SOFIA - A simulation tool for bottom founded and floating offshore structures

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Abstract

This paper presents a recently developed simulation tool, SOFIA (Simulation Of Floaters In Action), suitable for modeling slender bottom founded and moored/freely floating space frame structures exposed to environmental loads. In contrast to traditional rigid body formulations of floating structures, the finite element method is utilized in the implemented numerical approach, which allows for direct output of local section forces and displacements for joint analysis and fatigue calculations. The numerical approach builds upon a partitioned solution procedure, constituted by individual fluid and structure domains, which are coupled through the structural equation of motion. The structural domain is handled by means of the finite element method, while large displacements and stress stiffening effects, exhibited by moored floating structures, are inherently included due to a co-rotational element formulation. The fluid domain is modeled by an appropriate water wave theory, and the hydrodynamic loads are evaluated at the instantaneous fluid-structure interface by means of a relative Morison equation. The equation of motion is solved in time domain, which makes SOFIA capable of handling bottom founded and floating space frame structures that may experience non-linear behavior. To demonstrate the applicability of the simulation tool, numerical examples of a bottom founded and a floating space frame structure are presented.

1. Introduction

Numerical methods are widely used within structural and offshore engineering with respect to analysis of offshore structures. Because of rather complex interaction with the environment, it often requires the coupling of interdisciplinary topics such as aerodynamics, hydrodynamics and structural mechanics. The ability to predict the load-bearing capacity of offshore structures has always been of interest, but even more accurate methods are needed as structures, and their interactions with the environment, are getting even more complicated to analyze. Especially within renewable energy, floating concepts have received great interest during the latest decade. Floating offshore wind turbines have been investigated by means of state-of-the-art methods in [1]. Bottom founded structures, such as the well-known jacket structures used within the oil & gas sector, differ from floating offshore structures because they, in many cases,
behave linearly from a structural perspective. However, this is not always the case for floating offshore structures. These structures may experience non-linearities, such as large displacements and/or stress-stiffening effects, which may alter the structural behavior considerably and must therefore be taken into account.

Different simulation tools have been developed for the purpose of including the non-linearities [1]. Despite the currently developed tools an in-house simulation tool, abbreviated SOFIA (Simulation Of Floaters In Action), has been developed as an initial computational framework for future research and education within analysis of floating offshore structures in the Division of Structural and Offshore Engineering at Aalborg University (Denmark). In the current version, SOFIA is capable of simulating bottom founded and floating offshore structures exposed to wave forces from linear and non-linear waves, while geometrical non-linearities of the structure can be included. The paper presents the simulation tool, SOFIA, in the following manner: initially the overall methodology is introduced in section 2. In section 3 the numerical implementation is briefly touched upon, while also application examples are illustrated in relation to ongoing research. Finally, in section 4 concluding remarks are made regarding the present stage of SOFIA, while possible future developments are stated as well.

2. Methodology

Three overall topics are considered in SOFIA; the modeling of structures, waves and the interaction of these. These topics are introduced in the following subsections, where the theoretical principles are briefly summarized. A more comprehensive description can be found in [3].

2.1. Structural equation of motion

The structural domain is modeled by means of Timoshenko beam theory, while the governing differential equations are discretized by use of the finite element method. As offshore structures often consist of multiple tubular members, the application of beam elements is well-suited for this purpose. After discretizing the structure by a finite number of beam elements the structural equation of motion becomes

$$M\ddot{x} + C\dot{x} + Kx = f_w + f_g + f_b$$

where \{\ddot{x}, \dot{x}, x\} are the structural response vectors, \(f_w\) is the hydrodynamic force vector, \(f_g\) is the gravitational force vector, \(f_b\) is the buoyancy force vector, \(M\) is the structural mass matrix, \(C\) is the structural damping matrix and \(K = K_e + K_g\) is the structural stiffness matrix constituted by an elastic and geometric part. The equation of motion (1) is solved in time-domain by using either a linear or non-linear Newmark time integration scheme. In the case of non-linear Newmark time integration, the system matrices are updated during the analysis by means of a co-rotating beam element formulation [2]. An example of a jacket structure modeled in SOFIA is given in figure 1.

The environmental force vectors are described in section 2.3, whereas further details regarding the co-rotating beam element formulation and Newmark time integration can be found in [2]. For the environmental loading to be defined, particle kinematics are needed. The particle kinematics are formulated in the following section by means of linear and non-linear water wave theory.

Fig. 1. A jacket substructure [4] modeled in SOFIA.
2. Water wave mechanics

For the modeling of fluid-structure interaction, the main interests are the free surface elevation and particle kinematics. These parameters are modelled by means of water wave theory. In SOFIA it is possible to model various sea states ranging from multi-directional waves to steep regular waves, which are necessary for fatigue limit state and ultimate limit state analysis, respectively. The incoming flow is assumed to be incompressible, irrotational and inviscid. This allows the flow to be described by a velocity potential \( \phi \) from which the particle kinematics are derived as

\[
\dot{u} = \frac{\partial \phi}{\partial x}, \quad \dot{v} = \frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z}
\]

where the convective terms are neglected for linear waves. The overall principles are briefly summarized in the subsequent subsections for the implemented linear and non-linear wave theories, respectively.

2.2.1. Multi-directional waves

For linear solutions of the governing mathematical problem, the principle of superposition is valid. Therefore, the free surface elevation of a linear multi-directional sea state is represented by

\[
\eta(x,y,t) = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} \cos \left( \omega_i t - k_i \left( x \cos(\theta_j) + y \sin(\theta_j) \right) + \delta_{ij} \right)
\]

while the velocity potential is also determined by superimposing linear solutions as

\[
\phi(x,y,z,t) = U x + \sum_{i=1}^{n} \sum_{j=1}^{m} - \frac{a_{ij} g}{\omega_i} \frac{\cosh(k_i(z+d))}{\cosh(k_i d)} \sin \left( \omega_i t - k_i \left( x \cos(\theta_j) + y \sin(\theta_j) \right) + \delta_{ij} \right)
\]

where \( k_i \) is the wavenumber, \( U \) is the current velocity (assumed constant), \( \omega_i \) is the cyclic frequency, \( g \) is the gravitational acceleration, \( d \) is the water depth, \( \theta_j \) is the propagation direction, \( \delta_{ij} = [0, 2\pi] \) are random phase angles and \( a_{ij} \) are the amplitudes. The latter are determined from

\[
a_{ij} = \sqrt{2S(f_i, \theta_i) \Delta f \Delta \theta}
\]

where \( S(f, \theta) = S_{\eta}(f)D(\theta) \) is a directional wave spectrum, \( S_{\eta}(f) \) is a wave energy spectrum and \( D(\theta) \) is a spreading function, for which \( \int_0^{2\pi} D(\theta) d\theta = 1 \) must hold. In SOFIA the well-known JONSWAP spectrum is implemented alongside with a cosine-power distribution as spreading function [3]. Wheeler stretching [5] is applied for linear water waves, so that the particle kinematics can be approximated by stretching the profile towards the free surface.

2.2.2. Stream function waves

In ultimate limit states the hydrodynamic forces are based on particle kinematics from steep regular waves. For this purpose stream function wave theory is implemented in SOFIA. The free surface elevation can be evaluated by

\[
\eta(x,t) = 2 \sum_{j=1}^{N} a_j \cos (jk(x - ct)) + a_N \cos (Nk(x - ct))
\]

where

\[
a_j = \frac{2}{L} \sum_{i=1}^{N+1} \eta_i \cos (jk_i \Delta x_i), \quad c \text{ is the wave celerity, and the stream function may be converted to a velocity potential, so that }
\]

\[
\phi(x,z,t) = U x - \sum_{j=1}^{N} B_j \frac{\cosh(jk(z+d))}{\cosh(jk d)} \sin(jk(x - ct))
\]

where \( N \) is the stream function order. \( B_j, k, U, \eta_i \) and \( c \) are unknown parameters to be determined by means of Newton-Raphson iterations on the governing equations, as explained in [3,6].
2.3. Fluid-Structure Interaction

The environmental loading is, in the current version of SOFI A, modeled by two contributions; namely $f_w$ hydrodynamic forces and $\{f_g, f_b\}$ gravitational forces. As the finite element method is used for the structural domain, the environmental loadings are formulated as distributed loads along the submerged element length, as illustrated in figure 2, by means of polynomial regression. The different force contributions are introduced in the subsequent subsections. Tangential forces will be applied in a similar manner as the forces normal to the structural members.

2.3.1. Hydrodynamics

As structural elements are assumed slender, and as such, do not exhibit neither diffraction nor radiation, the hydrodynamic forces are expressed by means of relative Morison equations as

$$
\begin{align*}
  f_w = \begin{bmatrix} f_{wn} \\ f_{wt} \end{bmatrix} = & \begin{bmatrix} -\rho_w C_a A_s \ddot{x} + \rho_w C_M A_s \ddot{u}_n + \frac{1}{2} \rho_w C_D D (\ddot{u}_n - \ddot{x}_n) |\ddot{u}_n - \ddot{x}_n| \\ \frac{1}{2} \rho_w C_D D (\ddot{u}_t - \ddot{x}_t) |\ddot{u}_t - \ddot{x}_t| \end{bmatrix} 
\end{align*}
$$

where $\ddot{u}$ is the particle acceleration, $\ddot{x}$ is the structural acceleration, $\ddot{x}$ is the structural velocity, $C_D$ is the drag coefficient, $C_a$ is the added mass coefficient, $C_M$ is the inertia coefficient, $A_s = \pi R_o^2$ for tubular cross-sections and $D = 2R_o$ is the outer diameter. In (8) the subscripts $n$ and $t$ indicate directions normal and tangential to the considered structural member. Global kinematics are transformed into the local frame of reference to determine the normal and tangential force components. This is easily handled by use of the co-rotating element bases, which, as illustrated in figure 2, transform global variables to the local frame of reference.

2.3.2. Buoyancy and gravity

Especially for floating offshore structures the motion, and thereby structural response, depends highly on the relation between buoyancy and own weight. Hence, in addition to the hydrodynamic forces, also forces related to gravitation are introduced. The buoyancy forces are expressed as

$$
  f_b = \rho_w C_b A_s g
$$

where $\rho_w$ is the density of water and $C_b$ is a buoyancy coefficient, which can be tuned to model flooded members. As the structure may experience large displacements during the analysis, the buoyancy will vary based on the instantaneous wetted length of each structural member. It must also be stated that this formulation is considered acceptable as all members are assumed slender, and for deep water applications local stability issues, due to high hydrostatic pressure, should be taken into account. Gravity forces are expressed in a similar manner as the buoyancy force, namely

$$
  f_g = \rho_s A g
$$

Fig. 2. Hydrodynamic forces distributed over the instantaneous wetted length, $L_s$, of a structural member with total length $L$ [3].
where \( \rho_s \) is the structural mass density and \( A \) is the actual cross-sectional area of the structural member. This gravity force will also be updated during the analysis if non-linear time integration is enabled. In reality, the force vector is established by taking the matrix-vector product between the mass matrix and a gravitational acceleration vector.

3. Implementation in SOFIA

SOFIA is primarily being developed for research and educational purposes. In order to comply with demands for reproducibility and transparency, the computational framework is written in MATLAB, which makes it relatively easy to use for both experienced and less experienced users within scientific computing.

3.1. MATLAB classes and overall data structure

The overall data structure is shown in figure 3, which consists of four classes and two additional scripts for handling of input and postprocessing. It is conceptually straight-forward to use. Initially, the structure is modeled by defining traditional input for finite element analysis (nodes, topology and material properties) in the structures-class, while the wave field is defined in the fluid-class by specifying the wave characteristics, such as wave height, wave period, water depth, etc. In the interaction-class the coupling between the structures- and fluid-class is specified. At present stage the coupling is constituted by a relative Morison equation, as stated in section 2.3.

3.2. Example: Monopile exposed to linear regular wave

In this example a bottom founded monopile of constant diameter is exposed to a single-component, regular linear wave. This example validates the calculation of hydrodynamic forces (assumed: \( C_{D} = 1 \) and \( C_{M} = 2 \)) while piercing the free surface. In figure 4 an analytical solution is compared to the numerical results from SOFIA, and good agreement is found between the results.

3.3. Ongoing research and developments

Current research is targeted applications within the wind and wave energy sector. Offshore structures used within these sectors, as illustrated in figure 5, constitute offshore substructures for which the application of SOFIA is intended. Wave energy devices and floating offshore wind turbine structures are examples of application contexts, in which non-linear behavior may be introduced, and could necessitate more detailed analysis. Therefore, the modeling of non-linear wave-structure interaction is of great interest to develop in the future versions of SOFIA. At the moment a fully non-linear potential flow solver, including structural boundary conditions, is under development. Recently,
SOFIA has been used in relation to investigation of methods for detection and localization of damages in offshore structures [7], where the jacket structure in figure 5 was modeled.

4. Concluding remarks

The current implementations in SOFIA were presented and the hydrodynamic load calculation was validated by a simple example. Future investigations should be carried out, in order to test the tool against experimental results. In later versions SOFIA may become open-source and available online. This will allow users to use either of the MATLAB classes defined in SOFIA for individual purposes; whether it is for research or education. In addition to further improvements of the basic computational framework, additional features will be developed. At the moment a fully non-linear wave model is being developed, which eventually will allow for non-linear wave simulation with irregular domains and structural boundaries, in addition to the currently implemented Morison equations.

References