Abstract—Based on the hierarchical control structure in islanded Micro-Grid (MG) systems, the coupling/tradeoff effects in different levels are analyzed in details. In the primary level, analyses of the coupling effects among droop control gains, line impedance differences, output reactive power and voltage magnitudes are provided specifically. In the secondary level, the tradeoffs between accurate reactive power sharing and voltage magnitudes regulation are further detailed. The analysis results can provide a guideline for the design of MG structure and its control parameters. In addition, novel containment-based controller is proposed to control the voltage into a reasonable range which is the first time to apply this algorithm in MG. Furthermore, dynamic-consensus-based controller is used to guarantee accurate reactive power sharing. The combination of controllers offers a coordinated distributed operation and enhanced system performance. Finally, experimental results are shown to validate the effectiveness of the proposed method.

Keywords—Coupling/Tradeoff analysis, Containment-based Voltage Control, Reactive power sharing, Micro-Grid

I. INTRODUCTION

Based on the concept of MG, hierarchical control structure consisted of three levels, namely primary level, secondary level and tertiary level, is proposed [1] and successfully applied [2]. The primary level deals with the inner voltage/current control and power sharing control. Droop control is commonly applied to establish the linear function between active power and frequency (P-f) and reactive power and voltage (Q-V) achieving power management in a decentralized manner [3]. The secondary level control is used to restore the system frequency and voltage to nominal values following load change. The tertiary level deals with the energy management and optimization issues.

In the primary level, even though the frequency droop control can achieve accurate active power sharing, output reactive power and voltage magnitudes are sensitive to the line impedance differences and localized voltage information. The virtual impedance method [4] was proposed firstly to solve the coupling problem between active power and reactive power. In [5], the reactive power sharing problem is analyzed and a two-step estimation method is proposed to calculate the local reactive load and regulate the droop gains accordingly to achieve better reactive power sharing. Similarly, adaptive virtual impedance method can be implemented with the centralized EMS system to achieve the same objective [6]. In [7], based on the physical information of networked MG, the relationship among active load change, reactive load change and reactive power sharing error is analyzed and genetic algorithm is applied to optimize the virtual impedance, achieving reactive power sharing. However, it’s an offline parameter optimization method with the network information of MG. Furthermore, the problem of voltage magnitudes deviation has been rarely considered in existing methods, while, the using of virtual impedance may enlarge the voltage magnitude deviation.

To evaluate this issue, paper [8] gives a brief tradeoff analysis between the reactive power sharing and the voltage magnitude regulation and an averaging-based method has been proposed to achieve a tunable compromise. However, the effects of droop gains and line impedance differences are not considered for analysis in details. In addition, fixed average voltage regulation (at nominal value) is debatable under some conditions according to the standard [9]. Accordingly, a more flexible regulation scheme is required.

In this paper, the coupling effects from droop gains, line impedance differences to output reactive power sharing and voltage magnitudes are analyzed in the primary level in details with two conditions. Furthermore, the tradeoff between accurate reactive power sharing and voltage magnitudes regulation in the secondary level is evaluated. The analysis results can provide a guideline for the design of MG and its control parameters. To handle the tradeoff properly in the secondary control level, a novel containment-based controller is proposed to control output voltage magnitudes into a prescribed range while guarantee the accurate reactive power sharing by dynamic-consensus-based controller.

Fig. 1 Simplified model of the microgrid
The paper is organized as follows. In section II, the couplings within the primary level are analyzed. In Section III, the tradeoff in the secondary level is analyzed. In section IV, containment-based controller and dynamic-consensus-based controller are proposed. Experimental results are presented in Section V. Finally, the paper is concluded in Section VI.

II. COUPLING ANALYSIS WITHIN THE PRIMARY LEVEL

CONTROL

The simplified structure of an islanded MG is shown in Fig. 1, which shows a typical islanded MG with two DGs supplying public loads. For convenience, the analyses are based on the two DGs, which can be easily extended to more DGs.

During the islanding operation, DG units can be operated under conventional reactive power-voltage magnitude droop control, defined as:

\[ E_r = E^N - n_i Q_i \]  

(1)

where \( E_r \) is the reference voltage for the inner control loop, \( E^N \) is the nominal voltage magnitude, \( n_i \) is the droop gain, \( Q_i \) is the output reactive power.

In Fig. 1, it is shown that each DG unit is connected to the public load bus through a feeder which is usually distinguished in types with different line impedances. According to the line impedances, the voltage drop can be calculated as:

\[ E_r - V_{pcc} = \Delta U = (R_i + jX_i) \left( \frac{P - jQ}{V^N} \right) \]

\[ = \frac{R_i P + X_i Q_i}{V^N} + j \frac{P X_i - Q_i R_i}{V^N} \approx \frac{R_i P + X_i Q_i}{V^N} \]  

(2)

where \( V^N \) is the nominal voltage in the system, \( R_i, X_i \) is the line impedance from the \( i \)-th DG connected to the public load.

Compared with the real part of equation (2), the imaginary part of equation (2) is so small that this term is ignored normally. Meanwhile, it assumed that the \( X_i >> R_i \), thus the equation (2) can be simplified furthermore as:

\[ E_r - V_{pcc} = \frac{X_i Q_i}{V^N} \]  

(3)

From equation (3), it can be seen that due to the line impedance differences, the voltage drop will be different.

In the following, two conditions are considered.

A. Two DGs with same droop gains \( n_1 = n_2 = n \)

For this condition, combing the equation (1) and (3), the ratio of output reactive power is as

\[ \frac{Q_1}{Q_2} = \frac{X_2 + n V^N}{X_1 + n V^N} \]  

(4)

Meanwhile, the ratio of output voltage magnitudes is as

\[ \frac{E_1}{E_2} = \frac{X_1 E^N + n V^N V_{pcc} * n V^N + X_1}{X_2 E^N + n V^N V_{pcc}} \]  

(5)

The coupling effects of droop gains and line impedance differences on output reactive power sharing and voltage magnitudes is analyzed by the control variate method, as shown in Fig. 2. In order to make the figure more clearly, the logarithm horizontal axis is used.

Fig. 2 (a.1) shows the coupling effects of droop gains and the ratio of line impedance differences on reactive power sharing. To be more clearly, Fig. 2 (a.2) shows the relationship between droop gains and the error of reactive power sharing. It is shown that the reactive power sharing error can be considerably reduced by increasing droop values regardless of line impedance ratios. Fig. 2 (a.3)
shows the relationship between the ratio of line impedance differences and the error of reactive power sharing. It is shown that with the increasing ratio of line impedance, the reactive power sharing error is also increased, while large droop gain can decrease this effect significantly.

From the other standpoint, Fig. 2 (b.1) shows the effect of droop gains and the ratio of line impedance differences on deviation of output voltage magnitudes. As shown in Fig. 2 (b.2), when the droop gains are very small or relatively large, the deviation of output voltage magnitudes is effectively decreased. However, during this change process, there exist several extreme deviations for different line impedances conditions. Fig. 2 (b.3) proves the above argument. Thus, combination with the results gotten from Fig. 2, the relative larger droop gains within the stability margin can relieve the reactive power sharing error and voltage magnitude error effectively at the same time.

To be further discussion, the tradeoff relationship can be identified by comparing the black and red lines in Fig. 2 (a.2) with Fig. 2 (b.2). The black line shows that when the line impedance $X_{01}$ is larger than $X_{02}$, the output voltage magnitude $E_1$ is larger than $E_2$ and the output voltage $Q_1$ is smaller than $Q_2$. By comparison, the red line shows that when $X_{01} < X_{02}$, $E_1 < E_2$ and $Q_1 > Q_2$.

B. Two DGs with different droop gains $n_1$ and $n_2$

The above analysis gives an evaluation of absolute droop gains on voltage magnitudes and reactive power sharing, while this part conducts an analysis with different droop gain ratios. The coupling effects of the ratio of droop gains and the ratio of line impedance differences on output reactive power and voltage magnitudes can be also analyzed by the control variate method as shown in Fig. 3. The logarithm horizontal axis is used.

For convenience, the error of reactive power sharing is expressed as:

$$\frac{n_1Q_1}{n_2Q_2} = \frac{n_1(X_2 + n_1V^*)}{n_2(X_1 + n_1V^*)}$$

(6)

Meanwhile, the ratio of voltage magnitudes is defined as:

$$\frac{E_1}{E_2} = \frac{X_1E^1 + n_1V^*V_{pcc}}{n_1V^* + X_1} \cdot \frac{n_2E^2 + n_2V^*V_{pcc}}{X_2E^2 + n_2V^*V_{pcc}}$$

(7)

In this paragraph, the effect about ratio of droop gains on the error of reactive power sharing is analyzed. In Fig. 3 (c.2), it is shown that with the increasing droop gain ratio, the error of reactive power sharing is reduced. However, as shown in Fig. 3 (c.3), even though the black line and the red line have the same ratio between droop gains $n_1$ and $n_2$ (either $n_2/n_1=0.1$ or $n_2/n_1=0.1$), the errors of reactive power sharing have different variation features with increasing line impedance differences. To illustrate, in this analysis program, the droop gain value $n_1$ is always fixed, while the absolute value of $n_2$ is varied according to the ratio which means the absolute droop gains are increased with the increasing ratio. Thus the reason why the ratio of reactive power sharing of red line in Fig. 3 (c.3) is kept at 1 is because the absolute droop gains are increased. The ratio of reactive power sharing in black line in Fig. 3 (c.3) is enlarged because the absolute droop gains are decreased.

In line with results from Fig. 2 (a.2) and (a.3), the above results also validate that larger droop gains offers more accurate power sharing and the ratio of droop gains has much less effect on that.

The effect about the ratio of line impedance differences on the error of reactive power sharing is analyzed in this paragraph. As shown in Fig. 3 (c.3), with the increasing line impedance differences, the error of reactive power sharing is...
increased naturally. Similarly with the analysis in last paragraph, the curves in Fig. 3 (c.2) show that the absolute value of the line impedances really matters the sharing error compared with the relative ratio between the impedances. Take the black and red lines in Fig. 3 (c.2) as an example, in black line, if the absolute values of two line impedances are quite small, even though the ratio is quite large, it will not cause large power sharing error between DGs. By comparison, the red line with large absolute line impedance difference shows that the error of reactive power sharing is quite large.

As shown in Fig. 3 (d.2) and (d.3), with the same analysis process, it can be concluded that the voltage magnitude differences also depends on the absolute values of droop gains and line impedances rather than the pure ratios between units. In addition, from the Y-axis in Fig. 3 (c.2), (c.3) and (d.2) (d.3), the deviations of reactive power sharing are always more serious than that of voltage magnitudes.

Furthermore, comparing the blue lines with $X_2/X_1=1$ in Fig. 2 (a.2) and Fig. 3 (c.2), it is shown that even though the line impedances are same, reactive power sharing error also exists if the droop gains are different.

Based on above analyses, six main conclusions can be obtained as follows:
1). Larger droop gains can weaken the effect of line impedance differences and decrease the error of reactive power sharing accordingly;
2). Both larger and smaller droop gains can help to decrease the error of voltage magnitudes;
3). Based on conclusion 1) and 2), larger droop gain is more suitable to decrease both the two errors under the assumption that the droop gains satisfy the stability requirements;
4). The problem of reactive power sharing is more serious than that of the deviations of voltage magnitudes;
5). The absolute values of the droop gains and line impedances have decisive influence over the voltage magnitude deviations or reactive power sharing errors rather than the relative ratios;
6). It is better to connect DGs with same capacities to achieve the reactive power sharing with less deviation. If it is impossible, it is better that DGs with high rating powers should be connected with relative high line impedances according to the equation (6) in order to achieve the reactive power sharing, which means long transmission lines in the system.

III. TRADEOFF ANALYSIS WITHIN THE SECONDARY RESTORATION CONTROL

Even though the deviation of voltage magnitudes is small, the absolute values are deviated from nominal value due to the feature of droop control. Thus, the objectives of secondary regulation are as follows: 1) to enhance voltage magnitudes into a nominal range ($\pm 1\%$ nominal value) [9]; 2) to achieve accurate reactive power sharing in proportional at the same time. In other words, while considering the reactive power sharing issue, local buses voltage magnitudes deviation should be allowed, but a boundary according to standards is necessary.

Before going to the proposed control method, this section gives an insight about the tradeoff effects between accurate reactive power sharing and local bus voltages magnitudes regulation, as shown in Fig. 4-Fig. 7 (in which black lines denote the condition before secondary regulation, red lines stand for the condition after secondary control). The analyses are divided into two parts, Fig. 4 and Fig. 5 are based on only the voltage magnitudes restoration (restore DG output voltage magnitudes to nominal value), while Fig. 6 and Fig. 7 are based on only the reactive power sharing regulation (achieve accurate reactive power sharing).
magnitudes to the nominal value, $Q_1$ becomes much smaller and $Q_2$ becomes much larger. In addition, as shown in Fig. 5 with larger droop gains and smaller line impedance differences, after voltage restoration, even though $Q_1$ becomes larger than before, the deviation between $Q_1$ and $Q_2$ becomes larger. As shown in Fig. 4 and 5, no matter what kind of parameter conditions are in the system, the tradeoff effect is very obviously.

In Fig. 6, with smaller droop gains and larger line impedance differences, the deviation of voltage magnitudes become slight larger after reactive power regulation. Compared with the Fig. 4 and Fig. 5, the deviation degree of reactive power sharing is more serious than the deviation degree of voltage magnitudes. As shown in Fig. 7, with relatively larger droop gain and smaller line impedance difference, after the regulation, the deviation of voltage magnitudes is almost no change.

**IV. CONTAINMENT-BASED AND DYNAMIC-CONSENSUS-BASED CONTROLLER FOR VOLTAGE AND REACTIVE POWER**

In this section, a new concept for voltage magnitudes control is proposed namely containment-based control, achieving to limit the voltage magnitudes into a prescribed range. In addition, the dynamic-consensus-based control is applied to realize accurate proportional reactive power sharing according to DG capacities.

**A. Definition and Notations**

Let $C$ be a set in a real vector space $V \subseteq R^p$. The set $C$ is called convex if, for any $x$ and $y$ in $C$, the point $(1-z)x + zy$ is in $C$ for any $z \in [0,1]$. The convex hull for a set of points $X = \{x_1, \ldots, x_i\}$ in $V$ is the minimal convex set containing all points in $X$. We use $Co(X)$ to denote the convex hull of $X$. In particular, when $V \subseteq R^1$, $Co(X) = \{x | x \in [\min x_i, \max x_i]\}$. Two adjacency matrices are introduced: $A = [a_{ij}] \in R^{m \times m}$ with $a_{ij} > 0$ if node $i$ can receive information from node $j$ otherwise $a_{ij} = 0$, $B = [b_{ij}] \in R^{m \times m}$ with $b_{ij} > 0$ if node $i$ can receive information from one of the two reference leaders. For the $n$-agent system, an agent is called a reference leader if the agent does not receive information from neighbors. An agent is called a follower if an agent has one or more neighbors. We use $N_i$ to denote the set of $i$-agent’s neighbors chosen from followers and use $R_i$ to denote the set of reference leaders which can give its information to $i$-agent directly.

**B. Containment-based Control for Voltage Magnitudes**

This controller generates a correction term $\delta e_{vi}$ to control the output voltage within a prescribed range. Each DG controller based on the containment control is defined as:

$$\delta e_{vi} = - \sum_{j \in N_i} a_{ij} (v_i - v_j) - \sum_{i \in R_i} b_{ij} (v_i - v_j)$$ \hspace{1cm} (8)

where $v_i$ and $v_j$ are the voltage magnitude of DG $i$ and DG $j$ respectively, $v_i$ is the voltage boundary set given by two boundaries, which can be either upper boundary $V_{\text{max}}$ or lower boundary $V_{\text{min}}$. $\beta$ is a constant value. The range is chosen as follows [9]:

$$V_{\text{max}} = 325 \times (1+1\%) \approx 330 V, \quad V_{\text{min}} = 325 \times (1-1\%) \approx 320 V.$$  

Only the information from direct neighboring units has been used by the controller which can be seen as the distributed controller.

The output of equation (8) is then fed to a dynamic integrator to calculate the voltage compensation $\int \delta e_{vi}$.

**C. Dynamic-Consensus-Based Control for Reactive Power Sharing**

This controller is used to calculate the reactive power mismatch $\delta e_{qij}$ with the direct neighboring units, defined as:

$$\delta e_{qij} = - \sum_{j \in N_i} a_{ij} (n_Qi - n_Qj)$$ \hspace{1cm} (9)

where $n_i$ is the reactive power droop gain for DG $i$. Then this load mismatch is fed to an integrator to generate another voltage compensation $\int \delta e_{qij}$ to achieve reactive power sharing in proportional.

Finally, the nominal voltage value in droop controller is shown in equation (10):

$$\delta e_{vi}^{\ast} = E_{\text{droop}} + \int \delta e_{vi} + \int \delta e_{qij}$$ \hspace{1cm} (10)

The configuration of the proposed controller is shown in Fig. 8.

**V. EXPERIMENTAL RESULTS**

The proposed control scheme is implemented and tested in an experimental MG setup operated in islanding mode, as shown in Fig. 9. The setup consists of four parallel-configured Danfoss FC302 inverters, a real-time
dSPACE1006 platform, LCL filters and two RL loads. Communication structure is shown in Fig. 9. Rated power the DG converters have the ratio 2: 2: 1: 1 from DG1-DG4. The nominal voltage magnitude is set to 325 V with ±1% voltage boundary for the containment-based control. The experimental results are shown in Fig. 10 and 11. At t=T0, four DGs are controlled by conventional droop control and at t=T1, the proposed controller is activated.

VI. CONCLUSION

The coupling/tradeoff effects in different levels are analyzed in details in islanded MG systems. In the primary level, detailed analyses of the coupling effects among droop control gains, line impedance differences, reactive power sharing and output voltage magnitudes are provided. Furthermore, the tradeoffs between the accurate reactive power sharing and voltage magnitudes regulation are analyzed in the secondary level. Based on the analysis results, it can provide a guideline for the design of MG structure and its control parameters. Furthermore, the novel containment-based controller and dynamic-consensus-based controller are proposed to properly regulate output voltage magnitudes into a prescribed range and achieve accurate reactive power sharing respectively, offering an autonomous operation and enhanced system performance. Experimental results are shown to prove the effectiveness of the proposed method.

VII. REFERENCES