Bottom-up fabrication of graphene antidot lattices (GALs) has previously yielded atomically precise structures with subnanometer periodicity. Focusing on this type of experimentally realized GAL, we perform density functional theory calculations on the pristine structure as well as GALs with edge carbon atoms substituted with boron or nitrogen. We show that $p$- and $n$-type doping levels emerge with activation energies that depend on the level of hydrogenation at the impurity. Furthermore, a tight-binding parametrization together with a Green’s function method are used to describe more dilute doping. Finally, random configurations of impurities in moderately doped systems are considered to show that the doping properties are robust against disorder.

DOI: 10.1103/PhysRevB.93.245420

1. INTRODUCTION

Since its discovery [1], graphene has shown many interesting properties such as ultrahigh electron mobility [1–3], high transparency [4], and record-breaking mechanical strength [5]. However, one major drawback is the lack of a band gap, which is required for obtaining high on/off ratios in field-effect transistors [6]. Therefore, immense effort has been put into turning graphene into a semiconductor while preserving as much as possible its intrinsic characteristics. A popular method is dimensional narrowing, forming graphene nanoribbons (GNRs), which has been shown to introduce a tunable band gap dependent on the width and chirality [7–9]. Another promising and widely studied method is to periodically alter graphene in two dimensions. Hydrogen adsorption onto graphene on an iridium surface has been shown experimentally to create a periodic pattern and open a band gap [10]. Periodic gating has been studied as well, but was found to not open a band gap large enough for practical applications [11]. Finally, graphene antidot lattices (GALs) can be defined by means of periodic two-dimensional patterning in the form of perforations, which opens a widely tunable band gap depending on the geometry, characteristic dimensions, and chirality that define each element (unit cell) of these superlattices [12].

The above-mentioned methods for opening a band gap have been studied experimentally to a great extent using top-down methods [13–18]. However, fabricating GNRs along this route can lead to scattering from edge imperfections, which has been shown to degrade the transport properties [19,20]. GNRs may also be fabricated by unzipping carbon nanotubes which leads to much more regular edges [21,22]. Electron-beam lithography has been utilized to create GALs with periods down to a few dozen nanometers, and experimentally determined gaps as high as 102 meV have been reported [16]. However, GALs suffer from the same problems as GNRs when fabricated using top-down methods. The structures lack full periodicity and imperfections lead to scattering. Calculations have shown that disorder is detrimental to the electronic properties of GALs, as the band gap vanishes or is significantly lowered [23]. Transport calculations support this finding and show that leakage currents may form through disordered graphene antidot devices [24].

A promising method that can overcome the problems of disorder is to use bottom-up self-assembly for fabrication, which provides much better control of the formed structures. However, research utilizing bottom-up methods to fabricate graphene nanostructures is still in its infancy. Nonetheless, several groups have successfully synthesized various atomically precise nanostructures using such methods. Cai et al. [25] have fabricated GNRs and chevron-shaped GNRs, so-called graphene nanowiggles (GNWs), using surface-assisted coupling of two different precursors on an Au(111) surface followed by cyclodehydrogenation. This yielded narrow, fairly long GNRs and GNWs on the surface. Modified versions of the GNW precursor with pyridinelike nitrogen at one or two sites have been used by Bronner et al. [26] to fabricate doped GNWs. Later, Cai et al. [27] used these precursors to fabricate GNW heterojunctions and heterostructures by changing between pristine and doped precursors during synthesis. These structures were recently studied theoretically by Lherbier et al. [28], who reported reasonably high mobilities as well as charge carrier separation. Two-dimensional structures have also been prepared using bottom-up procedures. A nitrogenated GAL with $C_2N$ stoichiometry has been synthesized by Mahmood et al. [29] via a wet-chemical technique, producing a network of aromatic rings with nitrogen between them, where they measured a band gap of 1.96 eV. Sánchez-Sánchez et al. [30] utilized cyclodehydrogenation to produce BN-substituted heteroaromatic networks from another precursor. Finally, Bieri et al. [31] have used the precursor hexaiodo-substituted macrocycle cyclohexa-m-phenylene (CHP) to produce a GAL on an Ag(111) surface with subnanometer periodicity. These new results on bottom-up techniques for producing atomically precise and even doped graphene structures bring hope that graphene could be used for semiconductor electronics. Despite the high level of control on the atomic scale, these methods have some drawbacks as well. The domain size is still limited, and the synthesis takes place on metallic surfaces, requiring the structures to be transferred after fabrication. However, further optimization of the synthesis could improve the structures for device feasibility.

As mentioned above, doping has been actively pursued in graphene nanostructures in order to fabricate, e.g., junctions for device application. Usual dopants are boron and nitrogen,
as they fit in the lattice easily, but other types of doping have also been studied, such as aluminum, sulfur, and phosphorus [32]. Nitrogen doped graphene has been synthesized by several groups from methods such as chemical vapor deposition (CVD) on copper using methane and ammonia [33] or CVD on nickel using triazine [34]. The transport properties of boron or nitrogen doped graphene were studied theoretically by Lherbier et al. [35], while the effect of unbalanced sublattice nitrogen doping was studied by Lherbier and other co-workers [36]. Isolated boron and nitrogen doping in graphene and near graphene edges has also been studied theoretically [37,38]. Nitrogen doped carbon nanotubes [39] and GNRs [40] have been realized experimentally, and doped GNRs have been studied theoretically to a large extent [6,41–43]. It was shown that the most stable configuration of boron and nitrogen doping is at the edges of the nanoribbon and that nitrogen doping can be either pyridine or pyridiniumlike. Scanning Raman spectroscopy has indicated $p$-type doping in GALs after fabrication from electron-beam lithography and oxygen reactive ion etching [44]. These GALs were similar to other top-down fabricated ones, and it was suggested that the doping stems from the patterning process.

In this paper we study the effect of introducing doping in the GAL synthesized by Bieri et al. [31] in the form of boron or nitrogen impurities. As pointed out by Sánchez-Sánchez et al. [30], the method of cyclodehydrogenation may be extended to more complex systems, provided the precursor can be synthesized. For our study, we assume that a precursor similar to CHP used by Bieri et al. [31] can be synthesized, the only difference being that one of the inner carbons of each molecule is replaced by a nitrogen or boron impurity. We study the electronic properties of these structures using density functional theory (DFT) and employ a tight-binding (TB) parametrization to study the case of more dilute doping. Additionally, a Green’s function formalism is used to determine the activation energy for isolated dopants at a low computational cost. We also introduce impurities randomly and compare the density of states (DOS) with the ordered case to determine the effect of disorder. To our knowledge, there has been no theoretical work on doped GALs, and we thus report the first theoretical evidence of $p$- and $n$-type GAL semiconductors.

II. THEORY AND METHODS

The atomic structure of the pristine GAL used in our study is shown in Fig. 1(a), where the dashed red lines mark the primitive unit cells. The properties of GALs have been studied theoretically by several groups [12,23,45–52]. In the notation in Ref. [48], one synthesized by Bieri et al. is a rotated GAL (RGAL). It turns out that two thirds of RGALs are semimetals, while every third is a semiconductor. Petersen et al. [48] have presented a rule based on structural parameters determining if an RGAL is a semimetal or semiconductor and, according to their rule, the antidot lattice described here is semiconducting. Here we will not study doping in other types of antidot lattices than the experimentally realized one in Fig. 1(a). Therefore, we refer to this type of antidot lattice simply as GAL through the rest of the paper.

![Image](https://example.com/image.png)

**FIG. 1.** Structural unit cells and corresponding Brillouin zones of the GALs studied in this paper. (a) Unit cell of the pristine system and (c) its Brillouin zone and $k$ path for the band structure. (b) Unit cell for a doped system (in the case of no hydrogen termination at the impurity) with (d) the $k$ path for this structure.

We construct the doped systems from modified CHP molecules, where one carbon atom on the inner edge of the molecule is replaced with either boron or nitrogen. We choose the edge site, as this has been shown to be the most stable site for doping in GNRs [6,42,43]. An example of the structure for this unit cell is shown in Fig. 1(b). The figure also shows the Brillouin zones and corresponding band structure $k$ paths for both the pristine and doped systems. Because of broken symmetry in the unit cell containing an impurity, the route for the band structure is different than for the pristine system. We place the impurity at an edge site and vary the hydrogen termination between zero, one, and two hydrogen at the impurity. Previously, Huang et al. [53] have made theoretical studies of boron and nitrogen doping at graphene edges and shown that the favorable termination for edge doping is one hydrogen (pyridiniumlike) both for boron and nitrogen doping. However, Wang et al. [43] have shown that the most stable configuration may be changed to no hydrogen at the impurity (pyridinlike) by varying the ratio between monohydrogenated and dihydrogenated edge carbon. This suggests that the synthesis may be controlled to yield different degrees of hydrogenation at the impurity, for which reason we choose to study all three kinds of hydrogen termination. Doped structures are studied in the fully ordered configuration, meaning that all precursor molecules are oriented in the same direction. Systems with more dilute doping are also studied, for which some molecules are left undoped. We realize that the orientation would be random for practical synthesis, but we focus on ordered cases to keep the computational cost manageable. All structures are planar except for those with dihydrogenated impurities, where only the two hydrogen atoms on the impurity are out of the plane.
The pristine and fully ordered doped GALs are studied via DFT using the ABINIT package [54–57], in which a plane-wave basis set is used to expand the wave function. We use the Perdew-Burke-Ernzerhof generalized gradient approximation (PBE-GGA) functional [58] and the projector augmented wave (PAW) method [59] to solve for the eigenstates of the systems. We use a plane-wave cutoff energy of nearly 600 eV together with an 11 × 11 × 1 Monkhorst-Pack k-grid sampling. The distance between the layers is 10 Å in order to decouple them electronically, and we use a fairly low Fermi smearing of 68 meV. We perform full structural relaxation of all unit cells before calculating band structures. The structures are relaxed until the maximum force is less than 2.6 meV Å⁻¹.

In order to investigate the effects of more dilute doping, we employ a π-orbital TB model to describe the system, meaning that we concentrate on the electronic processes arising from hopping between the pπ, orbitals at each carbon/impurity atom, and disregard bands arising from other orbitals further removed in energy from the Fermi level. The Hamiltonian of the pristine system is given by

$$H_0 = \sum_{\substack{i \neq j}} \varepsilon_p |i\rangle\langle i| + \sum_{i,j} t_{ij} |i\rangle\langle j|,$$

where \(\varepsilon_p\) is the carbon on-site energy and \(t_{ij}\) is the hopping integral between atoms \(i\) and \(j\). We include interactions up to third-nearest neighbors and allow for nonorthogonality in the overlap matrix \(S\). The impurity is modeled solely by shifting the on-site potential on the impurity with respect to \(\varepsilon_p\). The impurity Hamiltonian, which must be added to Eq. (1), then becomes \(H_I = \Delta |l\rangle\langle l|\), where the impurity is located at site \(l\) and \(\Delta\) is the shift of the on-site potential. Other reports include a change in the hopping integral between the impurity and up to its third-nearest neighbors [28,36]. However, we find that this only changes the fit marginally.

We begin by calculating the band structure of the pristine system using DFT and obtaining the optimal TB parametrization. The fit is carried out using the three valence and three conduction bands closest to the Fermi level. Figure 2 shows that the DFT band structure can be fitted with excellent agreement by this TB parametrization. We find it necessary to include third-nearest neighbors in a nonorthogonal model for the fit to be in good agreement with DFT. The parameters for the TB model are listed in the table in Fig. 2. Here subscripts 1, 2, and 3 denote nearest, second-nearest, and third-nearest neighbors, respectively. The structure is a semiconductor, consistent with the rule presented in Ref. [48], and has a rather large band gap of 2.30 eV. This is good agreement with previous DFT calculations for the same structure, i.e., 2.34 eV [60] and 2.48 eV [61]. For reference, we have also fitted the band structure of pristine graphene to this TB model and, again, find excellent agreement. Moreover, the obtained parameters are in good agreement with those reported by Grüneis et al. [62].

Having an accurate TB parametrization of the electronic structure allows us to efficiently employ a Green’s function formalism to analyze the doping level in the case of completely isolated dopants. This formalism for nonorthogonal models was previously developed and used to describe bulk and edge doping in graphene, see Refs. [37,38] for further details on the derivation. Although the Green’s functions in these papers are simplified due to the nearest neighbor TB model employed, Green’s functions may be calculated for nonorthogonal TB models with an arbitrary number of neighbors. Therefore, we may use this formalism together with our third-nearest neighbor model as well. The theory shows that modeling the impurity by only adjusting its on-site potential yields the following particularly simple expression for the impurity perturbed Green’s function at lattice site \(l\):

$$G_{il}(z) = \frac{G_{il}^0(z)}{1 - \Delta G_{ll}^0(z)},$$

where the Green’s functions are given by \(G_{il}(z) = (z - S^{-1}H_0)^{-1}\) and \(G_{il}^0(z) = (zS - H_0)^{-1}\). For a semiconductor, the doping level shows up as a pole contribution in the band gap of the impurity local density of states (LDOS). In the limit of vanishing broadening, this approaches a Dirac delta function. The energy of this state, i.e., the doping level, may be evaluated in a simple manner by considering Eq. (2). The impurity LDOS is given by \(L(\omega) = -\pi^{-1} \text{Im}[G_{ll}(\omega)]\), which diverges when \(\text{Re}[G_{ll}^0(\omega)] = 1/\Delta\) and \(\text{Im}[G_{ll}^0(\omega)] = 0\) are both satisfied. This means that the doping level may be determined by evaluating the Green’s function, assuming the impurity on-site energy shift is known.

The above methods treat independent electrons only. The theory may be elaborated to include electron-electron interactions. This gives rise to a complex self-energy manifesting itself in a quasiparticle energy shift and a finite lifetime. This
was studied in Ref. [38], where it was found that the occupancy changes marginally for nitrogen doping and up to a few percent for boron doping. Because the changes are so small, we restrict our analysis to treat independent electrons only.

III. RESULTS

We now proceed to study the effect of replacing one edge carbon in the unit cell with either a boron or nitrogen atom as illustrated in Fig. 1(b). Figure 3 shows DFT band structures for both boron and nitrogen doped GALs with different hydrogen termination. From the top panels, we see that, as expected, boron doping introduces an acceptor level near the highest valence band, which moves closer to the conduction bands as the number of hydrogen atoms on the impurity increases. In the case of two hydrogen atoms, the doping level has even moved across the Fermi level of the pristine structure. Similarly, nitrogen doping introduces a donor level close to the lowest conduction band which moves towards the valence bands as the number of hydrogen atoms at the impurity increases. In the case of no impurity hydrogenation, the doping level is very close to the conduction band edge. Furthermore, we note that the remaining band structure does not change appreciably.

We use the parametrization of the pristine system as the basis for the TB model describing the perturbed systems. In our effort to make a good, yet simple model for the perturbed systems, we model the impurity by only adjusting the on-site element at the impurity site, making \( \Delta \) the only fitting parameter for the perturbed systems. Examples of fits for boron and nitrogen doping, both terminated by one hydrogen (corresponding to Figs. 3(b) and 3(e), respectively), are shown in Fig. 4. In the fit we include only the two valence (conduction) bands closest to the Fermi level for boron (nitrogen) doping. The fits demonstrate that such a simple model still yields a very good description of the bands in the vicinity of the Fermi energy for both types of doping. The fitted values of \( \Delta \) are listed in Table I. Only for boron terminated by two hydrogen atoms were we unable to obtain a satisfactory fit. From the trend of nitrogen doping and boron with lower degrees of hydrogenation, we expect the on-site energy shift to be large for boron terminated by two hydrogen. This suggests that the correction from electron-electron interactions due to a complex self-energy could be more significant. However, especially for low values of the on-site energy shift as found for most of the systems studied here, the correction will be small [38].

![Figure 3](image-url) **FIG. 3.** DFT band structures of boron or nitrogen doped GALs for different hydrogen termination on the impurity. (a)–(c) Boron doping and (d)–(f) show nitrogen doping, in both cases terminated by zero, one, and two hydrogen atoms at the impurity, respectively.

![Figure 4](image-url) **FIG. 4.** DFT and fitted TB band structures of boron (left) and nitrogen (right) doped GALs, both terminated by one hydrogen atom at the impurity. Full colors/lines show the bands used for the TB fit, while weak colors/lines show the rest of the band structure.
TABLE I. Fitted values of $\Delta$ for boron and nitrogen doping and for different hydrogen terminations at the impurity.

<table>
<thead>
<tr>
<th>Impurity hydrogenation</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boron $\Delta$ (eV)</td>
<td>3.22</td>
<td>8.20</td>
<td>–</td>
</tr>
<tr>
<td>Nitrogen $\Delta$ (eV)</td>
<td>−0.88</td>
<td>−2.12</td>
<td>−6.94</td>
</tr>
</tbody>
</table>

The unit cells in Fig. 1(b) used for the DFT calculations are relatively small and place the impurities only 12.8 Å apart. This is also evident from the significant dispersion of the impurity bands seen in the band structures of Fig. 3. The TB parametrization allows us to go comfortably beyond that, and calculate band structures for supercells consisting of $N \times N$ precursor unit cells, where each supercell contains only one impurity. Specifically, we study the doping level as the doping concentration decreases. Figure 5 shows band structures of $1 \times 1$, $2 \times 2$, and $3 \times 3$ supercells containing only one impurity for which $\Delta = -2$ eV. It is clear that the midgap impurity band becomes increasingly flatter as the cell size increases. In this case, a $3 \times 3$ supercell is enough to get a nearly dispersionless impurity band. However, for values of $\Delta$ closer to zero, the convergence is worse and a much larger cell is required. This is not surprising because the extent of the wave function associated with these impurity levels is determined by their distance to the nearest band and, consequently, shallower donors/acceptors tend to be hybridized over larger spatial scales. Note that the most striking impact of changing the supercell size takes place in the impurity band. The denser nature of the conduction and valence bands as we go from Figs. 5(a) to 5(c) is simply due to band folding, as the unit cell size is increased.

Once the doping level in the TB band structure is sufficiently flat, we are able to determine the activation energy of the system. However, we may also calculate the doping level of completely isolated impurities using the Green’s function technique described in Sec. II. The local Green’s function at the impurity site is shown in Fig. 6, where the band gap is clearly seen as the region where the imaginary part is zero. The real part of the Green’s function is used to calculate the activation energy. For negative values of $\Delta$ (corresponding to $n$-type doping), the activation energy is given by $E_A = E_c - Ed$, while it is $E_A = Ed - Ev$ for positive values (corresponding to $p$-type doping). Here $Ed$ refers to the energy of the doping level, while $Ev$ and $Ec$ are the highest (lowest) energy of the valence (conduction) bands, respectively. The doping level is found by solving $\text{Re}\{\tilde{G}_0(z)\} = 1/\Delta$ for the energy $z$ within the band gap region. This calculation is very fast when a converged Green’s function is provided. Due to the relatively simple structure of the pristine GAL, calculating the Green’s function is computationally straightforward. The activation energy as a function of $\Delta$ is shown in Fig. 7, where the fitted values from Table I are marked with green asterisks. The activation energies from the supercell band structures are also shown (green asterisks).

**FIG. 5.** TB band structures of supercells containing (a) $1 \times 1$, (b) $2 \times 2$, and (c) $3 \times 3$ precursor unit cells [see Fig. 1(b)] with only one impurity per supercell. The impurity is modeled using $\Delta = -2$ eV.

**FIG. 6.** Green’s function for the impurity site of the GAL with a broadening of 5 meV.

**FIG. 7.** Activation energy for different values of the impurity on-site energy shift $\Delta$, where the Green’s function method (solid blue line) is compared with the supercell band structure method (red circles). The actual values of $\Delta$ obtained $ab\; initio$ and listed in Table I are also shown (green asterisks).
Disordered of disorder in moderately doped systems. Figure 8 shows the Therefore, we study the effect of introducing various types doping in GALs. However, in an experiment, the doped molecules are not expected to be evenly separated or ordered. 4.1 meV. hydrogenation results in a very low activation energy of provided. We also note that nitrogen doping with no impurity dispersionless impurity level requires very large supercells for values of Δ close to zero, making them extremely time consuming. The results from the supercell band structures (red circles) are in excellent agreement with the curve obtained using the Green’s function method, thus verifying the result. Because of the slow convergence for Δ close to zero, the supercell method has only been used for values outside the ±0.5 eV regime. This is shown in the inset of the figure, where the agreement is seen to continue for all the values provided. We also note that nitrogen doping with no impurity hydrogenation results in a very low activation energy of 4.1 meV.

In this paper we have focused on very dilute and ordered doping in GALs. However, in an experiment, the doped molecules are not expected to be evenly separated or ordered. Therefore, we study the effect of introducing various types of disorder in moderately doped systems. Figure 8 shows the density of states (DOS) for various types of disorder in a GAL, where one out of nine molecules contain an impurity, meaning that 0.31% of the carbon atoms are replaced by impurities. We consider three types of disorder: (a) fixed position and random orientation, (b) random position and fixed orientation, and (c) full disorder with both random position and orientation. For the first type of disorder (a), the impurities are never close together and their coupling is therefore weak. The average DOS for the disordered structures shows a peak at the doping level that is nearly identical to the ordered case and a slightly smoothed curve for other energies. In the case of random position and fixed orientation (b), the dopants may be much closer and a broadening of the peak is observed. Notably, the peak is broadened more to the right, where a small peak is observed, which is attributed to the cases where doped molecules are adjacent. Full disorder (c) introduces two smaller peaks on either side of the main peak. However, for this doping concentration, these peaks are much smaller than the main peak. The extra peak in (b) is not observed for full disorder. Impurities in adjacent molecules in (b) will always be separated by the same distance, giving rise to a more pronounced feature in the DOS, whereas the distance between impurities in adjacent molecules in (c) may take several different values because of random orientation. This causes the disorder in (b) to seemingly affect the peak more than in (c) even though the type of disorder in (c) resembles the expected result from experiments more. The results from Fig. 8 show that introducing doped molecules randomly in an otherwise pristine lattice only affects the doping properties to a small extent.

The above results point to the advantage of using the Green’s function method even when the system is not in the strictly dilute limit. Its application is not limited in any way to the specific structure we considered here, and is applicable to any system of dilute impurities in a crystal lattice, such as other antidot lattice geometries, provided an accurate TB model is available. We are convinced that our analysis of the properties of doped GALs will be useful for future studies of electronic and transport properties of junctions in graphene nanostructures.

IV. CONCLUSION

We have studied the effect of substituting an edge carbon atom in a GAL with either boron or nitrogen. By means of DFT, we calculate electronic band structures for GALs where the impurity is terminated by zero, one, or two hydrogen atoms. We perform TB parametrizations describing both the pristine and doped systems with high accuracy, which are used together with a Green’s function method to study more dilute doping. Boron doping introduces an acceptor level near the valence band edge, which moves towards the conduction bands as the hydrogenation on the impurity increases. Similarly, nitrogen introduces a donor level near the conduction band, which moves towards the valence bands with increasing hydrogenation. This indicates that the properties of doped GALs may be tuned, provided the impurity hydrogenation is controllable during synthesis, as suggested in Ref. [43]. Furthermore, we studied the effect of disorder at moderate doping concentration and found that such systems are fairly robust against disorder. Our work is the first step on the way to understanding doping in GALs. We believe that the parametrizations reported here, together with the activation energy analysis, are useful tools for further studies of this and other types of doped GALs.
ACKNOWLEDGMENTS

S.J.B. and T.G.P. gratefully acknowledge the financial support from the Center for Nanostructured Graphene (Project No. DNRF103) financed by the Danish National Research Foundation and from the QUSCOPE project financed by the Villum Foundation. S.J.B. further acknowledges the hospitality and support of the NUS Centre for Advanced 2D Materials, where part of this research was conducted. V.M.P. acknowledges the support by the National Research Foundation (Singapore) through the CRP grant “Novel 2D materials with tailored properties: beyond graphene” (R-144-000-295-281).