Analytical Performance Verification of FCS-MPC 
Applied to Power Electronic Converters: 
A Model Checking Approach 

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Abstract—Since the introduction of finite control set model 
predictive control (FCS-MPC) in power electronics the algorithm 
has been missing an important aspect that would speed up 
its implementation in industry: a simple method to verify the 
algorithm performance. This paper proposes to use a statistical 
model checking (SMC) method for performance evaluation of 
the algorithm applied to power electronics converters. SMC is 
simple to implement, intuitive and it requires only an operational 
model of the system that can be simulated and checked against 
properties. Device under test for control algorithm application in 
this paper is a standard 2-level voltage source converter (VSC) 
with LC output filter used for uninterruptible power supply (UPS) systems. The performance of control algorithm is verified 
using the UPPAAL SMC toolbox and the behavior is compared to 
simulation results obtained from equivalent MATLAB/Simulink 
model and measurements from experimental set-up. Performance 
results are presented in terms of probabilities with corresponding 
uncertainties for calculated difference between the reference 
capacitor voltage value and the measured output voltage, and a 
simple moving average value. Algorithm’s performance is tested 
with parameter uncertainties introduced in the model as well. 

I. INTRODUCTION 

Rapid development of the digital control systems and 
exponential rise of their computing power has led to emergence 
of new control algorithm designs such as model predictive 
control (MPC) in various sectors of the industry [1]. Com-
pared to the traditional control algorithms based on linear or 
hysteresis controllers, the MPC concept has an intuitive design 
methodology and it suits the non-linear systems. Constraints of 
the system and multiple objectives can simply be implemented 
but with the cost of higher calculation time compared to the 
linear control. Nevertheless the computing power of today’s 
commercially available DSP’s is sufficient for execution of 
large number of calculations. Therefore, main obstacle pre-
venting a wide spread implementation of MPC algorithms to 
power converters is the non-existence of tools for the stability 
and performance evaluation, which is mandatory before any 
application in industry [1]–[4]. Stability and performance evaluation have to prove that below all transients the total harmonic distortion (THD) factor of the converter current and 
voltage will stay under the maximum allowed values defined 
in application standards. Transient system performance should 
also be evaluated to ensure the stability. Well established 
methods for linear system stability and performance evaluation 
can not be used and a closed loop stability analysis is typically 
very complex. Ref. [5] presents an attempt to solve this issue, 
yet the method remains quite complicated as it is based on 
nonlinear control theory. The developed method should be 
simple and intuitive like the control design itself so that 
new MPC control algorithms could easily and quickly be 
verified for various applications. SMC approach could offer 
the needed simplicity and at the same time be a powerful 
tool to evaluate the behavior of the system. This approach 
has been successfully used in various industrial systems e.g. 
sensor networks and communication systems, aeronautic and 
avtomotive embedded systems to solve problems that are 
beyond the scope of classical formal techniques [6]–[8], and 
this method is applied in this paper as well. 

UPPAAL SMC toolbox for model checking is appropriate 
for systems that can be modeled as a collection of non-
deterministic processes with finite control structure and real-
valued clocks that can evolve with rates specified by differ-
ential equations [9]. It can validate the performance of a de-
terministic or stochastic controller in a stochastic environment 
and run several consecutive simulations very fast in order to 
estimate the probability of the specified event with confidence 
levels and its probability distribution. The toolbox allows 
the user to visualize the results in the form of probability 
distributions, evolution of the number of runs with timed 
bounds and computation of expected values. Some successful 
implementations of SMC with UPPAAL toolbox include ap-
lications like schedulability analysis and quantitative aspects 
of scheduling systems as well as performance evaluation of 
controller strategies [6]–[10]. These facts make the UPPAAL 
SMC toolbox a very promising candidate for another field 
of implementation - the analytical performance verification 
of FCS-MPC algorithms in power electronics. Using SMC, 
the probability of system state variables staying within certain 
boundaries during transitional states could easily and quickly 
be assessed.
The paper is structured as follows: in first section the state-space equations of the system model are presented along with the control algorithm to be tested. The cost function is defined and the principle for weighting factor selection is explained. Section III gives an insight into the SMC approach and why it can be used to perform a performance validation of a FCS-MPC algorithm. In Section IV the FCS-MPC algorithm is first tested in MATLAB/Simulink simulation and then on an equivalent experimental set-up. Finally, the algorithm performance is verified in UPPAAL SMC for transient load changes using UPPAAL SMC’s Verifier toolbox. The results are presented graphically and summed up in a table of tested query results. In the last section, conclusions are drawn from obtained results.

II. SYSTEM MODEL

The power converter selected for demonstration of the SMC approach is a two-level VSC with LC output filter used for UPS application which is connected to a linear load [11] (see Fig. 1 and Fig. 2). Converter control is based on the following finite control set MPC (FSC-MPC) algorithm:

1) Measurement of the converter output current i.e. inductor current (i_l) and capacitor voltage (v_c) and load current (i_o)
2) Voltage prediction for the next sampling instant for all possible switching states
3) Cost function evaluation for each prediction
4) Selection of the switching vector S_{abc} defining the converter output vector (v_o) that minimizes the cost function
5) Application of the selected switching state

\begin{align*}
S_{abc} & = \begin{cases} 1, & \text{if } S_{1x} \text{ is on and } S_{2x} \text{ is off} \\ 0, & \text{if } S_{1x} \text{ is off and } S_{2x} \text{ is on}, x \in \{a, b, c\} \end{cases} \\
\end{align*}

Dynamics of the system can be expressed by two differential equations of the converter LC filter in stationary αβ frame, one for the converter current i.e. filter current i_{fαβ} and the second one for the converter output voltage v_{iαβ}:

\begin{align*}
\frac{d}{dt} i_{fαβ} &= C \frac{dv_{cαβ}}{dt} + i_{oαβ} \\
\frac{d}{dt} v_{iαβ} &= L \frac{di_{fαβ}}{dt} + v_{cαβ} 
\end{align*}

The primary control objective of the algorithm is voltage control with minimization of switching frequency as secondary objective to limit commutation number of power switches. Therefore the cost function of the presented system is defined as:

\begin{equation}
g = (v_{iα}^+ - v_{iα}^-)^2 + (v_{iβ}^+ - v_{iβ}^-)^2 + \lambda_{sw} \cdot n^2
\end{equation}

where \(v_{iα}^+\) and \(v_{iβ}^+\) represent the real and imaginary parts of reference voltage vector \(v_{iα}^*(k) = v_{iα}^+(k) + jv_{iβ}^+(k)\) and \(v_{iα}^-\) the real and imaginary parts of predicted voltage vector \(v_{iα}^-(k + 1) = v_{iα}^-(k) + jv_{iβ}^-(k)\). \(\lambda_{sw}\) is the weighting factor of secondary objective and \(n\) is the number of switches that change state when the switching state \(S_x(k), x \in \{a, b, c\}\) is applied and it is calculated using the following expression:

\begin{equation}
n = |S_a(k) - S_a(k - 1)| + |S_b(k) - S_b(k - 1)| \\
+ |S_c(k) - S_c(k - 1)|
\end{equation}

As there are no analytical or numerical methods to determine the weighting factors, two performance values are defined: THD factor of the capacitor voltage and average switching frequency \(f_{sw\text{-avg}}\) calculated as a sum of the switching frequencies in each phase divided by the number of switches during the simulation time interval:

\begin{equation}
f_{sw\text{-avg}} = \frac{1}{n} \sum_{i=1}^{n} f_{sw_{iα}} + f_{sw_{iβ}} + f_{sw_{iγ}}
\end{equation}
where \( i \in 1, 2, n \) represents the number of switches in each phase leg. The permitted THD factor for UPS is defined by the IEC 62040-3 standard, hence the selected weighting factor for switching frequency in the cost function should not produce a larger THD than defined in the standard [12]. Simulations in MATLAB/Simulink were performed for different values of \( \lambda_{sw} \) to calculate the performance values and the results are presented in Table I. The weighting factor \( \lambda_{sw} = 0.3 \) was chosen to be used for further performance evaluation as it is giving a good trade-off between the output voltage harmonic distortion and the average switching frequency. For \( \lambda_{sw} = 0.3 \) the average switching frequency was 5 kHz lower than for \( \lambda_{sw} = 0 \) but with a cost of 1% higher THD factor of the output voltage. In discrete system model, predicted voltage vector \( v^F(k+1) \) is calculated using the measurements of converter output current, capacitor voltage and load current. To obtain discrete-time model of the system, forward Euler method was used. Simulations were performed in MATLAB/Simulink with the following system parameters: \( V_{DC} = 300 \) V, \( L = 2.4 \) mH, \( C = 25 \) \( \mu \)F, \( T_s = 25 \) \( \mu \)s, \( R_{load} = 60 \) \( \Omega \), \( \lambda_{sw} = 0.3 \) and \( V_c_{ref} = 100 \) V. This also corresponds to experimental set-up values. Weighting factor value was selected to keep the THD factor of the UPS output voltage in accordance with IEC 62040-3. Control algorithm in experimental set-up also includes a delay compensation as presented in [13].

### III. MODEL CHECKING USING UPPAAL SMC

SMC can be defined as a series of techniques that monitor several runs of the system with respect to some property and afterwards use the results from the statistics theory to get an overall estimate of the design correctness [9], [14]. For engineers the technique is easy to understand, to implement and to use, particularly for industrial applications. The probability distributions that drive the timed behaviors of the system can be specified through user interface and the estimated probability can be compared to a value or to another probability. In other words the system \( S \) will be simulated for a finite number of executions, resulting in a number of samples which will be used to test the specified hypothesis e.g. does the property \( \phi = \Delta V \) (system voltage deviation) stay under threshold \( \theta = 0.5 \% \) during the transient load change? The result of this test will be the probability of satisfaction or violation of the specified property \( \phi \) [15]. The toolbox support-engine used for the hypothesis testing relies on the results coming from the statistics like sequential hypothesis testing or Monte Carlo simulation. Let us illustrate how one of the algorithms e.g the Monte Carlo algorithm will estimate the probability \( \gamma \) of satisfying property \( \phi \) [16]. First the user will specify the number of random simulations \( \rho_1, \rho_2...\rho_{N-1}, \rho_N \), and then estimation \( \gamma \) will be estimated as:

\[
\tilde{\gamma} = \frac{1}{N} \sum_{i=1}^{N} 1(\rho_i \models \phi)
\]

where \( 1 \) is an indicator function that will return 1 if the property \( \phi \) is satisfied for simulation run \( \rho_i \), otherwise it will return 0. The simulation will stop either when the simulation number \( N \) or a state violating property \( \phi \) is reached. The iteration number \( N \) can be determined by Chernoff’s bound [17] to obtain a confidence interval. The bound will determine the required number of simulations to be performed so that a confidence \( \delta \) with precision \( \epsilon \) entered by the user are obtained for the estimated probability.

As mentioned in the introduction, the idea behind this paper is to use the UPPAAL SMC toolbox to create a model of the MPC controller for a 2-level VSC in order to check the algorithm performance during transient load changes and parameter uncertainties. Differential equations presented in the system model (4) specify the evolving rates of clocks i.e. system voltage and current. The controller has a finite structure, there are eight possible converter output voltage vectors and the one selected is the product of the cost minimization function (5). Build model can now be used to make performance analysis for different FCS-MPC algorithms just by changing the cost function. In order to check how the secondary objective in the cost function affects the performance of the model, two \( \lambda_{sw} \) values were used for result comparison. Model parameters were adjusted to fit the MATLAB/Simulink model.

### IV. EVALUATION AND COMPARISON

Simulation results obtained from the MATLAB/Simulink model were used as a benchmark data to check the behavior of the model designed in UPPAAL SMC. The implemented MPC algorithm in Simulink shows good voltage reference tracking performance for linear load as it can be observed in Fig.2. The algorithm was also tested on an experimental converter set-up as displayed in Fig. 4 using the MicroLabBox with DS1202 PowerPC DualCore 2 GHz processor board and DS1302 I/O board from dSpace. Set-up parameters: DC supply voltage \( V_{DC} = 300 \) V, output filter values \( L = 2.4 \) mH, \( R_L = 50 \) m\( \Omega \) and \( C = 25 \) \( \mu \)F, control algorithm time sample \( T_s = 25 \) \( \mu \)s, linear load \( R_{load} = 60 \) \( \Omega \) and capacitor voltage reference \( V_{c_{ref}} = 100 \) V. Calculation time of the control algorithm is approximately 15 \( \mu \)s, and to compensate this delay the predictions are calculated one step further ahead and applied at the beginning of the next time sampling interval. Fig. 5 and Fig. 6 show the measured capacitor line to line voltages for \( \lambda_{sw} = 0 \) and \( \lambda_{sw} = 0.3 \). It can be observed how higher weighting factor for secondary objective resulted with increased capacitor voltage ripple.
Using the UPPAAL SMC verifier system, currents and voltages can also be presented graphically as shown in Fig. 7. By comparing the simulated capacitor voltage to the controller reference, the tracking ability of the algorithm was again proved to be satisfying, meaning the modeling in UPPAAL was correct, hence the model can be used for analytical performance validation during the transient load. Algorithm performance was assessed through two queries for the time interval of 40 ms:

\[ \text{diff} = (v_{c\alpha}^* - v_{c\alpha}^m)^2 + (v_{c\beta}^* - v_{c\beta}^m)^2 \]  

(9)

where \( v_{c\alpha}^* \) and \( v_{c\beta}^* \) represent the real and imaginary parts of reference voltage vector \( v^*(k) \), \( v_{c\alpha}^m \) and \( v_{c\beta}^m \) are the values of measured capacitor voltage \( v_{c}^m(k) = v_{c\alpha}^m + j v_{c\beta}^m \) and the second query was the simple moving average value (SMA) of the difference calculated on \( n = 10 \) sample subsets.

\[ \text{SMA} = \text{SMA}_{\text{prev}} + \frac{\text{diff}}{n} - \frac{\text{diff}_{M-n}}{n} \]  

(10)

As it can be seen from the queries focus of the performance evaluation is on reference tracking ability, i.e. the difference between reference and output values is calculated. The load is changing stochastically during the time interval in UPPAAL simulations so that through several runs all possible transients can be evaluated in the queries. An example of the load current waveform during one simulation run is shown in Fig. 8. Simulation for every query is finished when the number of simulations needed for calculated probability confidence level \( \delta = 0.95 \) is achieved or a state violating the property is reached. Some of the tested probabilities with corresponding uncertainty of \( \epsilon = 0.05 \) for transient load with parameter uncertainty are shown in Table II. This parameter example uncertainty is introduced into the model as a false estimation of the system inductance.

The queries were tested for \( \lambda_{sw} = 0 \) with the capacitor voltage as primary objective and \( \lambda_{sw} = 0.3 \) with switching...
TABLE II
QUERY PROBABILITY RESULTS OBTAINED FROM UPPAAL SMC TOOLBOX VERIFIER

<table>
<thead>
<tr>
<th>Query</th>
<th>Parameter uncertainty</th>
<th>Probability ($\lambda_{sw} = 0$)</th>
<th>Probability ($\lambda_{sw} = 0.3$)</th>
<th>Number of runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$diff &lt; 8%$</td>
<td>0</td>
<td>$0.699 - 0.799$</td>
<td>$0 - 0.097$</td>
<td>326/36</td>
</tr>
<tr>
<td>$diff &lt; 10%$</td>
<td>0</td>
<td>$0.902 - 1$</td>
<td>$0.107 - 0.206$</td>
<td>362/17</td>
</tr>
<tr>
<td>$diff &lt; 12%$</td>
<td>0</td>
<td>$0.902 - 1$</td>
<td>$0.620 - 0.720$</td>
<td>363/37</td>
</tr>
<tr>
<td>$diff &lt; 15%$</td>
<td>0</td>
<td>$0.902 - 1$</td>
<td>$0.902 - 1$</td>
<td>36/36</td>
</tr>
<tr>
<td>$diff &lt; 12%$</td>
<td>30%</td>
<td>$0.596 - 0.696$</td>
<td>$0 - 0.097$</td>
<td>369/36</td>
</tr>
<tr>
<td>$diff &lt; 15%$</td>
<td>30%</td>
<td>$0.902 - 1$</td>
<td>$0.667 - 0.767$</td>
<td>36/328</td>
</tr>
<tr>
<td>$SMA &lt; 8%$</td>
<td>0</td>
<td>$0.774 - 0.874$</td>
<td>$0 - 0.097$</td>
<td>238/36</td>
</tr>
<tr>
<td>$SMA &lt; 10%$</td>
<td>0</td>
<td>$0.902 - 1$</td>
<td>$0 - 0.097$</td>
<td>36/36</td>
</tr>
<tr>
<td>$SMA &lt; 12%$</td>
<td>0</td>
<td>$0.902 - 1$</td>
<td>$0.632 - 0.732$</td>
<td>36/351</td>
</tr>
<tr>
<td>$SMA &lt; 10%$</td>
<td>30%</td>
<td>$0.222 - 0.322$</td>
<td>$0 - 0.097$</td>
<td>322/36</td>
</tr>
<tr>
<td>$SMA &lt; 12%$</td>
<td>30%</td>
<td>$0.654 - 0.754$</td>
<td>$0.013 - 0.11$</td>
<td>337/88</td>
</tr>
<tr>
<td>$SMA &lt; 15%$</td>
<td>30%</td>
<td>$0.902 - 1$</td>
<td>$0.641 - 0.741$</td>
<td>36/345</td>
</tr>
</tbody>
</table>

Fig. 8. Load current under transient load change in the UPPAAL model of the VSC

Fig. 9. Difference between the reference and capacitor voltage in UPPAAL model during one simulation run under transient load change

Fig. 10. SMA of the reference and capacitor voltage in UPPAAL model during one simulation run under transient load change

Frequency minimization applied as the secondary objective. The last column in the Table II represents the number of runs that the UPPAAL SMC toolbox performed during query processing. If the query probability is close to 0 or 1, the number of runs necessary for assessment is low. The second query $diff < 10\%$ for $\lambda_{sw} = 0$ will be presented in detail. Number of simulations needed to provide the level of uncertainty $\epsilon = 0.05$ was 36, during each simulation run the load value changed stochastically which results in current like shown in Fig. 8, difference and SMA values were calculated for each run. An example of the calculated values for one run is presented in Fig. 9 and Fig. 10. The first transient ($t = 0 - 4$ ms) which occurs because of the system’s initial states were set to 0 was omitted from the calculation. It can be observed that the difference is staying below 10% threshold, meaning that the system voltage is always tracking the reference even under transient load change and there are no significant degenerations in the reference tracking.

As presented in the Table II the system with $\lambda_{sw} = 0.3$ showed higher probability to produce larger difference value during the transients. The lowest difference guaranteed to have a probability close to 1 was 10% for first function, which means that during the load transients at no time the difference
between the reference (100 V) and system voltage will be larger than 10 V. Both $\lambda_{sw}$ values produced a probability close to 1 for differences < 15%. With the introduction of the parameter uncertainty it can be seen that the the probability of exceeding the difference threshold for system with $\lambda_{sw} = 0.3$ was 30% lower. It can be deduced that the parameter uncertainty has a larger impact on this type of cost function. The obtained results are also useful for weighting factor selection as the performance degradation/improvement of each secondary term in the cost function can easily be evaluated. If the calculated SMA value of the system needs for instance to stay during transients below 12% a lower $\lambda_{sw}$ value should be selected for (5).

V. CONCLUSION

In this paper the performance evaluation of the FSC-MPC algorithm for 2-level VSC was done using UPPAAL SMC toolbox. A conversion from the Simulink system model to UPPAAL model was simple and the process of performance evaluation was uncomplicated and fast. Created model in the toolbox proved that the reference tracking error of the designed control algorithm will stay below 15% with the corresponding probability certainty of 0.95 even with introduction of the parameter uncertainty under transient load. Cost function with weighting factor $\lambda_{sw} = 0.3$ presented two times higher probability that the difference will be higher than 12% because the applied switching state was chosen not only to minimize reference error but also to choose the state with lower number of commutations. Overall, during the performance evaluation no significant degradation of reference tracking was found during transients nor under model parameter uncertainty. The presented MPC performance validation approach showed many possibilities for future work with focus on analytical performance evaluation of other MPC algorithms and converter topologies plus more complex load models. Further research will show if the SMC is the new powerful tool to solve the problem of performance and stability evaluation for MPC algorithms in different types of power converters.

REFERENCES