Channel Characterization Using Large Scale Uniform Arrays with Sidelobe Suppression
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Channel Characterization Using Large Scale Uniform Arrays with Sidelobe Suppression

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Abstract—A general form of Dolph-Chebyshev weighting methods is summarized for uniform linear arrays, uniform rectangular arrays and uniform cube arrays. In this paper, we adopt beamforming technique with both uniform amplitude weighting method and Dolph-Chebyshev amplitude weighting method to detect the channel multipaths. Numerical simulation results demonstrate the effectiveness of the sidelobe suppression technique.

Index Terms—Dolph-Chebyshev weighting, beamforming, multipath, detect.

I. INTRODUCTION

As the demand for high data rate mobile communications continuously increases, the exploitation of underutilized spectrum at high frequencies, especially the mm-Wave frequencies are proposed [1]. The implementation of massive antenna arrays and beamforming techniques becomes feasible due to the smaller wavelength at higher frequencies. Accurate information of the multipath channels, e.g., amplitude, angle and delay, are very important for system design and performance evaluation of future 5G systems. Uniform linear array (ULA) has gained its popularity due to simplicity in design and analysis. Further, its 2D and 3D extended version, i.e., the uniform rectangular array (URA) and the uniform cube array (UCuA) are extensively investigated. To exploit the multipaths for mm-wave communications, a dynamic range up to 35dB (i.e., power with respect to the strongest path) is often targeted. This requirement is quite demanding in practice. For high gain antennas, it is difficult to achieve due to the antenna size limitations. For antenna arrays, sidelobes are typically high (e.g., around -13.5dB for ULAs with uniform amplitude weights).

To achieve low-level side lobes for the ULAs, non-uniform amplitude weighting is widely utilized in practice. The binomial amplitude weighting theoretically eliminates the side lobes by using the binomial coefficients [3], [4]. However, this method is in general impractical because it results in larger beamwidth (compared to Dolph-Chebyshev method and Taylor method, etc.) and a very low efficient antenna elements [5]. Consequently, other more efficient amplitude weighting methods with sidelobe levels comparable to the sidelobes of binomial method are more desirable. Two of the most widely applied amplitude weighting methods are Dolph-Chebyshev method and Taylor method [6], [7]. Dolph-Chebyshev amplitude weights for ULAs were calculated based upon the properties of Chebyshev polynomials [6]. This approach results in all the sidelobes at the same level and a possible minimum beamwidth for the given sidelobe level. Taylor method is similar to the Dolph-Chebyshev method that it is also developed for the ULA and the maximum sidelobe level can be specified but with the decaying sidelobes [7]. For the specified sidelobe level, the sidelobes of Taylor weighting are smaller than that of Dolph-Chebyshev weighting, while the mainlobe beamwidth of Taylor weighting is broader than that of Dolph-Chebyshev weighting. In addition, the one-dimension Dolph-Chebyshev weighting for ULAs can be transformed into two-dimension and three-dimension weights for URA and UCuA, respectively.

To estimate the aforementioned channel parameters, an array has many advantages over a single directive antenna element [2]. For example, the array is able to steer beams electronically and achieve higher spatial resolution. A virtual planar array of $10 \times 25$ elements was applied to measure the multipath channel at the urban base station [8]. To improve the accuracy of the angle of arrival (AOA) and delay of arrival estimation, a modified likelihood function with window method was utilized for the measurement data. In [9], an extreme size virtual UCuA was used to estimate the multipath parameters in an indoor office environment. The analysis is based on conventional beam-forming with Hanning window weighting method. However, the weighting method was not discussed and amplitude weighting effect was not explained in the results.

In this paper, we derive the Dolph-Chebyshev weighting method for uniform arrays (UAs) in a general form [10]. Then the UAs are applied to detect parameters of multipaths channel, where the dynamic range is up to 30dB. In order to recover the weak paths (buried by the sidelobes of the strong paths with uniform weighting) and obtain more accurate amplitude estimation, the Dolph-Chebyshev weighting method is applied. Numerical simulations results are provided to demonstrate the performance of the sidelobe suppression method. In this paper, we only discuss and demonstrate Dolph-Chebyshev weighting method, but the general weighting approaches are also applicable by replacing the Chebyshev polynomial with other window polynomials.
II. DOLPH-CHEBYSHEV WEIGHTING FOR UNIFORM ARRAYS

In this paper, we discuss Dolph-Chebyshev weighting for ULAs, URA and UCuAs in a general form. The inter-element spacing in wavelength is \( d_x \), \( d_y \) and \( d_z \) (\( d_x, d_y, d_z \leq 1/2 \)) in the \( x \) direction, \( y \) direction and \( z \) direction, respectively.

Consider a uniform array (UA) composed of \( L \times M \times N \) isotropic antennas with \( L \), \( M \) and \( N \) elements in \( x \)-dimension, \( y \)-dimension and \( z \)-dimension, respectively. We assume the UA forms a beam in the direction of scan \( (\theta_0, \phi_0) \), with elevation angle \( \theta \) measured from z-axis and azimuth angle \( \phi \) measured counter-clockwise from the \( x \)-axis on the \( xy \) plane, as illustrated in Fig. 2 and Fig. 3. The beam pattern can be measured from the \( x \)-axis on the \( xy \) plane, as illustrated in Fig. 2 and Fig. 3. The beam pattern can be expressed in a general form as,

\[
B_{\psi_0}(\theta, \phi) = \frac{1}{LMN} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} w_{l,m,n} \cdot \exp(jl\psi_0) \\
\cdot \exp(jm\psi_y) \cdot \exp(jn\psi_z)
\]

where \( w_{l,m,n} \) is the amplitude weight of the \((l, m, n)\)-element and \((\psi_x, \psi_y, \psi_z)\) is defined as,

\[
\begin{aligned}
\psi_x &= u_{x0} - u_x \\
\psi_y &= u_{y0} - u_y \\
\psi_z &= u_{z0} - u_z
\end{aligned}
\]

with \((u_x, u_y, u_z)\) expressed as

\[
\begin{aligned}
u_x &= 2\pi d_x \sin \theta \cos \phi \\
u_y &= 2\pi d_y \sin \theta \sin \phi \\
u_z &= 2\pi d_z \cos \theta
\end{aligned}
\]

Assume \( P \) is an integer satisfied \( P > 1 \), ULA, URA and UCuA are special cases of UA, where

- With \( L = M = 1, N = P \), (1) is the beam pattern of a ULA with \( P \) elements located on the z-axis, as illustrated in Fig. 1;
- With \( L = 1, M = N = P \), (1) is the beam pattern of a URA with \( P^2 \) elements lying in the \( xy \) plane, as depicted in Fig. 2;
- With \( L = M = N = P \), (1) is the beam pattern of a UCuA with \( P^3 \) elements, as shown in Fig. 3.

By mapping the beam pattern to a Chebyshev polynomial [6], (1) can be given as,

\[
B_{\psi_0}(\theta, \phi) = \frac{1}{R} T_{P-1}(x_o \cos(\psi_x/2) \cos(\psi_y/2) \cos(\psi_z/2))
\]

where \( T_{P-1}(\cdot) \) is the \((P-1)\)th-degree Chebyshev polynomial, \( R \) is the sidelobe level (SIL) defined as \( R = \frac{\text{ sidelobe-peak}}{\text{ mainlobe-peak}} \), \( x_o \) can be determined by

\[
x_o = \cosh\left(\frac{1}{P-1} \cosh^{-1}(R)\right)
\]

According to [6], [10]-[12], we define \( B_{kx,ky,kz} \) as

\[
B_{kx,ky,kz} = B_Z^R(\psi_{kx}, \psi_{ky}, \psi_{kz}) \\
\cdot \exp\left(-\frac{P-1}{2}(\psi_{kx} + \psi_{ky} + \psi_{kz})\right)
\]

where \( \psi_{kx} = (kx - \frac{L-1}{2}) \frac{\pi}{M}, \psi_{ky} = (ky - \frac{M-1}{2}) \frac{\pi}{N} \) and the subscripts \((\cdot)\) denotes conjugate operation.

Then the corresponding weights \( w_{l,m,n} \) can be determined by

\[
w_{l,m,n} = b_{l,m,n} \cdot \exp(-j\frac{P-1}{P} \pi (l + m + n))
\]
The weights $w$ are independent of scanning angle $(\theta_0, \phi_0)$;

- The sidelobes are constant with the level of SIL;
- As the SIL increased, the mainlobe width will be wider;
- The width of the mainlobe changes with scanning angle, and the variation tendencies are same for uniform weighting (all the weights equal to 1) patterns and Dolph-Chebyshev weighting patterns;
- The inter-element spacing $d_x$, $d_y$ and $d_z$ could be unequal;
- Comparing with the separable weighting approach (i.e. $w_{l,m,n} = w_l \cdot w_m \cdot w_n$) for the 2D and 3D UAs, the aforementioned weighting method can achieve a Chebyshev beam pattern in any section, since the beam patterns of the URAs and UCuAs are mapped to a single Chebyshev polynomial, respectively.

### III. MULTIPATH DETECTION WITH SIDELOBE SUPPRESSION

Similar to the derivation in the last section, we discuss the multipath detection problem based on a general model of the UAs.

Assume there are $K$ paths impinging at a UA. The frequency response of the $(l,m,n)$-th element is:

$$H_{l,m,n}(f) = \sum_{k=1}^{K} \alpha_k \exp(-j2\pi f \tau_k) \cdot \exp(jlu_{zk}) \cdot \exp(jmu_{yk}) \cdot \exp(jnu_{zk})$$ \hspace{1cm} (8)

where $K$ is the number of paths, $\alpha_k$ and $\tau_k$ are the complex amplitude and the delay of the $k$-th path of the UA center, respectively, $f$ is the frequency of interest and $(u_{zk}, u_{yk}, u_{zk})$ is defined in (3). The main target is to detect path parameters $\alpha_k$, $\tau_k$, $\theta_k$. By applying conventional beam forming with Dolph-Chebyshev amplitude weighting, we have the estimated pattern,

$$\hat{H}(f, \theta, \phi) = \sum_{k=0}^{K} \alpha_k \exp(-j2\pi f \tau_k) \cdot B_{\psi_k}(\theta, \phi)$$ \hspace{1cm} (9)

where, the term of $\{ \cdot \}$ is substituted by (1) of scan angle $(\theta_k, \phi_k)$. So the total pattern can be obtained by summing the pattern of each path, where the pattern of each path is scaled by $\alpha_k \exp(-j2\pi f \tau_k)$.

The path parameters $\{\alpha_k, \tau_k, \theta_k, \phi_k\}$ for $k \in [1, K]$ can be obtained, according to the following:

- For $\theta = \theta_k$ and $\phi = \phi_k$, we have $B_{\psi_k}(\theta_k, \phi_k) = 1$ and $B_{\psi_i}(\theta_k, \phi_k)$ approaches $\varepsilon(\varepsilon \in [0, 1/SIL])$, for $i \neq k$;
- For $\theta \neq \theta_k$ or $\phi \neq \phi_k$, we have $B_{\psi_k}(\theta, \phi)$ approaches $\varepsilon$;
- The sidelobe level parameter $R$ should be chosen that the weaker paths are not buried by the sidelobes of other paths.

### IV. SIMULATION RESULTS

A ULA composed of 21 isotropic elements placing along $z$-axis with $d_z = \frac{4\lambda}{3}$ is utilized to demonstrate the idea, as shown in Fig. 1. Four paths impinging at the UA are assumed, where the parameters of the paths are summarized in Table I.

Four paths with widely different power levels (up to 30dB) and angles are targeted to demonstrate the effectiveness of sidelobe suppression technique. The power-angle profile of the ULA with uniform weighting (i.e. $w_n = 1$, $n \in [0, N-1]$) and Dolph-Chebyshev weighting of sidelobe level $R_{d_B} = 30dB$ and 40dB are shown in Fig. 4. We can see that:

- With uniform weighting, path 2 and path 4 are buried and the estimated amplitude $\alpha_2$ and $\alpha_4$ are inaccurate due to the strong sidelobes of path 1 and path 3. That is the weaker paths can not be detected and the estimated

![Fig. 3. An illustration of the UCuA.](image)
path powers are not accurate due to the interference of the strong paths:

- With Dolph-Chebyshev weighting, all of the sidelobes are constant in a pre-defined level;
- With Dolph-Chebyshev weighting, all the paths can be detected and the estimated amplitudes are more accurate than that of uniform weighting; 
- As the sidelobe level parameter $R$ increases, the sidelobes become lower and the mainlobes become wider, as expected.

A URA composed of $21^2$ isotropic elements lying in the x-y plane with $d_x = d_y = \frac{1}{2} \lambda$ is utilized as an example for URAs, as illustrated in Fig. 2. Similarly, four paths impinging upon the URA are assumed, where the parameters of the paths are summarized in Table II.

The power-angle profile of the URA with uniform weighting (i.e. $w_{l,m} = 1, l,m \in [0,N-1]$) and Dolph-Chebyshev weighting with sidelobe level $R_{dB} = 40dB$ are shown in Fig. 5. Similarly, we can see that with Dolph-Chebyshev weighting, the sidelobes can be effectively suppressed, with widened mainlobes. Further, all the four paths can be clearly detected with correct power levels with sidelobe suppression. Note that as the elevation angle decreases, the resolution of the elevation angle improves but the resolution of the azimuth angle degrades, where the URA is placed on the x-y plane.

A UCuA composed of $21^3$ isotropic elements with $d_x = d_y = d_z = 0.4\lambda$ is utilized as an example for UCuAs, as illustrated in Fig. 3. Four paths impinging upon the UCuA are assumed, where the parameters of the paths are summarized in Table III.

The power-angle profile of the UCuA with uniform weighting (i.e. $w_{l,m,n} = 1, l,m,n \in [0,N-1]$) and Dolph-Chebyshev weighting of sidelobe level $R_{dB} = 50dB$ are shown in Fig. 6. This Fig. shows the similar effectiveness of sidelobe reduction as Fig. 4 and 5. Comparing with Fig. 5, we can see that the resolutions of azimuth and elevation angle of UCuA are different to that of URA, which due to the configuration differences of the arrays.

V. CONCLUSION

We summarize the Dolph-Chebyshev weighting methods in a general form for ULA, URA and UCuA. Further, we apply beamforming technique with both uniform weighting and Dolph-Chebyshev weighting for channel multipath detections. Results demonstrate that:

- With uniform amplitude weighting, the weaker paths can not be detected and the estimated path powers are not accurate due to the high sidelobes of the strong paths;
- With Dolph-Chebyshev weighting, all the paths can be detected and the estimated amplitudes are more accurate with a proper pre-defined sidelobe level;
- Dolph-Chebyshev weighting results in all the sidelobes at the same level;
- The mainlobes widen as the sidelobes decrease.

This detection method is promising for the channel estimation of future 5G system.
Fig. 6. Power-angle profile for the UCuA with uniform weighting (left) and Dolph-Chebyshev weighting (right).

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