2-D Directional Topology Optimization
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Published in:
Proceedings of Conference on Architectural Research and Information Technology, 2001, Aarhus School of Architecture

Publication date:
2001

Citation for published version (APA):

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Abstract

Topology optimisation is a technique to optimally distribute a given amount of material in a predefined domain, thus creating a design that maximises some performance criterion, typically the structural stiffness. The scope of criterion functions in topology optimisation has so far been limited to performance-related technical aspects, for instance weight, stiffness, or thermal properties. In this work, we extend its use into the architectural field. This paper presents topology optimisation software developed for directional optimisation in the case of two-dimensional structures. In addition to stiffness and weight, the visual direction of the structure has been added as a possible design criterion. This is obviously a small augmentation of the problem definition compared to the normal architectural vocabulary, and it is designed for investigational purposes only. The choice is based upon a study of this vocabulary leading to the definition of 10 classes that define the visual expression of built form. Some of these are affected by others while some are independent. An analysis shows that visual direction is one of the nearly independent classes and therefore well suited for further studies. The evaluation of the methodology and the directional criterion will be supported by three examples where we compare a compliance optimised structure with a similar direction optimised structure.

Introduction

When dealing with structural optimisation there are three different methods; Topology optimisation, Shape optimisation and Sizing optimisation, see Figure 1. The three different methods have their force at different phases in the design process. The topology optimisation method is a very effective tool in the conceptual phase, due to the very non-restrictive description of the design. In fact only design space, element type, loads and supports, and finally the objective function and constraints are needed to obtain a solution to the optimisation problem. This gives the opportunity to discover totally new designs/solutions to known problems. The topology optimisation method acts as a pre-processor to a refinement optimisation. Whether the shape optimisation method or the sizing optimisation method is to be used, must be decided in each case. The amount of material is determining the result of the topology optimisation, either it will be a rough shape of a two-dimensional structural domain, or the skeleton of a truss- or beam-like structure with slender members, where a shape- or a sizing optimisation, respectively, will detail the design [1].

Figure 1 The three different methods of optimisation and their position in the design process.

We have chosen to use the topology optimisation method as a testbench for the new direction criterion, due to the fact that it is a powerful tool in the conceptual phase of the design process, which is in most cases of most interest seen from an architectural point of view. In traditional topology optimisation, the shape of the...
structure is interpreted from the material density levels of its finite elements. Our approach is to treat the
greyscale picture formed by these elements by an image analysis based on a modified version of the image-
processing tool SUSAN [2]. It is based on detection of discontinuities in the grey levels between
neighbouring pixels. The result of the algorithm is assignment of a weight and an angle to each pixel. The
weight is a measure of the pixel’s tendency to be on an edge. The angle is the direction of the edge that
the pixel may belong to. By statistical treatment, it is possible to identify, among many other things, predominant
directions of shapes of the structure.

The following definitions are used in the explanations of the obtained results. The standard design
optimisation model is defined as follows: find an n-vector \( x = (x_1, x_2, \ldots, x_n) \) of design variables to minimise the
objective function \( f \):

\[
f(x_1, x_2, \ldots, x_n) \to \min\]

subject to

\[
\begin{align*}
&g(x_1, x_2, \ldots, x_n) = 0, \quad j = 1, \ldots, p \\
&h(x_1, x_2, \ldots, x_n) \leq 0, \quad i = 1, \ldots, m
\end{align*}
\]

where \( p \) is the total number of equality constraints and \( m \) is the total number of inequality constraints [3]. In
this paper the objective function is either compliance or mean of direction difference. A design space is an n-
dimensional space, where \( n \) is the number of variables. The design domain is the domain allocated for which
the design must lie within.

**Determination of criteria**

The following analysis motivates the choice of visual direction as a first attempt to incorporate visual
expression as a criterion in the design optimisation process. It must be clearly understood that the analysis
of the visual expression is only a small subset of the criteria handled by architects in the design process. The
following figure shows how the analysis of built form can be divided into classes [4]. Each class is analysed
for influences on other classes. This way, a hierarchical classification is obtained.

<table>
<thead>
<tr>
<th>Class</th>
<th>In</th>
<th>Out</th>
<th>Hierarchical level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color and Pattern</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Proportion</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Light</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Texture</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Size and scale</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Form and shape</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Movement and rhythm</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Composition</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Visual weight</td>
<td>6</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>Space</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

**Figure 2** The flowchart of the interacting classes.

**Table 1** High value in the hierarchical level means high complexity.

Figure 2 visualises the interaction between the 10 classes. Arrows indicate influence coming from other
classes. It is now possible to order the classes hierarchically after their dependency on - and their ability to
influence other classes such that a high degree of independence and high influence on other classes leads
to a low value in the hierarchical level. Light is the only class that is not influenced in any way by the other
classes. However, to minimise complexity it is chosen to work initially in only two dimensions where light
does not make sense, seen from an architectural point of view. The form and shape class is the next choice.
This class interacts with six other classes, indicating that it would be a fine basis if this class were formalised.
**Edge detection algorithm**

In the field of image processing, the analysis of two-dimensional pictures is a well-known technique. Therefore, this approach is used to develop an analysis tool that is capable of finding forms and shapes in a specified image. Today it is possible to identify edges, lines, and objects in an image. Thereby, it is possible to analyse the visual direction. The determination of visual direction of a structure is much influenced by its visible edges. An image-processing tool is used to determine where an edge is located. Several different algorithms have been tested for reliability. The following three statements characterise a good edge detection algorithm [3]:

1. Good detection. There should be a minimum number of false negatives and false positives.
2. Good localisation. The edge location must be reported as close as possible to the correct position.
3. Only one response to a single edge.
4. Speed. The algorithm should be fast enough to be usable in an optimisation system that uses iterative evaluation of criterion functions.

The algorithm chosen is the SUSAN (Smallest Univalue Segment Assimilating Nucleus) [3]. It is based on a very simple idea different from almost all other edge-detecting algorithms [5][6]. SUSAN has proved to be very reliable in tests we have performed. The output of SUSAN is two numbers for each pixel. The first number is a weight indicating to which extent this pixel can be considered to be on an edge. The second number is the angle of the direction perpendicular to the edge. By statistical processing of the output, such properties as perimeter, average direction, uniformity of direction, deviation from a desired direction, ordering into distinct directions, etc., can be deduced. The results are directly applicable as objective and constraint functions in a mathematical programming formulation of an optimal design problem.

The computed visual properties are used together with structural properties such as compliance and volume in a topology optimisation framework. When working with multiple and varying criteria, it is advantageous to solve the topology optimisation problem by means of mathematical programming rather than optimality criterion methods. This implies the use of sensitivity analysis with respect to the design variables, i.e., the material density of each element in the structure. To obtain these sensitivities by finite differences with repeated calls to the image analysis function would lead to prohibitive computation times. To avoid this problem, data structures are set up to link each design variable with the subset of the image in which the density influences the edge detection. This enables local recalculation of the picture in the sensitivity analysis and reduces the total cost of all sensitivities to the order of $n$ rather than $n^2$, where $n$ is the number of design variables.

**Evaluation of the method**

The image analysis algorithm has been reprogrammed and tested. The algorithm is implemented into a modified structural topology optimisation module of the Optimum DESign SYstem, ODESSY, developed at Aalborg University [7]. In the following three examples a comparison of the direction-optimised structure and the compliance-optimised structure will show the effect of the new direction criterion. The first example is a simply supported bridge with a constraint on the height. The second example is the same problem but with no restriction on the height. The last example concerns optimisation of the structure of a cantilever beam.

**Example 1. Bridge**

![Figure 3](image3.png) The design domain is simply supported

![Figure 5](image5.png) Structure optimised to 45 degree. Objective function: Minimise direction difference, inequality constraints: volume of material and compliance.
The problem treated in Figure 4 is only prescribed one restriction, namely the volume of material. If we add more constraints to the optimisation problem we narrow the design space. For that reason we have to relax some of the constraints, as the problem must converge to a black and white “picture”, material no material respectively. In Figure 5 the problem is formulated such as to minimise the difference between the wanted direction and the analysed direction. The two constraints; compliance and volume of material narrow the design space. It will be impossible to demand the same compliance to this problem as for the one obtained in Figure 4. This is because the compliance is narrowing the design space. The compliance and direction is to some extent conflicting criteria. Therefore we have to relax the compliance constraint in this case by 100%.

Example 2: Bridge with extended design domain

Example 3. Cantilever Beam
Conclusion
The intentions of formulating a criterion that governs the visual direction of a structure were succeeded. The criterion is based on a rewritten version of an edge-detecting algorithm SUSAN and was implemented into an existing topology optimisation code ODESSY [7]. Three examples of well-known problems, a cantilever beam, a bridge with a limited design domain and a bridge with an extended design domain show that the criterion has the wanted effect on the design. The bridge examples also show that the same problem has several solutions, in fact they are all optimal solutions to the optimisation problem stated. The bridge examples also show that the use of optimisation methods does not necessarily lead to the same solution to a given problem. But the variety of solutions depends of course on the possibility to formulate different design spaces to the same problem, like in the bridge example, where both the design domain and direction was used to define restricted design problems.

Perspectives
The software development so far includes the implementation of only one aesthetic criterion, namely the visual direction. This method will be extended in the near future, so it will be possible to handle 3 dimensional structures also. But still it would be only one new criterion several other criteria are needed to fulfil a wider range of statements formulated by the designer. Other possible criteria could be complexity and transparency, which are also to some extent analysable by means of image processing technique. The combination of both structural as well as aesthetic criteria will be a very powerful tool in creating integrated designs in the future.

References
7. ODESSY - Optimum DESign SYstem, Institute of Mechanical Engineering, Aalborg University, DK.

Acknowledgements
Thanks to S. M. Smith and J.M. Brady for handing over the source code to their edge-detecting algorithm SUSAN. The present work is partially supported by the Danish Centre for Integrated Design (CID).