. As with the results discussed in section 6.1 this may have to do with the evolution of the business cycle over the period and this will be explored below.

6.3 The business cycle

Figure 4 shows the results of decomposing productivity change over the three periods trough, expansion and contraction between 2000 and 2010. The decompositions are undertaken for each interval of one year and the results are sums for the relevant years. This makes the results comparable even though the periods are not of equal length. Results using the two-level technique of equation 9 are presented in figure 4 while results using both the one- and two-level techniques are presented in table 5 in the appendix.
The industry selection effect is positive for the trough and the contraction but negative for the expansion. This means that during the expansion resources were moved to low labour productivity industries while resources were moved in the opposite direction during the trough and the contraction. Firm selection is by far strongest during the trough which partially supports the argument that the selection mechanism should be stronger when firms cannot employ their full production capacity. The exit effect computed with the two-level technique does not show this variation but when using the one-level technique the exit effect is seen to be lower in the expansion. This suggests that there are relatively many exits from low productivity industries (services) during the trough and the contraction, and that the exiting firms do not have particularly low productivity relative to the rest of their industry.

It seems that entrants in the trough tend to have productivity near the mean of their particular industry’s distribution while it tends to be lower during the expansion and the contraction. The negative effects of entry are caused by the use of one year periods and the tendency for the one level technique to report a too low entry effect is clearly seen. In general, however, the results from equations 6 and 9 are in agreement.

### 6.4 Structural transformation

In the above analyses there have been a number of references to the structural transformation of the economy without any documentation of it. While such an analysis is beyond the scope of the paper some evidence is presented in table 2. It can be seen that the service sector has been growing at the expense of manufacturing in both periods and that construction increased as a share of the economy in the 1990s and declined in the 2000s. Relatively low labour pro-

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**Tab. 2: Structural transformation**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Manufacturing</th>
<th>Construction</th>
<th>Services</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992-1999</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry selection</td>
<td>0.21</td>
<td>−0.48</td>
<td>−0.49</td>
<td>−0.76</td>
</tr>
<tr>
<td>Δ employment share</td>
<td>−0.036</td>
<td>0.010</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>Industries</td>
<td>167</td>
<td>12</td>
<td>121</td>
<td>300</td>
</tr>
<tr>
<td>2000-2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry selection</td>
<td>3.87</td>
<td>0.40</td>
<td>−0.93</td>
<td>3.34</td>
</tr>
<tr>
<td>Δ employment share</td>
<td>−0.075</td>
<td>−0.020</td>
<td>0.095</td>
<td>0.000</td>
</tr>
<tr>
<td>Industries</td>
<td>175</td>
<td>16</td>
<td>170</td>
<td>361</td>
</tr>
</tbody>
</table>

The top row for each period is a further disaggregation of the industry selection effects in the two top parts of table 3 into contributions by sector. Industries is the number of included industries within each sector. For 1992-1999 this is based on four digit NACE 1.1 while it is based on NACE 2 for 2000-2010.
ductivity in construction thus entails that construction contributed positively to the industry selection effect in the 2000s and negatively in the 1990s. The number of industries within manufacturing and services is quite high and the number in table 2 naturally covers up a lot of diversity.

Inspecting the data shows that there are two sources of the negative contribution to industry selection from services as presented in table 2: declining high productivity services (especially shipping) and growing low productivity services (especially retail) but there are also growing high productivity services (for example software) in both periods. The relatively low contribution of manufacturing in the 1990s seems to be predominantly caused by decline in the highly productive dairy and production and processing of meat industries while the high contribution in the 2000s seems to come from a decline of low productivity furniture manufacturing and the rise of high productivity pharmaceutical manufacturing. Notice that a negative contribution to the inter industry selection effect does not mean that an industry contributes negatively to productivity growth. Firm selection, learning and entry and exit dynamics may outweigh the effect of industry selection. Figure 5 casts further light on the diversity across industries. It present the distributions of the individual industries’ contributions to the firm and industry selection effects. There are 300 and 361 industries respectively for the two periods. For each plot between 9 and 20 industries with extreme values are left out in order to keep the histograms legible. The industry selection effect for 2000-2010 is 3.34 (cf. tables 3 and 2) so the average industry only contributes 3.34/361 = 0.009 percentage points while the average is −0.003 for 1992-1999. Many industries clearly have contributions quite different from these averages. The industry selection effect is positive for one period and negative for the other (cf. figure 2) but from figure 5 is seems that the distributions of industry level contributions are similar; i.e. that inter industry heterogeneity seems to be pervasive.

7 Discussion and conclusions

The various results presented in section 6 both contribute to the understanding of earlier results from decomposition studies and added new insight.

Other studies do not necessarily underestimate the effect of economic selection because they do not take into account the multiple levels of selection in an economy. Whether the selection effect is over- or underestimated depends on the direction of structural change in the economy. Some applications of the single level technique, for example the studies by Foster and colleagues, decompose the evolution of productivity by industry and then report weighted averages. This mends the bias but it does not account for its source: the effect of industry selection.

The entry and exit effects are also affected by the structural transformation of the economy. However, whether this also constitutes bias in measuring is less clear. In the single level Price’s equation an entrant contributes to productivity growth if it has above mean productivity and this makes intuitive sense, but if
Fig. 5: Distributions of the industry level contributions to the selection effects
much entry is in labour intensive and thus low labour productivity industries, e.g. many service industries, then the single level technique might reach the conclusion that entrants are contributing negatively to productivity growth while in fact they are raising productivity in the economy’s expanding industries.

All in all these results suggest that the multilevel technique is preferable. However, some conclusions regarding structural transformation were only arrived at because the multilevel result was compared to the single level result. Thus doing an auxiliary single level decomposition to aid in interpretation may be a good complement to the multilevel decomposition.

The effect of decomposing shorter periods and reporting summed effects illustrated that one must be very careful when comparing results of decompositions spanning different time periods. They should arguably not be compared at all. One solution is to report sums over a number of one year periods. These results will be more comparable but can generally be expected to show that entry decreases productivity while selection and exit are important. What is then the ideal length of time between the censuses of the pre-evolution population and the post-evolution population is left as a question for future research.

The explorative study of variations in decomposition results over the business cycle illustrated that the results do in fact vary, and that to some degree they vary as expected. Industry selection is strongest when resources are scarce; i.e. in the trough and in the contraction. During these periods the structural transformation of the expansion is evaluated and resources are allocated away from industries that are found to be inferior. Firm selection, on the other hand, is particularly strong in the trough. This is consistent with firms having room for innovation and experimentation during the expansion, and with the contraction hitting firms in an unpredictable fashion where even well-performing firms may suffer.

These inferences from decomposing productivity growth over the business cycle are somewhat speculative. Further research with more data could help to substantiate them, or may refute them.

One final interesting point concerns the efficiency of selection. Several of the results presented in the paper reports a relatively large role in productivity growth for economic selection which would suggest that there is correlation between initial productivity of a firm, \( z_i \), and its growth over the ensuing period, \( w_i \). This is however, not necessarily the case. The weighted correlation for the entire economy of productivity in 2007 and growth between 2007 and 2010 is 0.03 with \( p \)-value 0.0001 while the weighted rank correlation is 0.18 with \( p \)-value < 0.0001. But if the evolution of labour productivity for this period is decomposed with equation 6 the firm selection effect is ascribed 83 per cent of total evolution. The apparent contradiction arises since \( \text{Cov}(w, z) \) may also be written as \( \beta_w z \text{Var}(z) \), where \( \beta_w z \) is the slope coefficient from a regression of \( w \) on \( z \). The means that economic selection may be strong even when the correlation between fitness and productivity is low as long as the variance of productivity is high. In other words, selection can be inefficient but if there is abundant fuel for evolution, then selection will still move the mean a long way.

A few avenues for further research have already been touched upon above. In
general, the tables in the appendix show that there are a number of exceptions to the general tendencies focused on here, and analyses of the causes of these discrepancies are needed.

Lastly, I have completely avoided the firm learning effect though it varies significantly across the various decompositions. Systematic analyses of variation in the effects, similar to the analyses of the selection effect in the current paper, are still lacking.

Acknowledgements

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Appendix tables
Tab. 3: One- versus two-level decomposition

<table>
<thead>
<tr>
<th>Eq.</th>
<th>Industry Selection</th>
<th>Firm Selection</th>
<th>Learning</th>
<th>Entry</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1992 to 1999, $\Delta Z/Z \times 100 = 3.42$ (1.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9)</td>
<td>-0.76</td>
<td>2.19</td>
<td>-0.22</td>
<td>1.18</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(-0.22)</td>
<td>(0.64)</td>
<td>(-0.06)</td>
<td>(0.34)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>(6)</td>
<td>1.81</td>
<td>-0.22</td>
<td>0.89</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000 to 2010, $\Delta Z/Z \times 100 = 22.61$ (1.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9)</td>
<td>3.34</td>
<td>4.86</td>
<td>6.89</td>
<td>6.28</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.22)</td>
<td>(0.30)</td>
<td>(0.28)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>(6)</td>
<td>8.11</td>
<td>6.89</td>
<td>5.21</td>
<td>2.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1993 to 1997, $\Delta Z/Z \times 100 = 0.81$ (1.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9)</td>
<td>-1.98</td>
<td>1.88</td>
<td>-0.09</td>
<td>0.18</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(-2.43)</td>
<td>(2.31)</td>
<td>(-0.11)</td>
<td>(0.22)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>(6)</td>
<td>0.23</td>
<td>-0.09</td>
<td>-1.83</td>
<td>2.49</td>
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<td></td>
<td>2003 to 2007, $\Delta Z/Z \times 100 = 9.86$ (1.00)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(9)</td>
<td>-2.36</td>
<td>1.99</td>
<td>8.14</td>
<td>1.12</td>
<td>0.97</td>
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<td></td>
<td>(-0.24)</td>
<td>(0.20)</td>
<td>(0.83)</td>
<td>(0.11)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>(6)</td>
<td>1.82</td>
<td>8.14</td>
<td>-0.74</td>
<td>0.64</td>
<td></td>
</tr>
</tbody>
</table>

Decomposition by the single level Price’s equation (equation 6) and the two level version, equation 9. Values are percentage point contributions to total growth. Values in parentheses are shares of total.
Tab. 4: Splitting up the data and summing over periods

<table>
<thead>
<tr>
<th># period(s)</th>
<th>Industry Selection</th>
<th>Firm Selection</th>
<th>Learning</th>
<th>Entry</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>of # year(s)</td>
<td>1992 to 1998, $\Delta Z/Z \times 100 = 2.52$ (1.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1 of 6</td>
<td>-0.52 (0.21)</td>
<td>3.59 (1.42)</td>
<td>-1.71 (-0.68)</td>
<td>-0.07 (-0.03)</td>
<td>1.24 (0.49)</td>
</tr>
<tr>
<td>2 of 3</td>
<td>-1.35 (-0.21)</td>
<td>4.09 (1.42)</td>
<td>-1.65 (-0.68)</td>
<td>-0.89 (-0.03)</td>
<td>2.32 (0.49)</td>
</tr>
<tr>
<td>3 of 2</td>
<td>-0.80 (-0.21)</td>
<td>9.80 (1.42)</td>
<td>-6.71 (-0.68)</td>
<td>-1.17 (-0.03)</td>
<td>1.40 (0.49)</td>
</tr>
<tr>
<td>6 of 1</td>
<td>-2.00 (-0.21)</td>
<td>7.15 (1.42)</td>
<td>-3.29 (-0.68)</td>
<td>-1.81 (-0.03)</td>
<td>2.48 (0.49)</td>
</tr>
<tr>
<td>2000 to 2008, $\Delta Z/Z \times 100 = 16.80$ (1.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 of 8</td>
<td>1.06 (0.06)</td>
<td>4.76 (0.28)</td>
<td>4.88 (0.29)</td>
<td>4.92 (0.29)</td>
<td>1.17 (0.07)</td>
</tr>
<tr>
<td>2 of 4</td>
<td>-0.05 (0.06)</td>
<td>8.46 (0.28)</td>
<td>3.21 (0.29)</td>
<td>1.92 (0.29)</td>
<td>3.25 (0.07)</td>
</tr>
<tr>
<td>4 of 2</td>
<td>0.59 (0.06)</td>
<td>9.22 (0.28)</td>
<td>2.46 (0.29)</td>
<td>0.51 (0.29)</td>
<td>4.02 (0.07)</td>
</tr>
<tr>
<td>8 of 1</td>
<td>0.17 (0.06)</td>
<td>17.19 (0.28)</td>
<td>-3.67 (0.29)</td>
<td>-1.07 (0.29)</td>
<td>4.18 (0.07)</td>
</tr>
</tbody>
</table>

Decomposition by equation 9. Values are percentage point contributions to total growth. Values in parentheses are shares of total. The data are split up into a number of shorter periods which are decomposed separately. The reported numbers are the sums by effect over the periods. For example, the sum of the entry effects for two periods of four years 2000-2004 and 2004-2008 is equal to 1.92 percentage points of total growth.
Tab. 5: Decomposition by business cycle stage

<table>
<thead>
<tr>
<th>Industry</th>
<th>Firm</th>
<th>Eq. Selection</th>
<th>Selection Learning</th>
<th>Entry</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trough: 2000 to 2003, $\Delta Z/Z \ast 100 = 6.35$ (1.00)</td>
<td>Eq. Selection</td>
<td>Selection Learning</td>
<td>Entry</td>
<td>Exit</td>
<td></td>
</tr>
<tr>
<td>(9)</td>
<td>3.55</td>
<td>11.56</td>
<td>−10.34</td>
<td>−0.10</td>
<td>1.69</td>
</tr>
<tr>
<td>(0.56)</td>
<td>(1.82)</td>
<td>(−1.63)</td>
<td>(−0.02)</td>
<td>(0.27)</td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>14.73</td>
<td>−10.34</td>
<td>−0.60</td>
<td>2.56</td>
<td></td>
</tr>
</tbody>
</table>

Expansion: 2003 to 2007, $\Delta Z/Z \ast 100 = 9.86$ (1.00)

| Expansion: 2003 to 2007, $\Delta Z/Z \ast 100 = 9.86$ (1.00) | Eq. Selection | Selection Learning | Entry | Exit |
| (9) | −4.16 | 4.52 | 8.14 | −0.62 | 1.97 |
| (−0.42) | (0.46) | (0.83) | (−0.06) | (0.20) |
| (6) | 3.67 | 8.14 | −2.88 | 0.92 |

Contraction: 2007 to 2010, $\Delta Z/Z \ast 100 = 4.95$ (1.00)

| Contraction: 2007 to 2010, $\Delta Z/Z \ast 100 = 4.95$ (1.00) | Eq. Selection | Selection Learning | Entry | Exit |
| (9) | 3.45 | 3.20 | −2.59 | −0.61 | 1.50 |
| (0.70) | (0.65) | (−0.52) | (−0.12) | (0.30) |
| (6) | 6.17 | −2.59 | −1.10 | 2.47 |

Decomposition by equations 6 and 9. Values are percentage point contributions to total growth. Values in parentheses are shares of total. The reported numbers are sums of three or four decomposition of one year each.

References


