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Published in:
IEEE Transactions on Energy Conversion

DOI (link to publication from Publisher):
10.1109/TEC.2017.2669031

Publication date:
2017

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
Analysis and Design of Improved Weighted Average Current Control Strategy for LCL-Type Grid-Connected Inverters

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Abstract—The LCL grid-connected inverter has the ability to attenuate the high-frequency current harmonics. However, the inherent resonance of the LCL filter affects the system stability significantly. To damp the resonance effect, the dual-loop current control can be used to stabilize the system. The grid current plus capacitor current feedback system is widely used for its better transient response and high robustness against the grid impedance variations. While the weighted average current (WAC) feedback scheme is capable to provide a wider bandwidth at higher frequencies but show poor stability characteristics under weak grid scenarios. To overcome this shortcoming, the improved WAC damping method is proposed with an additional capacitor current feedback loop. In this paper, a systematic parameter design guideline for the optimal selection of the current loop PR controller and the additional capacitor current feedback coefficient are presented for the improved WAC feedback control strategy. The satisfactory range of the system control parameters can be obtained under different delay conditions to meet the system performance specifications. The improved WAC method enhances system robustness under weak grid scenarios, and the stability and robustness have been enhanced under control delay. Finally, the experimental results are presented to validate the effectiveness of the proposed improved WAC control strategy and the parameter design method.

Index Terms—Active damping, grid-connected PWM inverter, current control, grid impedance, harmonic distortion, weighted average current control.

NOMENCLATURE

Abbreviations

PCC Point of common coupling.

Manuscript received July 23, 2016; revised December 27, 2016; accepted February 08, 2017. This work was supported in part by the National Natural Science Foundation of China under Grant 51307015, in part by the Open Research Subject of Sichuan Province Key Laboratory of Power Electronics Energy-Saving Technologies & Equipment under Grant szjj2015-067, in part by the Open Research Subject of Artificial Intelligence Key Laboratory of Sichuan Province under Grant 2015ZJ002, and in part by the Fundamental Research Funds of Central Universities of China under Grant ZYGX2015J087. Paper no. TEC-00619-2016.

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Digital Object Identifier ******/TEC.******

I. INTRODUCTION

The increasing global energy consumption has greatly accelerated the demand for renewable energy, such as wind and solar power. The grid-connected PWM inverters play an important role for interconnecting renewable energy and the utility grid [1]. Compared to L filter, high order LCL filter can

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>WAC</td>
<td>Weighted average current.</td>
</tr>
<tr>
<td>PR</td>
<td>Proportional resonant controller.</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-locked loop.</td>
</tr>
<tr>
<td>THD</td>
<td>Total harmonic distortion.</td>
</tr>
<tr>
<td>HC</td>
<td>Harmonic compensator.</td>
</tr>
<tr>
<td>GM</td>
<td>Gain margin.</td>
</tr>
<tr>
<td>PM</td>
<td>Phase margin.</td>
</tr>
<tr>
<td>$G_{\text{open},i_{g}}$</td>
<td>Open loop transfer function of $i_{g}$.</td>
</tr>
<tr>
<td>$G_{\text{open},i_{c}}$</td>
<td>Open loop transfer function of $i_{c}$.</td>
</tr>
<tr>
<td>$G_{\text{close},i_{g}}$</td>
<td>Closed-loop transfer function of $i_{g}$.</td>
</tr>
<tr>
<td>$G_{\text{close},i_{c}}$</td>
<td>Closed-loop transfer function of $i_{c}$.</td>
</tr>
<tr>
<td>$G_{\text{close},WAC}$</td>
<td>Closed-loop transfer function of improved WAC feedback method.</td>
</tr>
<tr>
<td>$G_{\text{open},WAC}$</td>
<td>Open loop transfer function of improved WAC feedback method.</td>
</tr>
<tr>
<td>$G_{\text{close},WAC}$</td>
<td>Closed-loop transfer function of improved WAC feedback method.</td>
</tr>
<tr>
<td>$G_{i_{g}}$</td>
<td>Loop gain of improved WAC feedback method.</td>
</tr>
<tr>
<td>$G_{i_{c}}$</td>
<td>Transfer function of PR controller.</td>
</tr>
<tr>
<td>$G_{a_{d}}$</td>
<td>Transfer function of digital control delays.</td>
</tr>
<tr>
<td>$i_{\text{ref}}$</td>
<td>Reference current.</td>
</tr>
<tr>
<td>$E_{a}$</td>
<td>Steady state error.</td>
</tr>
<tr>
<td>$K_{r,\text{PM}}$</td>
<td>$K_{r}$ constrained by PM.</td>
</tr>
<tr>
<td>$K_{r,\text{GM}}$</td>
<td>$K_{r}$ constrained by GM.</td>
</tr>
<tr>
<td>$V_{d}$</td>
<td>Amplitude of the input dc-link voltage.</td>
</tr>
<tr>
<td>$V_{m}$</td>
<td>Amplitude of the triangular carrier.</td>
</tr>
<tr>
<td>$i_{g}$</td>
<td>Injected grid current.</td>
</tr>
<tr>
<td>$i_{c}$</td>
<td>LCL inverter side current.</td>
</tr>
<tr>
<td>$i_{L}$</td>
<td>LCL capacitor current.</td>
</tr>
<tr>
<td>$K_{S}$, $K_{L}$</td>
<td>Sensor gain of grid current and LCL inverter side current.</td>
</tr>
<tr>
<td>$K_{a}$, $K_{r}$</td>
<td>Active damping coefficient.</td>
</tr>
<tr>
<td>$K_{p}$, $K_{r}$</td>
<td>Parameters of the PR controller.</td>
</tr>
<tr>
<td>$L_{1}$</td>
<td>Inverter-side inductance.</td>
</tr>
<tr>
<td>$L_{2}$</td>
<td>Grid-side inductance.</td>
</tr>
<tr>
<td>$L_{g}$</td>
<td>Equivalent grid inductance.</td>
</tr>
<tr>
<td>$C$</td>
<td>Filter capacitor.</td>
</tr>
<tr>
<td>$\omega_{0}$, $\omega_{s}$</td>
<td>Fundamental and cutoff frequencies.</td>
</tr>
<tr>
<td>$f_{c}$</td>
<td>LCL resonance frequency.</td>
</tr>
<tr>
<td>$T_{d}$</td>
<td>Delay time.</td>
</tr>
<tr>
<td>$T_{s}$</td>
<td>Sampling and switching period.</td>
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</tbody>
</table>
attenuate harmonics produced by the PWM inverter effectively with reduced cost and size, making LCL filter an popular solution in industrial applications [2]. However, the resonant peak produced by LCL-filter can seriously affect the system stability [3]. In order to stabilize the system, the passive damping scheme by adding a resistor in series with the filter capacitor has been widely adopted for its simplicity and high reliability. But it inevitably leads to higher power losses and decreasing system efficiency [4]. Therefore, the active damping methods which can be achieved by feeding back other control variables as damping terms to the current control loop are preferred due to its efficiency and flexibility [5-8].

Compared with single-loop current control [9], the dual-loop active damping control schemes are preferable which responses slower but with better output waveform quality [10]. Generally, the inner current loop is mainly used for active damping, and the outer grid current loop with current controller is used for tracking the reference current, and in synchronism with the voltage at the point of common coupling (PCC) [11], [12]. In retrospect, the grid current plus LCL capacitor current feedback control scheme was widely addressed [6], [10], [13-16]. And proportional feedback of the capacitor current has been proven to be equivalent to a virtual resistor connected in parallel with the filter capacitor [16]. In addition to LCL capacitor current, LCL inverter side current can also be used as the inner control variable [11], [17]. And it was mentioned in [14] that, from the dynamic point of view, the capacitor current plus grid current feedback control scheme is equivalent to the converter current plus grid current feedback control scheme.

On the other hand, the weighted average current (WAC) control method was derived by combing the conventional single converter current feedback and grid current feedback loops, which has been successfully introduced in the LCL grid connected inverter to damp the resonance [18, 19]. With this approach, the order of the system transfer function can be degraded from a third-order function to a first-order one [19], the current harmonics can be decreased and the dynamic performance can also be enhanced. A comparative analysis between two current sensor feedback methods was presented in [20], and it is found that the WAC control has high controller bandwidth but more sensitive to the LCL inductor variation, which may make the system unstable. Moreover, when the weighted average current is controlled, both the grid-side current and the inverter-side inductor current are critically stable even though the weighted average current can be easily stabilized [21].

In [22], an improved current feedback control method combined with the WAC feedback and capacitor current feedback is proposed. In the proposed control scheme, the capacitor current feedback is introduced at the inner loop without adding any extra current sensor. Consequently, the resonance damping and harmonic suppressing capability are improved. Besides, the improved WAC method is also capable of reducing the sensitivity to the disturbance and improving the quality of the grid current. However, in the WAC control method, only the weighted current is directly controlled, thus the injected grid current may have a steady state error at the output, which affects the system performance significantly. Besides, the LCL-filter resonance frequency will vary in a wide range due to the variation of the grid impedance, which challenges the design of capacitor current feedback coefficient and the current controller parameters [23, 24].

Based on the concept of the virtual resistor, a step-by-step method was proposed in [15] to design the active damping coefficient and the current control parameters of the grid current plus capacitor current feedback control method. This design method is useful when the delay effect is ignored, but the digital control delay is inevitable in the digital-controlled system and will change the system phase-frequency characteristic, thus the system stability will be affected [25, 26]. Therefore, the delay effect should be considered in the design procedure to determine the capacitor-current-feedback coefficient and the current controller parameters. However, the parameter design guidelines for the improved WAC control scheme has not been reported in the existing literatures.

The main contributions of this paper are summarized as follows.

1) This paper presents the comparison between the grid current plus capacitor current feedback method and the improved WAC feedback method. It was proved that with improved WAC feedback control method, a lager current controller proportional gain can be adopted to provide larger bandwidth at higher frequencies, and the system robustness under weak grid conditions can be enhanced.

2) A systematic method for designing PR controller and the capacitor current feedback coefficient are presented for the improved WAC feedback control scheme under different delay conditions. With this method, controller parameters and the active damping feedback coefficient can be easily obtained by specifying the system stability and dynamic performance indices, and it is more convenient and explicit to optimize the system performance according to the satisfactory region. The improved WAC method enhances system robustness under weak grid conditions, and the system becomes more robust against time delay.

3) Implementation of the improved WAC control strategy on a 2.2 kW three-phase experimental platform based on Dansfoss inverter has been presented. The control strategy, designed control parameters and system performance under weak grid and maximum delay time scenario are tested. The results are consistent with the simulation results.

This paper is organized as follows. In Section II, the modeling and analysis for dual-loop current control methods are presented. In Section III, a systematic method for designing PR controller and the capacitor current feedback coefficient in the improved WAC feedback control scheme is presented. The experimental results in Section IV validates the proposed control scheme and parameter design method. Finally, Section V concludes this paper.

II. MODELING AND ANALYSIS OF THE DUAL-LOOP CURRENT CONTROL SCHEME

Fig. 1(a) shows the main circuit of the three-phase LCL interfaced grid-connected inverter. $L_1$, $L_2$ and $L_g$ represents the inverter-side inductance, the grid-side inductance and the equivalent grid inductance. The voltages at the PCC are sampled for the feed-forward loop and the phase-locked-loop (PLL). The corresponding current control diagram is shown in Fig. 1(b), where $K_{P_{inv}}$ is the transfer function of the PWM.
inverter, which is defined as $V_{dc}/V_{tri}$, where $V_{dc}$ is the amplitude of the input dc-link voltage and $V_{tri}$ is the amplitude of the triangular carrier [15]. The injected grid current $i_g$ and the LCL inverter side current $i_c$ are sensed with the sensor gain $K_p$ and $K_i$ respectively. The gain $K_p$ and $K_i$ are equal to $L_2/(L_1+L_2)$ and $L_1/(L_1+L_2)$ respectively. A novel feedback variable $i_{wac}$ named weighted current has been constructed.

$$G_r(s) = K_p \left(1 + K_i \frac{2\omega_0 s}{s^2 + 2\omega_0 s + \omega_0^2}\right)$$

(1)

where $K_p$ is the proportional gain, $K_i$ is the integral gain at the fundamental frequency and $\omega_0$ is the bandwidth of the resonant part concerning -3 dB cutoff frequency.

The resonance frequency of the LCL filter $f_r$ is written as

$$f_r = \frac{1}{2\pi \sqrt{\frac{L_1 + L_2}{L_1L_2C}}}$$

(2)

The digitally controlled system contains the computation delay, sampler continuous approximation, and PWM delay [27], which can be expressed as

$$G_{ij}(s) = e^{-T_d s}$$

(3)

The delay effect with three typical values for $T_d$ can be used, with $T_d = T_{\pi}/2$ is defined as the minimum delay, with $T_d = T_{\pi}$ defined as the medium delay and with $T_d = 3T_{\pi}/s$ defined as the maximum delay.

$$I_{ref, off} = G_{0}(s) \left(I_{ref, on} - I_{dc}ight) + I_{c, off} = I_{wac, off} - I_{c, off}$$

Fig. 2. The s-domain block diagrams of traditional WAC feedback control scheme.

The s-domain block diagrams of traditional WAC feedback control scheme is shown in Fig. 2. From Fig. 2, the close-loop transfer function from the reference current $i_{ref}$ to grid current $i_g$ is expressed as (4), where the subscript “WAC” represents the transfer function with traditional WAC control.

$$G_{close, WAC}(s) = \frac{D_1}{s^3CL_1L_2 + s^2CL_2K_p D_1 + s(L_1 + L_2) + D_1}$$

(4)

where the parameters $D_1$ is denoted as

$$D_1 = G_r(s)K_{PWM} G_d(s)$$

(5)

Using the parameters listed in Table I [14], Fig. 3 shows the close-loop characteristics of the traditional WAC control method, where $K_p$ is the proportional gain in the current controller. As can be seen from Fig. 3, with the increase of $K_p$, the system bandwidth is improved. However, the peak at resonant frequency is also increased. As a result, if the noise disturbances especially those around $f_r$ are in existence, the resonance at harmonic frequencies will be amplified. Therefore, the WAC control method needs to be improved. To damp the resonance, the passive damping method can be used with a resistor in series with the capacitor. This method is simple and effective, but also result in additional power losses.

Conventional WAC feedback system shows a high controller bandwidth but the system robustness under LCL parameter variation is poor with a high steady state error [20]. The $i_c$ plus $i_g$ feedback scheme shows a high robustness against the grid impedance variations [28]. However, the controller bandwidth is affected. The improved control method using a combination of the WAC and the capacitor current feedback is shown in Fig. 4. Note that the active damping feedback of $i_c$ is obtained using the difference of $i_c$ and $i_g$ according to Kirchhoff’s current law, thus an additional high-precision current sensor can be avoided in order to reduce the hardware cost.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-link voltage</td>
<td>$V_{dc}$</td>
<td>650 V</td>
</tr>
<tr>
<td>Grid voltage(RMS)</td>
<td>$V_{tri}$</td>
<td>220 V</td>
</tr>
<tr>
<td>Amplitude of triangle carrier wave</td>
<td>$V_{tri}$</td>
<td>1 pu</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>$f_1$</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Inverter side inductance</td>
<td>$L_1$</td>
<td>1800 µH</td>
</tr>
<tr>
<td>Grid side inductance</td>
<td>$L_2$</td>
<td>1800 µH</td>
</tr>
<tr>
<td>Filter capacitance</td>
<td>$C$</td>
<td>25 µF</td>
</tr>
<tr>
<td>Sampling and switching period</td>
<td>$T_s$</td>
<td>100 µs</td>
</tr>
<tr>
<td>Fundamental angular frequency</td>
<td>$\omega_0$</td>
<td>100π rad/s</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>$f_r$</td>
<td>1.06 kHz</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_s$</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

Fig. 3. Amplitude frequency characteristics of the traditional WAC close-loop transfer function (Undamped system).
a larger $K_p$ results in a larger gain at the fundamental frequency, but the value of $K_i$ has no effects on the system bandwidth. When the feedback loop of the LCL filter capacitor current is introduced, a higher value of $K_i$ shows the better resonance damping capability.

![Fig. 5. Amplitude frequency characteristics of the improved WAC close-loop transfer function with additional capacitor current feedback (damped system).](image)

![Fig. 6. The s-domain block diagrams of $I_g$ plus $I_c$ dual-loop feedback control scheme [17].](image)
The main goals of the current control are to minimize the steady-state error, achieve the best dynamic performance and minimize the harmonics in the injected grid current. Generally, these system performance and stability margin can be specified by steady state error ($E_s$), crossover frequency ($f_c$), gain margin (GM) and phase margin (PM) of the system. In general, GM $\geq 3$ dB is adopted to ensure the system robust stability, and PM $\geq 15^\circ$ is adopted to achieve good dynamic response and robustness.

To make a comparison between $i_d$ plus $i_c$ feedback control and improved WAC feedback control, the open loop transfer functions of two feedback system are given in (8) and (9), where the subscript “igic” represents the transfer function with grid current plus capacitor current control.

$$G_{open,igic}(s) = \frac{G_d(s)K_{PWM}G_c(s)}{s^3CL_1L_2 + s^2CL_2G_d(s)K_{PWM}K_c + s(L_1 + L_2)}$$  

(8)

$$G_{open,WAC}(s) = \frac{D_c}{s^3CL_1L_2 + s^2D_c + s(L_1 + L_2)}$$  

(9)

The value of $K_p$ and $K_c$ can both affect GM, PM and $f_c$ of the system, in order to get the appropriate values of $K_p$ and $K_c$ to achieve the best dynamic performance and ensure system stability. According to (8) and (9), PM, GM and $f_c$ constrained by $K_p$ and $K_c$ can be represented in 3-D plots shown in Fig. 7 and Fig. 8.

As for the $i_d$ plus $i_c$ feedback control system, Fig. 7(a) shows the relationship between $K_p$, $K_c$, and GM, the possible $K_p$ and $K_c$ that satisfy GM $\geq 3$ dB are included in the shaded area. Fig. 7(b) shows the relationship between $K_p$, $K_c$ and PM, the possible $K_p$ and $K_c$ that satisfy PM $\geq 15^\circ$ are included in the shaded area. Take into account both GM and PM, the shaded area of $K_p$ and $K_c$ that both satisfying the aforementioned GM and PM are plotted in Fig. 7(c). From Fig. 7(c), a larger $f_c$ can be chosen to achieve a better dynamic performance.

Using the same main circuit parameters, the corresponding plots of the improved WAC feedback system are shown in Fig. 8. Compared with Fig. 7, it can be seen that a larger range of $K_p$ and $K_c$ can be chosen to satisfy GM $\geq 3$ dB and PM $\geq 15^\circ$ with the improved WAC feedback control. For example, if $K_p=0.05$ is adopted, in $i_d$ plus $i_c$ feedback control system, $K_c$ can be selected in the range of 0.04–0.05. However, in improved WAC feedback control system, $K_c$ can be selected in the range of 0.01–0.15. Similarly, with the same $K_c$, a larger $K_p$ can be adopted in the improved WAC feedback control system to provide larger bandwidth at higher frequencies.

Under weak grid scenario, the grid impedance $L_2$ is inductive and can be regarded as equivalent to the increasing of inductor $L_2$. In this study, $L_2$ is increased to 1200 $\mu$H to emulate the weak grid condition. When $L_2=1200$ $\mu$H, the areas that satisfying GM $\geq 3$ dB and PM $\geq 15^\circ$ are depicted in Fig. 7 and Fig. 8. It can be seen that the shaded areas of two control methods are both enlarged with larger grid impedance, and the enlarged area of improved WAC control is much larger than that of $i_d$ plus $i_c$ feedback control. Therefore, compared with $i_d$ plus $i_c$ feedback control scheme, the improved WAC method enhances system robustness under weak grid conditions.

### III. PARAMETER DESIGN GUIDELINES OF THE IMPROVED WAC CONTROL SCHEME

The improved control method contains a combination of the WAC and the capacitor current feedback loops. In order to achieve satisfactory active damping performance, parameter design procedures are of vital significance, which are outlined in this section.

$$I_{ref,\alpha\beta}^{*} \quad V_{g,\alpha\beta} \quad i_{g,\alpha\beta}$$  

Fig. 9. The equivalent transformation of s-domain block diagrams of the proposed improved WAC feedback control scheme.

The simplified equivalent results are shown in Fig. 9, where $G_d(s)$ and $G_c(s)$ are denoted as:

$$G_d(s) = \frac{G_d(s)K_{PWM}G_c(s)}{s^3CL_1L_2 + s^2D_c + s(L_1 + L_2)}$$  

(10)

$$G_c(s) = \frac{D_c}{s^3CL_1L_2 + s^2D_c + s(L_1 + L_2)}$$  

(11)

If the control parameters $K_p$ and $K_c$ are equal to $L_2/(L_1+L_2)$ and $L_1/(L_1+L_2)$, respectively, the system model with $LCL$ filter can be degraded from a third-order to a first-order system.

![Bode Diagram](image)

Fig. 10. Bode diagram of the improved WAC control scheme with different capacitor current feedback coefficients.

The loop gain is defined as $G_L(s)$, and can be expressed as

$$G_L(s) = G_d(s)G_c(s) = \frac{G_c(s)G_d(s)K_{PWM}}{s^3CL_1L_2 + s^2D_c + s(L_1 + L_2)}$$  

(12)

From the loop gain $G_L(s)$ defined in (12), the bode plot with different capacitor current coefficient is illustrated in Fig. 10. As can be seen from Fig. 10, with additional capacitor current feedback, the system amplitude frequency characteristics above the fundamental frequency can be modified. The magnitude curve crosses over 0 dB at $f_c$, thus the phase at $f_c$ should be adjusted above -180° to ensure PM>0, and the PM of the system can be expressed as

$$PM = 180^\circ + \angle G_L(s)_{f_c} \geq 15^\circ$$  

(13)

In Fig. 10, the phase curve crosses over -180° at the resonance frequency $f_c$. Therefore, the gain at $f_c$ needs to be adjusted below 0 dB to ensure GM >0. The gain margin GM...
can be expressed as
\[ GM = -20 \log |G_s(j2\pi f_c)| \] (14)

The influence of the filter capacitor can be ignored when calculating the magnitude of the loop gain lower or equal to \( f_c \). Thus \( G_1(s) \) and \( G_2(s) \) can be approximated as
\[ G_L(s) \approx T(s) = \frac{G_s(s)G_2(s)K_{PWM}}{s(L_s + L_L)} \] (15)
\[ G_f(s) \approx T_f(s) = \frac{1}{s(L_s + L_L)} \] (16)

where \( T(s) \) and \( T_f(s) \) are the approximation of \( G_1(s) \) and \( G_2(s) \) respectively. Therefore, with reference to Fig.9, the current injected into the grid can be approximated as
\[ i_g(s) = \frac{T(s)}{1 + T(s)}i_{ref}(s) - \frac{T_f(s)}{1 + T_f(s)}v_g(s) \] (17)

where \( i_{ref}(s) \) is the reference current and \( v_g(s) \) is the grid voltage.

The steady-state error of the grid-connected inverter includes the amplitude error \( E_A \) and the phase error \( \delta \). The PR controller can eliminate the static error at the fundamental frequency, thus the phase error \( \delta \) can be attenuated. Considering \( T(s) \gg 1 \) and \( |G_d(j2\pi f_0)| \approx 1 \), the amplitude of the grid current \( i_g \) at the fundamental frequency \( f_0 \) can be expressed as
\[ i_g(j2\pi f_0) = i_{ref}(j2\pi f_0) - \frac{T_f(j2\pi f_0)}{T(j2\pi f_0)}v_g(s) \] (18)

and the amplitude error \( E_A \) can be expressed as
\[ E_A = \left| \frac{i_g(j2\pi f_0) - i_{ref}(j2\pi f_0)}{i_{ref}(j2\pi f_0)} \right| = \frac{1}{G_s(j2\pi f_0)} \left| \frac{v_g(s)}{i_{ref}(j2\pi f_0)} \right| K_{PWM} \] (19)

where \( I_{ref} \) and \( V_g \) are the root mean square (rms) values.

At the fundamental frequency \( f_0 \), the PR controller can be approximated as
\[ G_s(j2\pi f_0) \approx K_p + K_r \] (20)

Substituting (20) into (19) yields
\[ K_{r,EA} = \frac{V_g}{E_A I_{ref} K_{PWM}} - K_p \] (21)

where \( K_{r,EA} \) is the critical value of the integral gain constrained by \( E_A \). The magnitude of the PR controller can be approximated to \( K_p \) at the frequencies equal or higher than \( f_c \), the magnitude frequency response of the system is zero at \( f_c \). Therefore, considering \( G_s(j2\pi f_c) \approx K_p \) and \( |T(j2\pi f_0)| = 1 \), the proportional gain \( K_p \) can be written as
\[ K_p \approx \frac{2\pi f_c(I_s + L_s)}{K_{PWM}} \] (22)

From (22), it can be inferred that \( K_p \) is proportional to \( f_c \), and a larger \( K_p \) ensures a faster dynamic response. When the expected crossover frequency \( f_c \) is selected, the proportional gain \( K_p \) can be determined.

Since \( f_c \) is higher than both \( f_0 \) and \( \omega_0/\pi \), \( G_s(s) \approx K_p + 2K_r\omega_0/s \) is reasonable to calculate the phase margin [15]. Substituting \( s = j2\pi f_c \) into \( G_1(s) \) in (15), the PM can be rewritten as
\[ K_{c,PM} = \frac{G_D^2 G_A + 4G_D G_E + \tan(\text{PM} + 180^\circ) \left[ 2\pi f_c(L_s + L_L) + 2\pi f_c f_L (f_c^2 - f_c^2) \right] - 2\pi^2 f_c^2 (L_s + L_L)(f_c^2 - f_c^2)}{4G_D \frac{K_{PWM} f_c}{2\pi L_L} - \tan(\text{PM} + 180^\circ) \frac{\pi f_c^2 (L_s + L_L)}{\omega_0 L_L}} \] (28)

\[ \text{PM} = -180^\circ + \arctan \left( \frac{f_L K_{PWM} K_{r, ref}}{2\pi L_L} \right) \] (23)

Equation (23) can be rewritten as
\[ K_c^2 G_A + K_c G_B + G_C = 0 \] (24)

where \( G_A, G_B \) and \( G_C \) are denoted as
\[ G_A = \tan(\text{PM} + 180^\circ) \frac{K_{PWM} \omega_0}{2\pi L_L} \] (25)

\[ G_B = \tan(\text{PM} + 180^\circ) (f_c^2 - f_c^2) - f_c K_p K_{PWM} \frac{1}{2\pi L_L} \] (26)

\[ G_C = \tan(\text{PM} + 180^\circ) \frac{\pi f_c^2 K_p}{\omega_0} \] (27)

As \( K_c > 0 \), the root \( K_{r,2} \) is invalid; substituting (19) and (20) into \( K_{r,1} \), the boundaries of \( K_c \) constrained by PM and \( E_A \) can be derived by (28), at the bottom of this page, and the subscript “PM” represents the \( K_c \) constrained by PM.

In (28), \( G_D \) and \( G_E \) are denoted as:
\[ G_D = \frac{V_g}{E_A I_{ref} K_{PWM}} \] (29)

\[ G_E = \tan(\text{PM} + 180^\circ) (f_c^2 - f_c^2) \] (30)

In general, the crossover frequency is set over a decade above the fundamental frequency \( f_0 \). Therefore, the PR controller can be approximated to \( K_p \) around \( f_c \). Considering \( |G_d(s)| = 1 \), and substituting \( s = j2\pi f_c \) into (14) yields
\[ K_{c,GM} = \frac{2\pi f_c f_L 10^{20} - 4\pi^2 f_c^2 f_L L_L C}{K_{PWM}} \] (31)

where the subscript “GM” represents the \( K_c \) constrained by GM.
From the above analysis, a design method can be obtained:

(i) If the upper and lower boundaries of GM and PM, and \( E_A \) are specified, the satisfactory region constrained by \( K_c \) and \( f_c \) can be obtained from (28) and (30).

(ii) With the possible value of \( f_c \) and \( K_c \), the proportional gain \( K_p \) can be calculated from (22).

(iii) Finally, with the obtained \( K_p \), the integral gain \( K_i \) can be calculated from (21).

To verify the proposed design method, a design example is given: If the upper and lower boundaries of PM and GM is chosen as \( 15^\circ \leq \text{PM} \leq 40^\circ \) and \( 3 \text{ dB} \leq \text{GM} \leq 20 \text{ dB} \), respectively, and the magnitude error \( E_A \) is set to be less than 0.2%. Based on (28) and (30), all the possible \( K_c \) and \( f_c \) under different delay conditions are plotted in the shaded area in Fig. 11. The delay \( T_d \) exists in the expression of \( K_c, \text{PM} \) in (28), thus the delay time affects the phase margin of the system. As can be seen from Fig. 11, from the minimum delay time to maximum delay time, the satisfactory region moves towards to the lower frequency.

Therefore, \( K_c \) and \( f_c \) should be selected inside the shaded area to ensure the system stability under different delay conditions.

From Fig. 11, point A is selected with \( f_c=800 \text{ Hz} \) and \( K_c=0.03 \). With (22), the calculated proportional gain \( K_p \) is 0.028. Substituting \( K_c, K_p \) and \( E_A \) into (21), \( K_i = 36.5 \) can be obtained. Considering the previous steps, the designed parameters can be obtained as:

\[
K_c = 0.03, \quad K_p = 0.028, \quad K_i = 36.5 \tag{31}
\]

Based on the parameters in (31), the system bode plots is shown in Fig. 12. The gain margin (GM) is 5.28 dB, the phase margin (PM) is 16.5\(^\circ\) and the crossover frequency \( f_c \) is 782 Hz, all satisfying the aforementioned specifications, which are consistent with the theoretical analysis.

The variation tendencies of traditional and improved WAC system performance indexes, PM, GM and \( f_c \), with respect to the system delay \( T_d \) variation are depicted in Fig. 13. In Fig. 13, PM, PM and \( f_c \) all decrease when \( T_d \) increases, the stability and transient responses of the system will be inevitably worse. However, when \( T_d \) increases from the minimum delay time (50 \( \mu s \)) to maximum delay time (150 \( \mu s \)), the improved WAC system remains stable while the traditional WAC system becomes unstable. Therefore, with capacitor current feedback, the WAC system become more robust against time delay.

To further validate the effectiveness of the proposed design method, simulation results with \( T_d=100 \text{ \( \mu s \)} \) and different \( K_c \) are presented in Fig. 14. Different \( K_c \) are selected in Fig. 11. With \( K_c=0.03, K_p=0.028 \) and \( K_i=36.5 \), which correspond to point A, are selected, the system is perfectly stable as shown in Fig. 14(a). The fundamental rms value of the injected grid current \( i_g \) is 2.002 A, the reference value is 2 A, thus the steady-state amplitude error is 0.1\%. Fig. 14(b) shows dynamic response when the reference of reference current jump from 2 A to 4 A. The percentage overshoot (PO) is 5\%, and the settling time is about 0.3 ms. With \( K_c=0.08, K_p=0.028 \) and \( K_i=36.5 \), which correspond to point B, are selected, on the edge of the satisfactory margin, the system remains stable, but there are small oscillations on the waveforms in Fig. 14(c). With \( K_c=0.12, K_p=0.028 \) and \( K_i=36.5 \) selected in point C, which is out of the satisfactory region, the system cannot work in stable state in Fig. 14(d). Hence the simulation results verified the correctness of the proposed design method.

Therefore, with the designed controller and active damping parameters, both the system stability and dynamic response can be ensured.
Fig. 14. Simulation results of improved WAC control with different capacitor current feedback coefficients (only phase-A voltage and current are shown) (a) Steady state response at point A with $K_c=0.03$ (b) Dynamic response at point A (c) At point B with $K_c=0.08$ and (d) At point C with $K_c=0.12$.

Fig. 15. Photo of the experimental setup.

To validate the analysis, the current control strategies are evaluated on a 2.2 kW three-phase experimental platform based on Danfoss inverter, as shown in Fig. 15. The experimental setup was built and tested in the Microgrid Research Lab of Aalborg University[29], and the parameters of the power stage and controllers are consistent with the theoretical analysis.

Fig. 16. Experimental results of improved WAC control system with different capacitor current feedback coefficients. With $K_c=0.03$, $K_p=0.028$ and $K_r=36.5$ which corresponds to point A in Fig. 11, the grid current oscillation is relatively small and the dynamic performance is also satisfactory, which verified the excellent performance of the improved WAC control under normal grid condition. With
$K_c = 0.08$ which correspond to point B are selected, the system is still in stable state, but there are obvious distortion in the grid current waveforms in Fig. 16(c). In Fig. 16(d), with $K_c = 0.12$ selected in point C, which is out of the satisfactory region, the system cannot work in stable state. Therefore, the experimental waveforms well match the simulation waveforms as shown in Fig. 14.

Fig. 17 shows the experimental results under weak grid conditions when the grid inductance is approximately 1.2 mH. In Fig. 17(a), when the $i_s$ plus $i_g$ current feedback control method is used, the system become unstable with a high current loop gain under weak grid scenario. In Fig. 17(b), the current is distorted with the THD of 8.32% under traditional WAC feedback control, and the system is still in stable state. Fig. 17(c) shows the experimental results for the improved WAC control scheme under weak grid scenario.

By improving the robustness against the grid impedance variation and eliminating the resonance effect, the THD of grid current is reduced to 2.32%, and the quality of the grid currents are significantly improved. The results verify the effectiveness of the proposed improved WAC control scheme under weak grid conditions, which can be preferred in the grid-integration of DGs to the weak grid conditions.

$$\text{THD}=8.23\%$$

$$\text{THD}=2.32\%$$

Fig. 18 shows the experimental results with approximately 150 $\mu$s delay time, when the control parameters are consistent with the analysis in the previous section. It can be seen from Fig. 18, the system with traditional WAC feedback method becomes unstable with an increased delay time of about 150 $\mu$s. Under the capacitor current feedback, the system with improved WAC method is still stable. Therefore, the robustness of the improved WAC method against time delay has been verified.

V. CONCLUSION

This paper investigates the active damping characteristics and system stability of weighted average current feedback control method. Compared with traditional grid current plus capacitor current feedback control scheme, the improved WAC feedback control method is capable to provide larger bandwidth at higher frequencies, and has higher robustness against the grid impedance variation.

In order to achieve optimal system performance, a systematic parameter design guideline for optimal selection of the current loop PR controller and the additional capacitor current feedback coefficients is presented. The satisfactory range of the system control parameters can be obtained under different control delay conditions to meet the system specifications. The improved WAC method enhances system robustness under weak grid scenarios, and the system become more robust against time delay, thus system stability and performance are remarkably improved. The experimental results are presented to validate the effectiveness of the proposed improved WAC control scheme and parameter design methodologies, which can be widely applied for the grid-integration of DGs to the grid with a long transmission cable with a large equivalent grid impedance and distorted grid voltage conditions.
REFERENCES


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