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# FINITE-LIFE FATIGUE CONSTRAINTS IN 2D TOPOLOGY OPTIMIZATION OF CONTINUA

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**Key words:** Topology Optimization, Fatigue Constraints, Adjoint Method.

**Summary.** Topology optimization of 2D continua with the objective of minimizing the mass while considering finite-life fatigue constraint is considered. The structure is subjected to proportional variable-amplitude loading. The topology optimization problem is solved using the density approach. The fractions of fatigue damage are estimated using the stress-based Sines fatigue criterion and  $S - N$  curves, while the accumulated damage is estimated using Palmgren-Miner's rule. The method is a natural extension of classical density-based topology optimization with static stress constraints, and thus utilizes many of the same methods. A benchmark example is presented.

## 1 INTRODUCTION

Since the seminal work by Bendsøe and Kikuchi<sup>1</sup>, the topology method has been applied to the optimal material distribution problem in a variety of fields. Most work has been done on minimizing compliance subject to an overall volume constraint, but the method has also been extended to e.g. stress-constrained optimization, fluid-structure interaction problems, and many complicated multi-physics problems. Limited research on fatigue-constrained topology optimization has been published. However, fatigue failure is one of the most common failure modes of structures subjected to repeated loading. A few works on fatigue-constrained topology optimization have been published, where most work either design for infinite life, e.g. Collet et al.<sup>2</sup>, or reformulate the fatigue problem into a static stress-constrained problem, see e.g. Holmberg, Torstenfeldt, and Klarbring<sup>3</sup>.

In the recently published work by Oest and Lund<sup>4</sup> a method is proposed where the entire fatigue analysis is included directly in the optimization problem, including the entire load-history. By utilizing an effective adjoint formulation of the design sensitivities, the computational cost of the finite-life fatigue-constrained problem is comparable to static stress-constrained topology optimization. The method is currently limited to linear quasi-static finite element analysis, linear elastic material behavior, and proportional loading. The method can sometimes experience gray-scale issues, which in the current work is addressed using the heaviside density filter.

## 2 PHYSICS

A reference load vector  $\hat{\mathbf{P}}$  is applied to a structure, and a reference displacement  $\hat{\mathbf{u}}$  is obtained by solving the static equilibrium state equation:

$$\mathbf{K}(\tilde{\mathbf{x}}(\mathbf{x}))\hat{\mathbf{u}} = \hat{\mathbf{P}} \quad (1)$$

$\mathbf{K}$  is the interpolated global stiffness matrix,  $\tilde{\mathbf{x}}$  is the vector containing the filtered (physical) variables, and  $\mathbf{x}$  is the design variables. The response for any other magnitude of the reference load can then be determined efficiently by linear superposition of the reference displacement vector. The global stiffness matrix is interpolated using the well-known modified SIMP with a penalization factor  $p = 3$ . Consequently, the effective Young's modulus  $E_e$  in each element  $e$  is given by:

$$E_e(\tilde{x}_e(\mathbf{x})) = E_{min} + \tilde{x}_e(\mathbf{x})^p(E_0 - E_{min}), \quad \mathbf{x} \in [0; 1] \quad (2)$$

Here  $E_{min} \ll E_0$  is a lower bound on the effective modulus representing the stiffness of a void region. The element reference stresses  $\hat{\sigma}_e$  are obtained using the reference displacement and relaxed using the  $qp$ -stress relaxation method<sup>5,6</sup>:

$$\hat{\sigma}_e = \tilde{x}_e(\mathbf{x})^q \mathbf{E} \mathbf{B} \hat{\mathbf{u}}_e \quad (3)$$

Here  $0 \leq q < 1$  is the stress interpolation exponent,  $\mathbf{E}$  is the constitutive matrix,  $\mathbf{B}$  is the strain-displacement matrix, and  $\mathbf{u}_e$  is the vector of element displacements.

To estimate each fraction of damage caused by each load cycle, the stress-based Sines criterion is applied. The Sines criterion includes all stress components and accounts for amplitude and mean stresses, which are determined using traditional rainflow-counting. A beneficial property of the Sines criterion is that it reduces the stress state to an equivalent uniaxial stress,  $\tilde{\sigma}_{e_i}$ , for each stress cycle  $i$ . This allows for the use of  $S - N$  curves and Palmgren-Miner's rule. In the present work, a linear  $S - N$  curve has been applied. Expressed in stress reversals, the  $S - N$  relation is:

$$\tilde{\sigma}_{e_i} = \sigma'_f (2N_{e_i})^b \quad (4)$$

$\sigma'_f$  and  $b$  are material parameters, and  $N_{e_i}$  is the expected amount of cycles to failure in each element  $e$  for stress cycle  $i$ . The fractions of damage are accumulated using Palmgren-Miner's rule by:

$$D_e = c_D \sum_{i=1}^{n_i} \frac{n_i}{N_{e_i}} \leq \eta \quad (5)$$

$c_D \geq 1$  is a scaling factor to make the data representative of a lifetime,  $n_i$  is the amount of cycles for a stress cycle, and  $\eta$  is the allowable damage. The above equation constitutes a fatigue constraint in every element. To reduce the computational costs, the  $P$ -norm method is applied to reduce the large amount of constraints to a single global constraint.

Due to the cumulative nature of Palmgren-Miner’s rule, it is possible to solve just one adjoint equation per load case<sup>4</sup>. Thus, for large load series, the computational costs are only increased slightly, as the amount of adjoint equations is independent of the size of the load cases.

In the original work, it was shown that grayscale issues may occur. To address this the regularization scheme has in this work been changed from the consistent density filter to the heaviside density filter<sup>7</sup>. The heaviside filter is given by:

$$\tilde{x}_e = 1 - e^{-\beta \bar{x}_e} + \bar{x}_e e^{-\beta} \quad (6)$$

$\bar{x}_e$  is the density as obtained using the classical density filter, and  $\beta$  is a filter parameter. For a  $\beta$  value of zero, the physical variables  $\tilde{\mathbf{x}}$  are similar to those obtained using the classical density filter. When  $\beta$  goes to infinity the design variables are all 0 – 1. Note that the implementation is using the standard element density, and not the mapped nodal design variables which reportedly improves the problem numerically.

### 3 EXAMPLES

An optimized cantilever beam with two holes added to introduce stress concentrations is shown in Figure 1. The asymmetric design is a direct consequence of the mean stress contributions caused by the loading condition. The example is obtained using careful optimizer settings, a very slowly increasing  $\beta$  parameter, and with a  $P$ -norm factor of  $P = 8$ . For comparison, the same problem is solved using the original formulation, i.e. with a density filter and a  $P$ -norm factor of  $P = 12$ . However, as compared with the original publication, more iterations are allowed in this framework in both examples. The

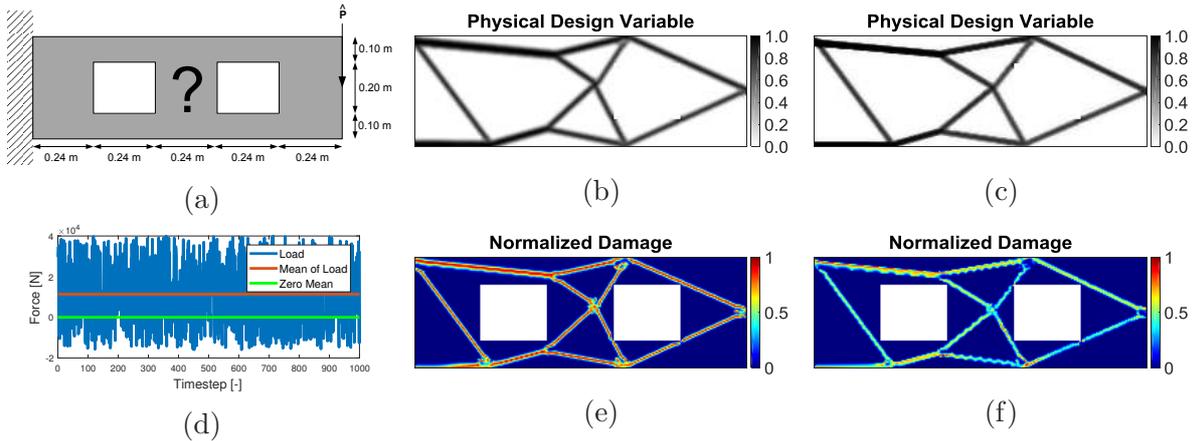


Figure 1: Minimization of mass problem with fatigue constraints demonstrated on a cantilever beam, sketched in (a). The time-varying load is shown in (d). The original formulation from Oest and Lund<sup>4</sup> is shown in (b) and (e), and the results obtained using Heaviside filtering is shown in (c) and (f).

design obtained using the heaviside is with  $\beta = 20$ , presents a slightly more 0 – 1 design. However, the formulation adds an additional tuning parameter and is more difficult to solve. Thus, a lower  $P$ -norm factor had to be applied, which resulted in a less well-distributed damage as compared with the original formulation.

### 3.1 CONCLUSION

A minimization of mass topology optimization problem constrained by finite-life fatigue is demonstrated using the heaviside filter. However, the added non-linearity makes the already very non-linear problem even more difficult to solve. Thus, the example shown here is solved with a  $P$ -norm parameter lower than in the original work. It is not necessarily worth the extra effort to use the heaviside filter for this specific purpose as compared with the density filter. However, if the optimization can be made more stable by other means, the heaviside filter may prove a good regularization approach for fatigue constrained topology optimization using the density method.

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