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An Economic Demand Response Model in Liberalized Electricity Markets with Respect to Flexibility of Consumers

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Abstract: Before restructuring in the electricity industry, the primary decision-makers of the electricity market were deemed to be power generation and transmission companies, market regulation boards, and power industry regulators. In this traditional structure, consumers were interested in receiving electricity at flat rates while paying no attention to the problems of this industry. This attitude was the source of many problems, sometimes leading to collapse of power systems and widespread blackouts. Restructuring of the electricity industry however provided a multitude of solutions to these problems. The most important solution can be demand response (DR) programs. This paper proposes an economic DR model for residential consumers in liberalized electricity markets to change their consumption pattern from times of high energy prices to other times to maximize their utility functions. This economic model is developed based on constant elasticity of substitution (CES) utility function known as one of the most popular utility functions in microeconomics. Simulation results indicate that the proposed model is adaptable to any group of residential consumers with any disposition toward participation in DR programs and can be adjusted for any time period according to the preference given by the residential consumer.

Keywords: Demand-side management, economic demand response model, consumer utility function, constant elasticity of substitution (CES).

Nomenclature

C_{peak}	Demand for the peak times after implementation of DR Programs
$C_{shoulder}$	Demand for shoulder times after implementation of DR Programs
$C_{off\text{-}peak}$	Demand for off-peak times after implementation of DR Programs
C'_{peak}	Demand for the peak times before implementation of DR Programs
$C'_{shoulder}$	Demand for shoulder times before implementation of DR Programs
$C'_{off\text{-}peak}$	Demand for off-peak times before implementation of DR Programs

$(1-\rho)^{-1}$	Elasticity of substitution
θ	Coefficient of relative risk aversion
U_t	Utility function
P_{peak}	Peak time price
$P_{shoulder}$	Shoulder time price
$P_{off\text{-}peak}$	Off-peak time price
B	Budget

1. Introduction

In the early years of formation of deregulated electricity market, the competition was only between generation companies as the dominant players of the market. In such environment, however, there was no interaction between the demand-side and the supply-side and the consumers did not have the choice for their retailers and better services. This was also augmented by lack of inelasticity on the demand-side and resulted in an increase in electricity prices and the emersion of bidding strategies called as “hockey-stick bidding”. Over the years, several solutions have been introduced in order to suitably address these issues and increase share of consumers’ participation in electricity markets. In [1], all those solutions have been reported and divided into three categories namely demand side management (DSM), purchase allocation and bidding strategy. As one of the aforementioned strategies, DSM can change an inelastic demand to an elastic one [2]. The implementation of a DSM plan results in numerous advantages in deregulated energy systems and provides beneficial effects on both supply and demand sides [3]-[4]. One of the techniques mostly used as a complement with DSM is demand response (DR) program which could be implemented in a price-wise or incentive-based manner [5]-[6]. Although application of DR programs has a long history goes back to 1970’s, the importance of such programs have endorsed when the United States Congress approved the Energy Policy Act of 2005 [7]**.** This matter led to a greater willingness to develop methods for further improvement of DR models.

Recently, a new participant in the electricity market is considered to assist better implementation of DR programs. This new participant is called Demand Response Provider (DRP) and is in charge of management and implementation of DR programs at demand-side [8]-[9]. Planning and executing a DR program for residential consumers requires access to complex models, at least when compared with what is needed for industrial and commercial consumers [10]-[12]. Retailer companies (or DR providers) seek to maximize their own profits in a given electricity market by adopting proper bidding strategies, and at the same time,

to reduce their risks by encouraging consumers to actively participate in DR programs [1]. But on the implementation, the use of these programs is associated with many barriers. One reason for this is the lack of an appropriate DR model which show consumer's reaction to price-based programs.

A good residential DR model should have two primary features: the first feature is defined as the adaptability to different consumers with different dispositions toward the DR program. As shown in Fig. 1, different consumers may behave differently towards DR program, and they can be categorized into at least three groups depending on their disposition toward participation. The ability of a DR model to adapt to and accommodate all groups of consumers is an imperative feature and is the point where current DR models show plain weakness.

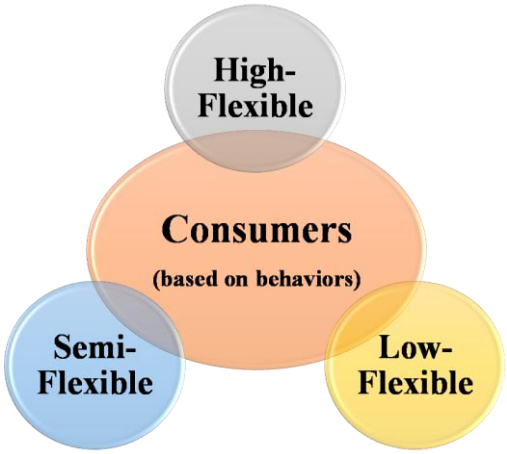


Fig.1. *consumers' reactions to DR programs*[13]

The second feature of a good residential DR model is defined as the adjustability to time preferences of consumers. This means that each consumer should be able to easily shift his/her demand from the high-price hours to the favourite hours according to his/her lifestyle. To be more specific, the concept of adjustability denotes the ability to merge the desire of consumers (according to their habits and lifestyle) with DR model to adjust consumption over the time periods. Hence, an efficient model should be able not only to express the extent of consumers' reactions to prices, but also to account for adjustment of consumption levels of different time periods.

Regarding the residential DR programs, several research works have been done to date and convincing results have been reported, however there is still room for modifying these programs to fit into a comprehensive model as described earlier. Researches in [13]-[23] attempted to simulate the DR model based on the concept of demand-price elasticity which is an idea extracted from the consumer theory in microeconomics that shows the change in the demand in reaction to the change in the price [20]. For example, a model for DR programs was introduced in [15] where it was illustrated that consumer's electricity demand depends on the price elasticity of the demand, reward and the penalty values determined for concerned DR

programs. Also, researchers in [16] proposed a model for ‘‘interruptible/curtailable service” and ‘‘capacity market program”. Both models introduced in [15]-[16] could satisfy the characteristics of a good DR model to wit adaptability and adjustability features but with sophisticated control settings. In [18], a DR model based on the consumers' behavior with the concept of demand-price elasticity was introduced. The main idea in creation of the model was adopted from the economic and psychological analysis that accept the differences between the impacts of applying reward and penalty programs and bate that reward is a superior method for habit establishment compared to the penalty. In [19], a model of price-responsive loads was concluded relying on the concept of consumer utility function and price elasticity. In a like manner, a DR model by using ideas of demand-price elasticity and based on incorporating time of use (TOU) and incentive based DR programs was introduced in [20]. This model had the potential of applying both penalty and reward in DR tariffs. Authors in [22] proposed a DR algorithm based on Stackelberg model for scheduling electrical loads. The results showed that the model is useful for attaining the optimal load control in reaction to real-time price changes. Finally, a new DR model was introduced in [23] with fuzzy subtractive clustering methods which was able to manage the controllable loads for consumers’ profit maximization.

Considering the reviewed DR models, it can be observed that majority of the recent works have not covered the features of a good DR model (i.e., adaptability and adjustability features). However, in a few cases such as [24], the authors have managed to address the adaptability and adjustability features. While having some time-related limitations, they have paved the way for development of more appropriate DR models. Moreover, most of the reviewed literature on price-elasticity based DR models have considered point elasticity concept and tried to linearize the demand curve at a particular operating point instead of the entire curve which might result in a discontinuous decision-making process.

This paper employs economic theories and mathematical formulations to introduce a new model for time-of-use (TOU)-based DR program that not only improves the consumption patterns of consumers over the time to save more money, but also enables adaptability and adjustability features for end-users. In the proposed model, the entire demand curve is also considered which results in increased flexibility and provides a continuous decision-making process. This flexibility would cause new parameters to emerge and guarantee the prerequisites for making a good DR model (i.e., adaptability and adjustability). With the above-mentioned features, the proposed model preponderates over the existing models for two reasons. First, with the help of these features, consumers can manage their consumption over the time periods according to their desire which is relatively unique for each consumer particularly in the residential sector. Second, these features can help to adopt a new decision-making strategy by participating in DR programs when a

new event affects the decision-making process. As a whole, the contributions of this paper could be summarized as follows:

- A new model for TOU-based DR program is proposed based on the economics theories to enable residential consumer response to price-based programs for profit maximization,
- An effective structure is presented for residential load management by considering different levels of participation in DR actions.
- Desire of residential consumers to reapportionment of consumption over the time periods is introduced into DR events.

The organization of this paper is as follows. The consumer theory is introduced in Section 2. Afterward, section 3 explains the mathematical formulation of the proposed DR model. Section 4 presents a sensitivity analysis to study the effect of model parameters on consumer participation in DR programs. Then, Section 5 provides numerical results and shows validity of model. Finally, Section 6 concludes the paper by summarizing the main results and discussing future work.

2. Consumer Theory

Consumer theory is one of the most important theories in economics. This theory concerns how consumers spend their money given their preferences and budget constraints [24]-[29]. The two primary tools of this theory are utility functions and budget constraints which allows the consumer to make a decision in regard to their consumption level. In this paper, we employ this theory and its tools to develop a model for residential DR programs. This model allows the consumer to make decisions based on their own maximum benefits while satisfying the budget constraints. Moreover, the constant elasticity of substitution (CES) function is used as the utility function of consumers, which is one of the most widely used utility functions in economic studies [30].

2.1. CES utility function

Utility function is one of the basic concepts of economics and reflects the interest to earn more profit. Utility function is derived from the concept of potential in physics and its maximization represents reaching an equilibrium from the economic perspective. Unlike the potential energy, there is no specific method to find utility function, and economists usually derive the related formulation through empirical methods. From the consumer's perspective, utility function expresses the extent of his/her satisfaction with consumption of one or more products. Researchers have developed several utility functions for microeconomics, among

them CES is considered as one of the most important functions. This function is especially popular for multiproduct scenarios. CES utility function for two different products is as follows [30]-[31]:

$$U(X_1, X_2) = (X_1^{\rho} + X_2^{\rho})^{1/\rho} \quad 0 \neq \rho < 1 \tag{1}$$

where X_1 and X_2 are the products and $(1-\rho)^{-1}$ is called the elasticity of substitution. In the extension of this function for the electricity market, electricity offered at different prices is assumed as multiple products. For instance, a market where electricity has three different prices is assumed as a market with three products. In the next section, this utility function is employed to develop a model for DR programs.

3. Economic DR Model

The proposed economic DR model in this paper, simulates the DR program based on TOU-rate program. The price of electricity varies depending on the time of energy consumption labelled as peak, shoulder and off-peak. In the absence of DR program, it is assumed that the power consumption during these three periods are C'_{peak}, $C'_{shoulder}$ and $C'_{off\text{-}peak}$., respectively while enabling DR actions changes these values to C_{peak}, $C_{shoulder}$ and $C_{off\text{-}peak}$, accordingly. The utility function of consumer in regard to a DR program is expressed as follows:

$$\begin{aligned} &U(C_{peak}, C_{shoulder}, C_{off-peak}) \\ &\mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\mathrm{P}_{shoulder} + \mathrm{C}_{off-peak}.\mathrm{P}_{off-peak} \end{aligned} \tag{2}$$

Based on the previously-mentioned elements of the consumer theory, i.e., utility function and budget constraint, consumers can present two different reactions to DR programs: 1) they can decrease their electricity usage in the peak times without any shift to the other times; or 2) they can shift their electricity usage from the peak times to other times. This model assumes that consumers do not intend to reduce their electricity usage (i.e., total energy consumption before and after executing the DR program remains the same) but to save money through changing the consumption pattern and reducing the electricity bill. With this assumption, consumer should shift a part of his consumption from high price hours to those with lower prices, thus:

- Case I: Consumer shifts a part of consumption from shoulder time to off-peak time. In this scenario, the following utility function should be maximized:

$$Max\left\{U\left(C_{shoulder},C_{off-peak}\right)=\left(C_{shoulder}^{\rho'}+C_{off-peak}^{\rho'}\right)^{\frac{1}{\rho'}}\right\}\ ;\ 0<\rho'<1$$
s.t.
$$B'=C_{shoulder}.P_{shoulder}+C_{off-peak}.P_{off-peak} \tag{3}$$

- Case II: Consumer shifts a part of consumption from peak time to shoulder time and off-peak time. In this scenario, the following utility function should be maximized:

$$Max\left\{U\left(C_{peak},U\left(C_{shoulder},C_{off-peak}\right)\right)=\left(C_{peak}^{\rho}+\left(U\left(C_{shoulder},C_{off-peak}\right)\right)^{\rho}\right)^{1/\rho}\right\}\ ;\ 0<\rho<1$$
$$B=C_{peak}.P_{peak}+C_{shoulder}.P_{shoulder}+C_{off-peak}.P_{off-peak} \tag{4}$$

Here, we are dealing with optimization problems with equality constraints (equations (3)-(4)). These types of problems can be solved by Lagrange multipliers [32]. As an example, let's consider the following optimization problem:

$$Max\ \{f(x)\};\ \text{s.t.}\ \ h_j(x)=0 \tag{5}$$

If x^* is considered as the local maximum, then there exists a new variable λ_j (j=1, 2, 3…) such that:

$$\begin{aligned}&\nabla f(x^*)+\sum_{j=1}^{l}\lambda_j\nabla h_j(x^*)=0\\&h_j(x^*)\le 0\ ;\ \ \forall\ j=1,2,...,l\\&\mu_i\ge 0\qquad ;\ \ \forall\ i=1,2,...,m\end{aligned} \tag{6}$$

By applying the same procedure to (3), we could create the Lagrange function of Case I as follows:

$$L=\left(C_{shoulder}^{\rho'}+C_{off-peak}^{\rho'}\right)^{\frac{1}{\rho'}}+\lambda\left[B'-C_{shoulder}.P_{shoulder}-C_{off-peak}.P_{off-peak}\right] \tag{7}$$

Finding the partial derivatives of (7) with respect to $C_{shoulder}$ and $C_{off\text{-}peak}$ and λ would yield:

$$\frac{dL}{dC_{off-peak}}=C_{off-peak}^{\rho'-1}.\left(C_{off-peak}^{\rho'}+C_{shoulder}^{\rho'}\right)^{\frac{1-\rho'}{\rho'}}-\lambda P_{off-peak}=0 \tag{8}$$

$$\frac{dL}{dC_{shoulder}}=C_{shoulder}^{\rho'-1}.\left(C_{off-peak}^{\rho'}+C_{shoulder}^{\rho'}\right)^{\frac{1-\rho'}{\rho'}}-\lambda P_{shoulder}=0 \tag{9}$$

From (8) and (9):

$$\frac{C_{off-peak}^{\rho'-1}}{C_{shoulder}^{\rho'-1}} = \frac{P_{off-peak}}{P_{shoulder}} \Rightarrow C_{off-peak} = C_{shoulder} \cdot \left(\frac{P_{off-peak}}{P_{shoulder}} \right)^{\frac{1}{\rho'-1}} \tag{10}$$

Substituting (10) into (3):

$$\begin{aligned} & U\left(C_{shoulder}, C_{off-peak}\right) = \\ & \left(C_{shoulder}^{\rho'} + \left(C_{shoulder} \cdot \left(P_{off-peak} \,/\, P_{shoulder} \right)^{\frac{1}{\rho'-1}} \right)^{\rho'} \right)^{1/\rho'} \\ & = C_{shoulder} \cdot \left[1 + \left(\frac{P_{off-peak}}{P_{shoulder}} \right)^{\frac{\rho'}{\rho'-1}} \right]^{\frac{1}{\rho'}} = \sigma \cdot C_{shoulder} \end{aligned} \tag{11}$$

By substituting (11) into (4), the optimization problem formulation for Case I is as follows:

$$\begin{aligned} & Max\left\{ U\left(C_{peak}, U\left(C_{shoulder}, C_{off-peak}\right)\right) = \left(C_{peak}^{\rho} + \sigma^{\rho} . C_{shoulder}^{\rho}\right)^{\frac{1}{\rho}} \right\} \\ & \text{s.t.} \\ & \mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\mathrm{P}_{shoulder} + \mathrm{C}_{off-peak}.\mathrm{P}_{off-peak} \end{aligned} \tag{12}$$

The budget constraint can be also formed as follows:

$$\begin{aligned} & \mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\mathrm{P}_{shoulder} + C_{shoulder} \cdot \left(\frac{P_{off-peak}}{P_{shoulder}} \right)^{\frac{1}{\rho'-1}} .\mathrm{P}_{off-peak} \\ & \mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder} \cdot \left[\mathrm{P}_{shoulder} + \left(\frac{P_{off-peak}}{P_{shoulder}} \right)^{\frac{1}{\rho'-1}} .\mathrm{P}_{off-peak} \right] = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\omega \end{aligned} \tag{13}$$

Now, we create the Lagrange function for Case II introduced in (12):

$$\begin{aligned} & Max\left\{ U\left(C_{peak}, U\left(C_{shoulder}, C_{off-peak}\right)\right) = \left(C_{peak}^{\rho} + \sigma^{\rho} . C_{shoulder}^{\rho}\right)^{\frac{1}{\rho}} \right\} \\ & \text{s.t.} \\ & \mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\omega \end{aligned} \tag{14}$$

Using the Lagrange multipliers, (14) can be rewritten as follow:

$$L = \left(C_{peak}^{\rho} + \sigma^{\rho}.C_{shoulder}^{\rho}\right)^{\frac{1}{\rho}} + \lambda\left[\mathrm{B}\text{- }\mathrm{C}_{peak}.\mathrm{P}_{peak} - \mathrm{C}_{shoulder}.\omega\right] \tag{15}$$

Calculating the partial derivatives of (15) with respect to C_{peak}, $C_{shoulder}$ and λ would result in:

$$\begin{aligned} \frac{dL}{dC_{peak}} &= C_{peak}^{\rho-1}.\left(C_{peak}^{\rho} + \sigma^{\rho}.C_{shoulder}^{\rho}\right)^{\frac{1-\rho}{\rho}} - \lambda P_{peak} = 0 \\ &\Rightarrow \lambda = \frac{1}{P_{peak}} C_{peak}^{\rho-1}.\left(C_{peak}^{\rho} + \sigma^{\rho}.C_{shoulder}^{\rho}\right)^{\frac{1-\rho}{\rho}} \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{dL}{dC_{shoulder}} &= \sigma^{\rho}.C_{shoulder}^{\rho-1}.\left(C_{peak}^{\rho} + \sigma^{\rho}.C_{shoulder}^{\rho}\right)^{\frac{1-\rho}{\rho}} - \lambda\omega = 0 \\ &\Rightarrow \lambda = \frac{1}{\omega}\sigma^{\rho}.C_{shoulder}^{\rho-1}\left(C_{peak}^{\rho} + \sigma^{\rho}.C_{shoulder}^{\rho}\right)^{\frac{1-\rho}{\rho}} \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{dL}{d\lambda} &= \mathrm{B}\text{- }\mathrm{C}_{peak}.\mathrm{P}_{peak} - \mathrm{C}_{shoulder}.\omega = 0 \\ &\Rightarrow \mathrm{B} = \mathrm{C}_{peak}.\mathrm{P}_{peak} + \mathrm{C}_{shoulder}.\omega \end{aligned} \tag{18}$$

Solving (17) and (18) gives:

$$C_{peak} = C_{shoulder}.\left(\frac{\sigma^{\rho}P_{peak}}{\omega}\right)^{\frac{1}{\rho-1}} \tag{19}$$

From (18) and (19) it can be concluded that:

$$C_{shoulder} = \frac{\mathrm{B}}{\left(\frac{\sigma^{\rho}P_{peak}}{\omega}\right)^{\frac{1}{\rho-1}}.\mathrm{P}_{peak} + \omega} \tag{20}$$

Taking (19) and (20) into consideration, the following can be derived:

$$C_{peak} = \frac{\mathrm{B}}{\left(\frac{\sigma^{\rho}P_{peak}}{\omega}\right)^{\frac{1}{\rho-1}}.\mathrm{P}_{peak} + \omega}.\left(\frac{\sigma^{\rho}P_{peak}}{\omega}\right)^{\frac{1}{\rho-1}} \tag{21}$$

Also, by considering (10) and (20), we can conclude that:

$$C_{off-peak} = \frac{\mathrm{B}}{\left(\frac{\sigma^{\rho} P_{peak}}{\omega}\right)^{\frac{1}{\rho-1}} .\mathrm{P}_{peak} + \omega} . \left(\frac{P_{off-peak}}{P_{shoulder}}\right)^{\frac{1}{\rho'-1}} \tag{22}$$

In this context, C_{peak}, $C_{shoulder}$ and $C_{off\text{-}peak}$ represent electric power consumed in peak, shoulder and valley period, respectively. To obtain the power consumption at each time interval of the study period, one must use the following equations:

$$\mathrm{C}_{t,peak} = C'_{t,peak} \times \left(C_{peak} / C'_{peak}\right) \tag{23}$$

$$\mathrm{C}_{t,shoulder} = C'_{t,shoulder} \times \left(C_{shoulder} / C'_{shoulder}\right) \tag{24}$$

$$\mathrm{C}_{t,off-peak} = C'_{t,off-peak} \times \left(C_{off-peak} / C'_{off-peak}\right) \tag{25}$$

4. Sensitivity Analysis

Regarding the economic DR model proposed in this paper, it has been demonstrated that each consumer is assigned with two control parameters ρ and ρ' that adjust his/her level of participation and preference over the time. For each consumer, the values of these parameters depend on his/her interest to participate in DR programs and his/her lifestyle. The proper ranges of these two parameters for each consumer can be obtained through a sensitivity analysis. As shown in Fig. 2 for example, when ρ (i.e., substitution parameter between peak consumption and consumptions in other times) is considered as a constant and ρ' (i.e., substitution parameter between shoulder consumption and off-peak consumption) is treated as a variable, the changes in the off-peak time consumption are small, but changes in the consumption of other time periods are significant. Therefore, consumption level of shoulder-time and peak-time can be easily controlled by parameter ρ', however its control over off-peak consumption is negligible. Hence, we can say that constant ρ and variable ρ' denote the consumption in the peak-time and shoulder-time can be adjusted.

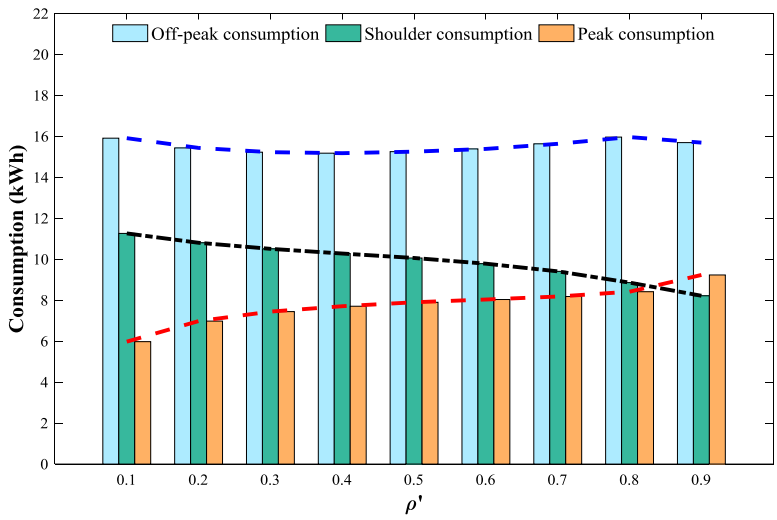


Fig.2. *Consumption in different time periods for constant ρ and variable ρ′*

In Fig. 3, changes in consumption level of different time periods are plotted for constant ρ' and different ρ values. It can be seen that shoulder-time consumption has a slight rise, however changes in consumption of other time periods are significant. Therefore, consumption level of off-peak and peak-times can be easily controlled by parameter ρ, however its control over shoulder-time consumption is negligible.

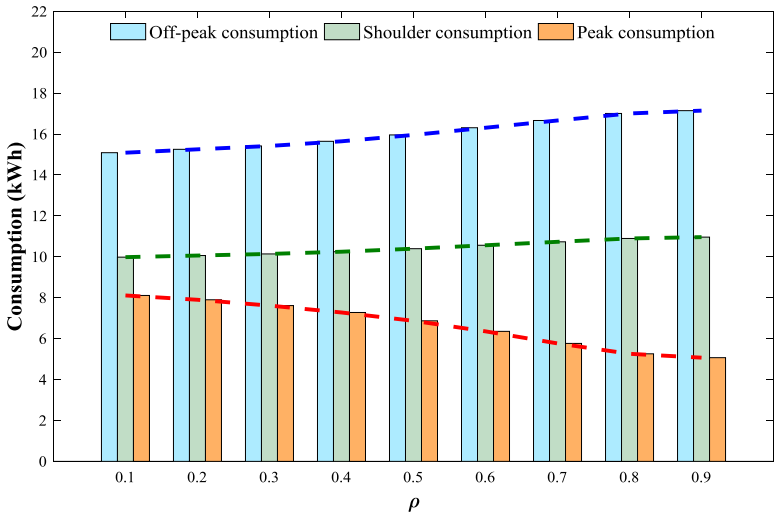


Fig.3. *consumption in different time periods for constant ρ′ and variable ρ*

As can be seen, these two control parameters provide a very powerful tool to adjust the participation and consumption levels in accordance with consumes' preferences. Here, we determine the suitable range of (ρ, ρ') for the consumer's decision-making process. As shown in Fig. 4 and can be expected, peak-time electricity demand after execution of DR program is decreased in all areas (except for $\rho' > 0.7$ where there is a discontinuity in consumer's decision pattern), meaning that main objective of the program, which is reducing peak-time consumption, has been accomplished. So, this sensitivity analysis shows that the suitable range of the mentioned parameters can be defined as: $0 < \rho < 1$ and $0 < \rho' < 0.7$.

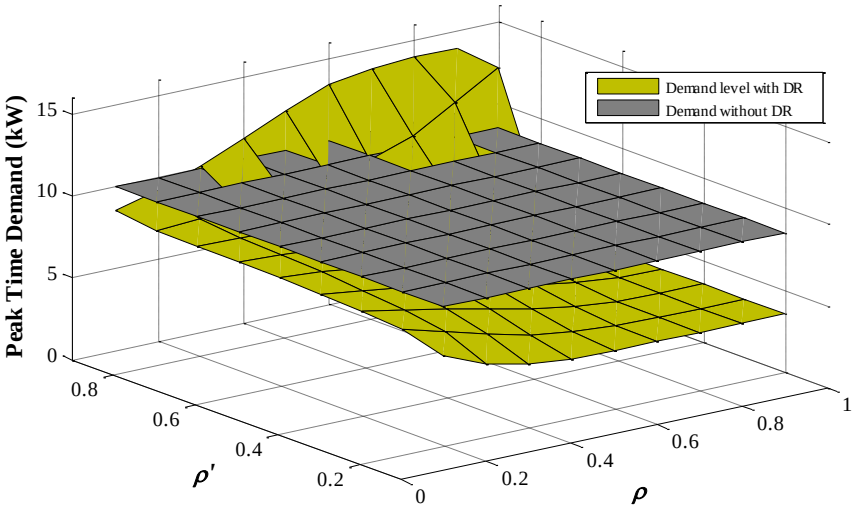


Fig.4. *Consumption levels for different rates of time preferences (peak period)*

In Fig. 5, changes in the off-peak time consumption are plotted against different ρ' and ρ values. Considering the feasible region which was previously found in the peak time curve, this step of sensitivity analysis is carried out for $0 < \rho < 1$ and $0 < \rho' < 0.7$. As can be seen, there is no point of discontinuity in the consumer's decision pattern, so the same ρ and ρ' value ranges can be also applied to this case. Also, as Fig. 5 shows, execution of DR program has increased the consumption in this time period, which is consistent with the program's objectives.

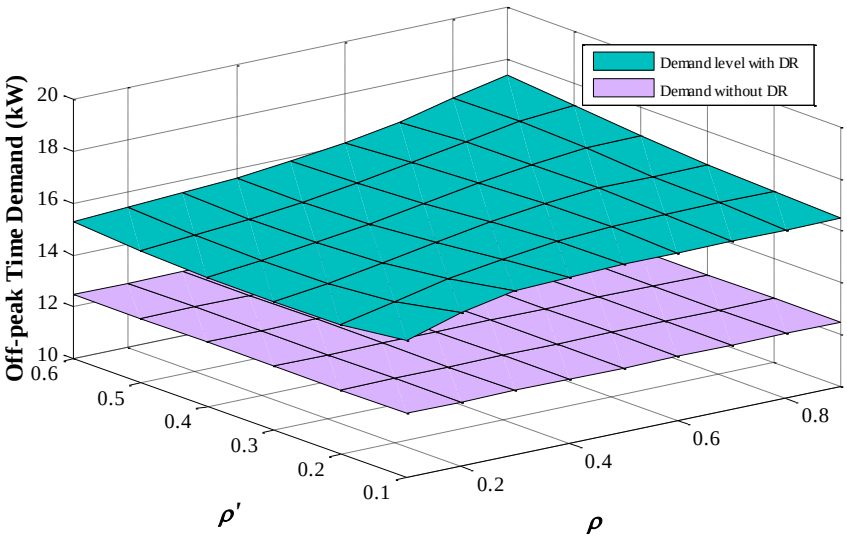


Fig.5. *Consumption levels for different values of substitution elasticity (off-peak period)*

Similarly, in Fig. 6, changes in the power consumption in shoulder-time are plotted against different ρ and ρ' values.

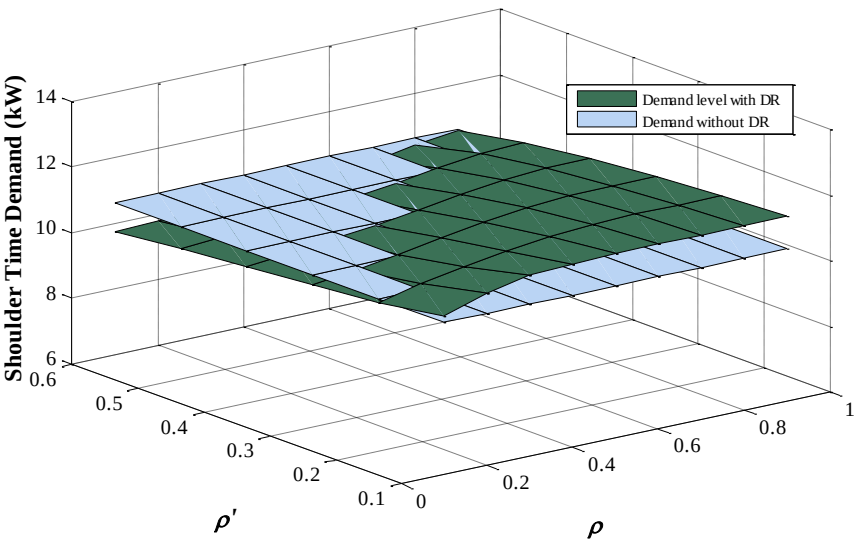


Fig.6. *Consumption levels for different values of substitution elasticity (shoulder period)*

As can be seen, the power consumed after the execution of DR program has increased in some areas and has reduced in some others; this does not imply a discontinuity in the consumer pattern, but rather the presence of two modes:

1) *Energy-saving mode*: user can choose this mode to shift some consumptions into the off-peak times due to lower energy price (the area below the intersection of the base load surface (without DR) and the load surface with DR actions),
2) *Consumption mode*: user can choose this mode to consume the energy saved during the peak time due to the lower energy price of this time compared to the one in peak-time (the area above the intersection of the base load surface and the load surface with DR actions)

5. Results and discussion

In this section, performance of the proposed model is investigated using a given load profile adopted from [33] and shown in Fig. 7. Based on the load profile, daily time horizon is divided into three time frames.

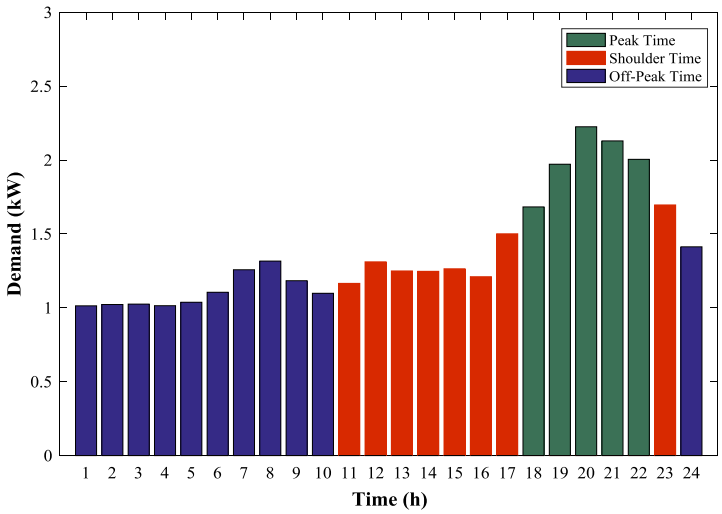


Fig.7. *the load Profile used for assessment[33]*

Electricity tariff is defined based on TOU scheme, which means that each time interval has its own unique price. The price information has been extracted from a residential sector in North Dakota electricity market in February 2016 [34]. These prices are summarized in Table 1. As can be observed form the table, the average price is 9.37 (*Cents/kWh*) in the examined residential sector. The off-peak period includes hours 1-10 and hour 24; the shoulder time includes hours 11-17 and hour 23; and the peak time includes hours 18-22.

Table 1 Price of electricity in different time periods [34]

Demand level	P_{peak}	$P_{shoulder}$	$P_{off\text{-}peak}$
Time Period	18:00-22:00	11:00-17:00 & 23:00	00:00-10:00
Price (*cents* per *kWh*)	13.8	10	6.9

Given the valid range of $0 < \rho < 1$ and $0 < \rho' < 0.7$ obtained in the sensitivity analysis, different scenarios for a residential consumer are defined and the effect of TOU-based DR program is investigated in each working scenario. To this end, the consumer is assigned with a pair of (ρ, ρ') depending on two

important features, i.e., the level of participation in the DR program and his lifestyle which influences the priority he gives to each time interval.

Table 2 shows the changes in the peak time power consumption for different values of (ρ, ρ'). The obtained results fully agree with the results of sensitivity analysis. As can be seen, for a given ρ', as ρ increases, the peak time consumption decreases which reflects the consumer's desire to participate in DR programs. On the other hand, for a given ρ, as ρ' increases, so does the peak time consumption.

Table 2 Changes in the power consumption during the peak time

Program	ρ'	D_{peak} (kWh)		
		ρ= 0.2	ρ= 0.5	ρ= 0.8
Base Case-Without DR	-	10.0140		
Proposed DR model	0.2	6.98064	5.3063	5.0666
	0.4	7.70692	6.3444	5.1073
	0.6	8.03462	7.3831	5.7726

Table 3 shows the changes in shoulder-time power consumption for different values of (ρ, ρ'). Similar to what has been shown in the sensitivity analysis, for a given ρ', increase in ρ leads to an increase in the shoulder time consumption, which reflects the consumer's desire to move towards the consumption mode. On the other hand, for a given ρ, increase in ρ' leads to opposite process, which reveals the consumer's desire for energy saving.

Table 3 Changes in the power consumption during the shoulder time

Program	ρ'	$D_{shoulder}$ (kWh)		
		ρ= 0.2	ρ= 0.5	ρ= 0.8
Base Case-Without DR	-	10.6436		
Proposed DR model	0.2	10.8045	11.4294	11.5395
	0.4	10.2796	10.7535	11.197
	0.6	9.7838	9.9656	10.4111

Changes in the off-peak time consumption for different values of (ρ, ρ') are also given in Table 4. As it was previously observed in the sensitivity analysis, for a given ρ, the effect ρ' on the user's consumption level is very limited and has no clear trend. However, for a given ρ', the off-peak time consumption is increased as ρ increases.

Table 4 Changes in the power consumption during the off-peak time

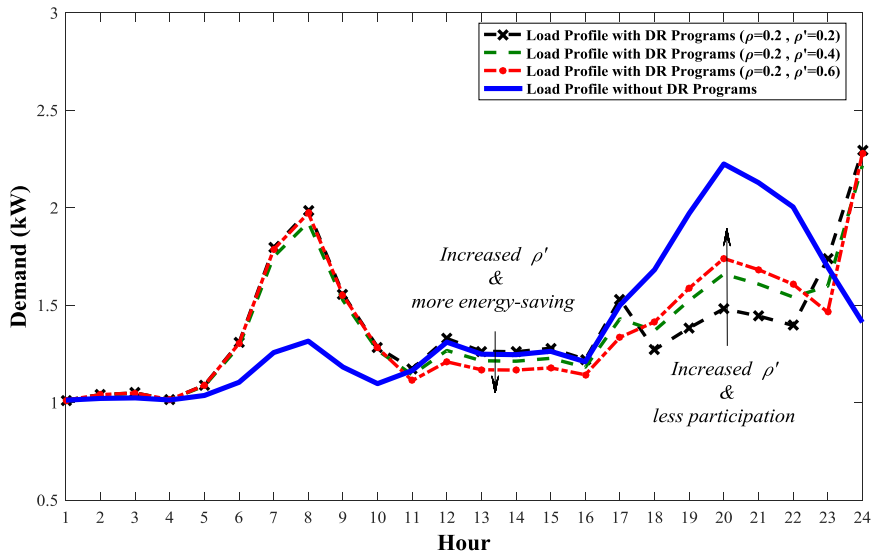
Program	ρ'	$D_{off\text{-}peak}$ (kWh)		
		ρ= 0.2	ρ= 0.5	ρ= 0.8
Base Case-Without DR	-	12.4847		
Proposed DR model	0.2	15.4367	16.4504	16.6055
	0.4	15.1797	16.0588	16.8826
	0.6	15.3877	15.8475	16.9738

Table 5 shows the budget changes and the total power consumption for different values of (ρ, ρ'). As mentioned, the total daily power consumption (D_{all}) in presence and absence of DR program is the same, as consumption is only shifted among different time periods. In the case of budget, as Table (5) shows, a change in (ρ, ρ') also changes the budge.

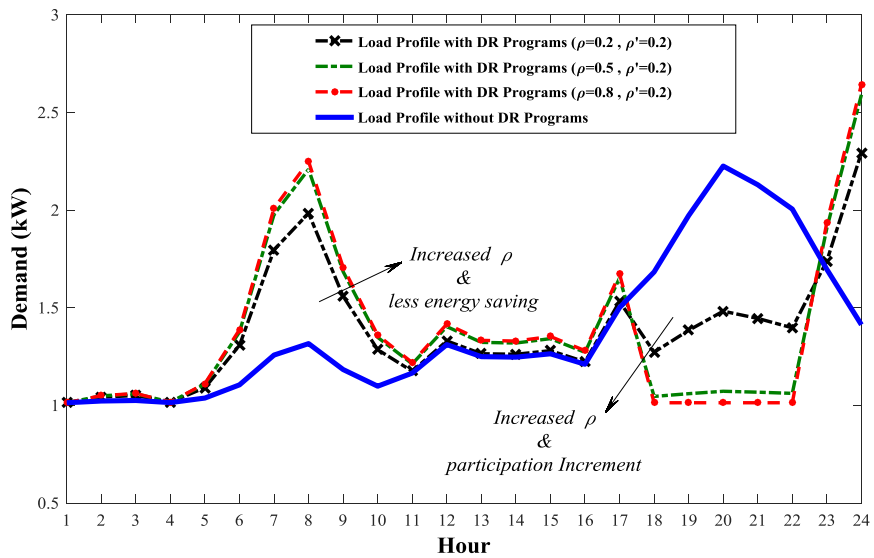
Table 5 Changes in the Budget ($)

Program	ρ'	D_{all} (*kwh*)			Budget ($)		
		ρ= 0.2	ρ= 0.5	ρ= 0.8	ρ= 0.2	ρ= 0.5	ρ= 0.8
Base Case-Without DR	-	33.14 kwh			3.30 $		
Proposed DR model	ρ'=0.2	33.22184	33.1861	33.2116	3.10	3.00	2.98
	ρ'=0.4	33.16622	33.1567	33.1869	3.13	3.06	2.99
	ρ'=0.6	33.20612	33.1962	33.1575	3.14	3.10	3.00

In this section, the consumption profiles for some of the case scenarios presented in Tables (2) -(4) are plotted. As shown in Fig. 8-(a), the shoulder-time and peak-time consumption levels can be adjusted simply by changing the values of ρ'. Also, as can be seen in Fig. 8-(b), the peak-time and off-peak time consumption levels can be adjusted by changing the values of ρ. This means that the proposed model can merge the desire of consumers (according to the lifestyle and tendency to shift the load over the time) into the DR model to adjust the consumption over the time in a cost-effective way.



(a)



(b)

Fig.8. *Residential load profile with and without DR programs: a) ρ = 0.2 and variable ρ', b) ρ' = 0.2 and variable ρ*

To show how the proposed DR model could affect the aggregated load profile in a larger area, we consider a case study where a target group of five residential consumers with distinct behaviours are participating in DR programs. As shown in Table (6), it is assumed that consumers can react differently to the price signals as each of the residential consumers has a unique set of preferences (ρ, ρ'). However, the values of such preference parameters are not something fixed or specific for each type of consumer (such as LFB, SFB and HFB) and could be different from case to case. Simulation results regarding the mentioned case study considering a given set of preferences for different end-users is shown in Fig. 9.

Table 6 Target group of five residential consumers with distinct behaviours

Consumer #	Consumer's behaviour	Preferences (ρ, ρ')
1	Low-Flexible Behaviour (LFB)	(0.04,0.2)
2	Low-Flexible Behaviour (LFB)	(0.08,0.2)
3	Semi-Flexible Behaviour (SFB)	(0.12,0.2)
4	Semi-Flexible Behaviour (SFB)	(0.16,0.2)
5	Highly Flexible Behaviour (HFB)	(0.20,0.2)

It is observed from the simulation results that consumers' participation in DR actions not only ends in peak shaving during the evening times, but also helps to valley filling during early hours in the day. Moreover, there is no rebound peak effect in the morning as part of the peak-load is shifted to shoulder time.

It is worth mentioning that the growth of residential electricity demand together with the emersion of smart grids have presented new horizons for application of energy management systems in residential sector (REMS). REMS could effectively enable DR programs for improvement of end-user's consumption profile through demand shifting and/or load curtailment [35]. In this regard, all equations and algorithms that were presented previously can be coded into a REMS unit in order to draw the best performance out of the examined system under different conditions, level of participation in DR programs and consumer's desires (ρ, ρ'), energy prices, and time intervals ranging from hours to months.

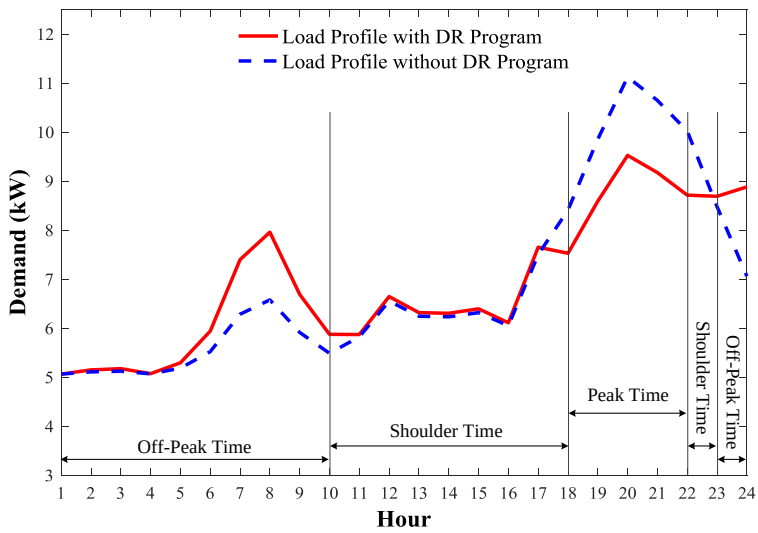


Fig.9. *Aggregated residential load profile with and without DR programs*

The level of participation in DR programs and residential consumer's desires can be set by the residential consumer in REMS at any time. However, other data like energy prices are read-only information that cannot be modified by user. Taking all these information into account, REMS can automatically schedule the consumption. On the other hand, retailer companies (or DR service providers) seek to maximize their own profits in a given electricity market by adopting proper bidding strategies, and at the same time, to reduce their risks by encouraging consumers to actively participate in DR programs (i.e., bidding strategy equipped with a DR model). Moreover, TOU-based DR programs allow consumers to voluntarily regulate their consumption based on electricity prices. Thus, when adopting optimal bidding strategies, retailer companies must consider fine-tuning control parameters of DR programs according to the parameters set by the consumer not their own benefits. Otherwise, they might fail to incentivise users for participation in DR actions and accordingly suffer from penalties imposed by the difference between the energy purchase and the demand (which must be compensated in the secondary market rather than the day-ahead market) [36]. Therefore, retailer companies must obtain these parameters correctly to avoid profit loss. Determination of these parameters could be done as follows:

- Consumers are equipped with REMS: The values of control parameters are determined by consumers and the retailer companies can use these parameters to adopt proper bidding strategies,
- Consumers are not equipped with REMS: In the absence of REMS, there is no justification to determine parameters values by consumers. Therefore, retailer companies must extract the values from consumer's reaction to price-based DR programs in order to adopt proper bidding strategies. In this relation, retailer companies can use the historical data related to a few similar days prior to the current date to calculate these parameters.

6. Conclusion

In this paper, the consumer theory and the CES utility function were employed to develop a new model for TOU-based DR program. Unlike the price-elasticity based DR models, the proposed model allowed a continuous decision-making process over the time that leading to increased flexibility. This flexibility could also address the features of a good demand response program (i.e., adaptability and adjustability) and fit the needs of any type of residential consumer by considering different levels of participation in DR actions and lifestyles. The proposed model also enabled two key functionalities of an effective DR program: *adaptability* to different residential consumers with different dispositions toward the DR program, and *adjustability* to time preferences of residential consumers meaning. The proposed model also demonstrated the ability of merging the residential consumers' desires (according to their habits and lifestyle) into DR actions for better adjustment of consumption over the time.

Future extensions of this study will be mainly focused on developing the economic model of DR programs based on alternative methods such as real-time pricing and critical peak-pricing for residential consumers. More analysis will also be conducted to assess the applicability and effectiveness of the model in different working environments with uncertain parameters and partial observable information.

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